Spatial and Temporal Coherence; Coherent Undulator Radiation

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(http://www.coe.berkeley.edu/AST/srms)
COHERENCE AT SHORT WAVELENGTHS

\[ l_{\text{coh}} = \frac{\lambda^2}{2\Delta\lambda} \text{ (temporal (longitudinal) coherence)} \] (8.3)

\[ d \cdot \theta = \frac{\lambda}{2\pi} \text{ (spatial (transverse) coherence)} \] (8.5)

or \[ d \cdot 2\theta_{\text{FWHM}} = 0.44 \lambda \] (8.5*)

\[ P_{\text{coh},N} = \frac{(\lambda/2\pi)^2}{(d_x \theta_x)(d_y \theta_y)} P_{\text{cen}} \] (8.6)

\[ P_{\text{coh,}\lambda/\Delta\lambda} = \frac{e\lambda \eta (\Delta\lambda/\lambda)^2 N^2}{8\pi\epsilon_0 d_x d_y} \left[ 1 - \frac{\hbar\omega}{\hbar\omega_0} \right] f(K) \] (8.9)

\[ P_{\text{coh}} = \frac{(\lambda/2\pi)^2}{(d_x \theta_x)(d_y \theta_y)} P_{\text{laser}} \] (8.11)
Persistence of fringes as the source grows from a point source to finite size.

\[ \lambda_{\text{coh}} = \frac{\lambda^2}{2\Delta\lambda} = \frac{1}{2} N_{\text{coh}} \lambda \]

\[ d \cdot 2\theta_{\text{FWHM}} \approx \frac{\lambda}{2} \]
Spatial and Spectral Filtering to Produce Coherent Radiation

Courtesy of A. Schawlow, Stanford.
Coherence, Partial Coherence, and Incoherence

Point source oscillator
\(-\infty < t < \infty\)

Source of finite size, divergence, and duration
Mutual coherence factor

\[ \Gamma_{12}(\tau) \equiv \langle E_1(t + \tau)E_2^*(t) \rangle \]  

(8.1)

Normalize degree of spatial coherence (complex coherence factor)

\[ \mu_{12} = \frac{\langle E_1(t)E_2^*(t) \rangle}{\sqrt{\langle |E_1|^2 \rangle \langle |E_2|^2 \rangle}} \]  

(8.12)

A high degree of coherence (\( \mu \to 1 \)) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence (\( \mu \to 0 \)) implies an absence of interference, except with great care. In general radiation is partially coherent.

Longitudinal (temporal) coherence length

\[ \ell_{\text{coh}} = \frac{\lambda^2}{2 \Delta \lambda} \]  

(8.3)

Full spatial (transverse) coherence

\[ d \cdot \theta = \frac{\lambda}{2\pi} \]  

(8.5)
Define a coherence length $\ell_{coh}$ as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength $\lambda$ propagating through $N$ cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \frac{1}{2})(\lambda + \Delta\lambda)$$

Equating the two

$$N = \frac{\lambda}{2\Delta\lambda}$$

so that

$$\ell_{coh} = \frac{\lambda^2}{2\Delta\lambda} \quad (8.3)$$
A Practical Interpretation of Spatial Coherence

- Associate spatial coherence with a spherical wavefront.
- A spherical wavefront implies a point source.
- How small is a “point source”?

From Heisenberg’s Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size “d” you can resolve, with wavelength $\lambda$ and half angle $\theta$, is

$$d \cdot \theta = \frac{\lambda}{2\pi}$$
Partially Coherent Radiation Approaches
Uncertainty Principle Limits

\[ \Delta x \cdot \Delta p \geq \frac{\hbar}{2} \quad (8.4) \]
Standard deviations of Gaussian distributed functions
(Tipler, 1978, pp. 174-189)

\[ \Delta x \cdot \hbar \Delta k \geq \frac{\hbar}{2} \]

\[ \Delta x \cdot k \Delta \theta \geq 1/2 \]

\[ 2 \Delta x \cdot \Delta \theta \geq \lambda/2\pi \]

\[ d = 2 \Delta x \]

Note:
\[ \Delta p = \hbar \Delta k \]
\[ \Delta k = k \Delta \theta \]

Spherical wavefronts occur in the limiting case
\[ \frac{d \cdot \theta}{\lambda/2\pi} \]
(spatially coherent)

\[ \left\{ \frac{1}{\sqrt{e}} \right\} \]
quantities

or
\[ (d \cdot 2\theta)_{\text{FWHM}} \approx \lambda/2 \]
FWHM quantities
Propagation of a Gaussian Beam

