Beam Parameters, Spectral Brightness, Harmonics and Wiggler Radiation