For a spherical wave propagating with a Gaussian intensity distribution, \( I/I_0 = \exp(-r^2/2r_0) \), where \( r_0 \) is the \( 1/\sqrt{e} \) waist radius at the origin \((z = 0)\), the intensity distribution grows with a \( 1/\sqrt{e} \) radius given by (Siegman, Lasers)

\[
r(z) = r_0 \sqrt{1 + \left( \frac{\lambda z}{4\pi r_0^2} \right)^2}
\]

In the far field, where \( z \gg 4\pi r_0^2/\lambda \), the \( 1/\sqrt{e} \) divergence half angle is

\[
\theta = \frac{r(z)}{z} = \frac{\lambda}{4\pi r_0}
\]

with waist diameter \( d = 2r_0 \), we have TEM\(_{00}\) radiation with \( d \cdot \theta = \lambda/2\pi \)
Spatially Coherent Undulator Radiation

\[ \lambda = 11.2 \text{ nm} \quad \lambda = 13.4 \text{ nm} \]

1 \( \mu \text{m} \) pinhole
25 mm wide CCD at 410 mm

Courtesy of Patrick Naulleau, LBNL.
Undulator central radiation cone \((\frac{\lambda}{\Delta \lambda} = N; \theta = \frac{1}{\gamma^{*}/N})\):

\[
\bar{P}_{\text{cen}} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2} f(K) \quad (5.41a)
\]

With spatial filtering (a pinhole and an angular aperture):

\[
\bar{P}_{\text{coh, } N} = \left(\frac{\lambda/2\pi}{d_x \theta_x}\right) \left(\frac{\lambda/2\pi}{d_y \theta_y}\right) \bar{P}_{\text{cen}} \quad (8.6)
\]

With eq.(5.28), \(\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right)\). Convert to photon energies where for \(\theta = 0\), \(\hbar \omega = \frac{\hbar \omega_0}{1 + K^2/2}\), and where \(\hbar \omega_0 \equiv 4\pi \hbar c \gamma^2 / \lambda_u\) corresponds to \(K = 0\). For small electron beam divergence, \(\sigma_{x,y}^2 \ll \theta_{\text{cen}}^2\), the spatially coherent power is

\[
\bar{P}_{\text{coh, } N} = \frac{e \lambda_u I N}{8\pi \epsilon_0 d_x d_y} \left(1 - \frac{\hbar \omega}{\hbar \omega_0}\right) f(\hbar \omega/\hbar \omega_0) \quad (8.9)
\]
Using a pinhole-aperture spatial filter, passing only radiation that satisfies $d \cdot \theta = \lambda/2\pi$

$$\tilde{P}_{\text{coh},N} = \left(\frac{\lambda/2\pi}{d_x \theta_x}\right) \left(\frac{\lambda/2\pi}{d_y \theta_y}\right) \tilde{P}_{\text{cen}}$$  \hspace{1cm} (8.6)

$$\tilde{P}_{\text{coh},N} = \frac{e \lambda_u I N}{8\pi \epsilon_0 d_x d_y} \left(1 - \frac{\hbar \omega}{\hbar \omega_0}\right) f(\hbar \omega/\hbar \omega_0)$$  \hspace{1cm} (8.9)

for $d_x = 2\sigma_x$, $d_y = 2\sigma_y$, $\theta_{Tx} \rightarrow \theta_x$, $\theta_{Ty} \rightarrow \theta_y$, and $\sigma'^2 \ll \theta_{\text{cen}}^2$. 

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In addition to the pinhole – angular aperture for spatial filtering and spatial coherence, add a monochromator for narrowed bandwidth and increased temporal coherence:

\[
\tilde{P}_{\text{coh, } \lambda/\Delta \lambda} = \eta \frac{(\lambda/2\pi)^2}{(d_x \theta_x)(d_y \theta_y)} \cdot N \frac{\Delta \lambda}{\lambda} \cdot \tilde{P}_{\text{cen}} \quad (8.10a)
\]

which for \( \sigma_{x,y}^2 << \theta_{\text{cen}}^2 \) (the undulator condition) gives the spatially and temporally coherent power \((d \cdot \theta = \lambda/2\pi)\; ; \; l_{\text{coh}} = \frac{\lambda^2}{2 \Delta \lambda}\):

\[
\tilde{P}_{\text{coh, } \lambda/\Delta \lambda} = \frac{e \lambda_u I \eta (\Delta \lambda/\lambda) N^2}{8\pi \epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar \omega}{\hbar \omega_0}\right) f\left(\hbar \omega / \hbar \omega_0\right) \quad (8.10c)
\]

which we note scales as \(N^2\).
• Pinhole filtering for full spatial coherence
• Monochromator for spectral filtering to $\lambda/\Delta\lambda > N$

Spatially and Spectrally Filtered Undulator Radiation

$$\tilde{P}_{\text{coh},\lambda/\Delta\lambda} = \eta \frac{(\lambda/2\pi)^2}{(d_x \theta_x)(d_y \theta_y)} \cdot \frac{N \lambda}{\Delta\lambda} \cdot \tilde{P}_{\text{cen}} \quad (8.10a)$$

$$\tilde{P}_{\text{coh},\lambda/\Delta\lambda} = \frac{e \lambda u I \eta (\Delta\lambda/\lambda) N^2}{8\pi \epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar \omega}{\hbar \omega_0}\right) f(\hbar \omega/\hbar \omega_0) \left(\sigma'^2 \ll \theta_{\text{cen}}^2\right) \quad (8.10c)$$
Coherent Soft X-Ray Beamline: Use of a Higher Harmonic (n = 3) to Access Shorter Wavelengths

- 8.0 cm period, N = 55
- 1.9 GeV, 400 mA
- \[ d \cdot \theta = \frac{\lambda}{2\pi} \]
- \[ \ell_{coh} = 1000 \frac{\lambda}{2} \]
- \[ \eta_{euv} = 10\%, \eta_{sXR} = 10\% \]

Coherent Power with a Monochromator

\[ \frac{\lambda}{\Delta\lambda} = 10^3 \]

\( n = 1 \)

\( n = 3 \)
Coherent Power at the ALS

U8

1.9 GeV, 400 mA

\[ \lambda_u = 80 \text{ mm}, \quad N = 55 \]

\[ 0.5 \leq K \leq 4.0 \]

\[ \sigma_x = 260 \mu\text{m}, \quad \sigma_x' = 23 \mu\text{r} \]

\[ \sigma_y = 16 \mu\text{m}, \quad \sigma_y' = 3.9 \mu\text{r} \]
1.9 GeV, 400 mA

\[ \lambda_u = 50 \text{ mm}, \ N = 89 \]

\[ 0.5 \leq K \leq 4.0 \]

\[ \sigma_x = 260 \ \mu\text{m}, \ \sigma_x' = 23 \ \mu\text{r} \]

\[ \sigma_y = 16 \ \mu\text{m}, \ \sigma_y' = 3.9 \ \mu\text{r} \]

\[ \eta = 10\% \]
Coherent Power for an EPU at the ALS

U5 EPU

1.9 GeV, 400 mA
\lambda_u = 50 \text{ mm}, N = 27
0.5 \leq K \leq 4.0
\sigma_x = 260 \mu m, \sigma_x' = 23 \mu r
\sigma_y = 16 \mu m, \sigma_y' = 3.9 \mu r
\theta_{cen} = 61 \mu r @ K = 0.87 (500 eV)
Coherent Power at MAX II

1.5 GeV, 250 mA

\[ \lambda_u = 52 \text{ mm}, \quad N = 49 \]

\[ 0.1 \leq K \leq 2.7 \]

\[ \sigma_x = 300 \mu m, \quad \sigma_x' = 26 \mu r \]

\[ \sigma_y = 45 \mu m, \quad \sigma_y' = 20 \mu r \]

\[ \eta = 10\% \]
Coherent Power at Elettra

2.0 GeV, 300 mA
\[ \lambda_u = 56 \text{ mm}, \; N = 81 \]
\[ 0.5 \leq K \leq 2.3 \]
\[ \sigma_x = 255 \mu \text{m}, \; \sigma_x' = 23 \mu \text{r} \]
\[ \sigma_y = 31 \mu \text{m}, \; \sigma_y' = 9 \mu \text{r} \]
\[ \eta = 10\% \]
Coherent Power at the Pohang Light Source

2.5 GeV, 180 mA  
\( \lambda_u = 70 \text{ mm}, N = 59 \)  
\( 0 \leq K \leq 6.5 \)  
\( \sigma_x = 433 \mu m, \sigma_x' = 43 \mu r \)  
\( \sigma_y = 27 \mu m, \sigma_y' = 6.8 \mu r \)  
\( \theta_{cen} = 33 \mu r @ K = 1 \)  
\( \eta = 10\% \)
Coherent Power at New Subaru

1.0 GeV, 100 mA
$\lambda_u = 54$ mm, $N = 200$
$0.3 \leq K \leq 2.5$
$\sigma_x = 450 \mu$m, $\sigma_x' = 89 \mu$r
$\sigma_y = 220 \mu$m, $\sigma_y' = 18 \mu$r
$\eta = 10\%$

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Photon Energy (eV)

$P_{\text{coh}, N}(\mu$W)

$P_{\text{cen}}$(W)

$\frac{\lambda}{\Delta \lambda} = 10^3$

$\frac{\lambda}{\Delta \lambda} = 200$
Coherent Power at the Canadian Light Source

CLS, 75 mm EPU

2.9 GeV, 500 mA
\( \lambda_u = 75 \text{ mm}, \ N = 21.5 \)
\( 0 \leq K \leq 5.2 \)
\( \sigma_x = 440 \ \mu\text{m}, \ \sigma_x' = 47 \ \mu\text{r} \)
\( \sigma_y = 88 \ \mu\text{m}, \ \sigma_y' = 21 \ \mu\text{r} \)
\( \theta_{\text{cen}} = 47 \ \mu\text{r} @ K = 1 \)
\( \eta = 10\% \)
Coherent Power Predicted with SPEAR 3 at SSRL

3.0 GeV, 500 mA
\( \lambda_u = 3.3 \text{ cm}, N = 105 \)
0 \( \leq K \leq 2.2 \)
\( \sigma_x = 436 \text{ \( \mu \)m}, \sigma_x' = 43 \text{ \( \mu \)rad} \)
\( \sigma_y = 30 \text{ \( \mu \)m}, \sigma_y' = 6.3 \text{ \( \mu \)rad} \)
\( \theta_{cen} = 17 \text{ \( \mu \)rad} @ K = 1 \)
\( \eta = 10\% \)
Coherent Power at the Australian Synchrotron

3.0 GeV, 200 mA
\( \lambda_u = 22 \text{ mm}, N = 80 \)
\( 0 \leq K \leq 1.8 \)
\( \sigma_x = 320 \mu \text{m}, \sigma_x' = 34 \mu \text{rad} \)
\( \sigma_y = 16 \mu \text{m}, \sigma_y' = 6 \mu \text{rad} \)
\( \theta_{cen} = 23 \mu \text{rad} @ K = 1 \)
\( \eta = 10\% \)
Coherent Power at the UK’s Diamond Synchrotron Facility

3.0 GeV, 300 mA
\( \lambda_u = 2.4 \text{ cm}, N = 82 \)
\( 0 \leq K \leq 1.4 \)
\( \sigma_x = 123 \mu m, \sigma_x' = 24 \mu r \)
\( \sigma_y = 6.4 \mu m, \sigma_y' = 4.2 \mu r \)
\( \theta_{cen} = 23 \mu r @ K = 1 \)
\( \eta = 10\% \)

Courtesy of Brian Kennedy (King’s College London), Susan Smith (Daresbury), and Yanwei Liu (LBNL)
6.0 GeV, 200 mA
\( \lambda_u = 42 \text{ mm}, N = 38 \)
\( 0 \leq K \leq 2.1 \)
\( \sigma_x = 395 \mu\text{m}, \sigma_x' = 11 \mu\text{r} \)
\( \sigma_y = 9.9 \mu\text{m}, \sigma_y' = 3.9 \mu\text{r} \)
\( \eta + 10\% \)

(high beta)
Coherent Power at the APS

APS

7.00 GeV, 100 mA
\( \lambda_u = 33 \) mm, \( N = 72 \)
\( 0.5 \leq K \leq 3.0 \)
\( \sigma_x = 320 \) µm, \( \sigma_x' = 23 \) µr
\( \sigma_y = 50 \) µm, \( \sigma_y' = 7 \) µr
\( \eta = 10\% \)

\[ \frac{\lambda}{\Delta\lambda} = 72 \]

\[ \frac{\lambda}{\Delta\lambda} = 10^3 \]
8 GeV, 100 mA
\( \lambda_u = 32 \text{ mm}, N = 140 \)
\( 0 \leq K \leq 2.46 \)
\( \sigma_x = 393 \mu \text{m}, \sigma_x' = 15.7 \mu \text{r} \)
\( \sigma_y = 4.98 \mu \text{m}, \sigma_y' = 1.24 \mu \text{r} \)
\( \eta = 10\% \)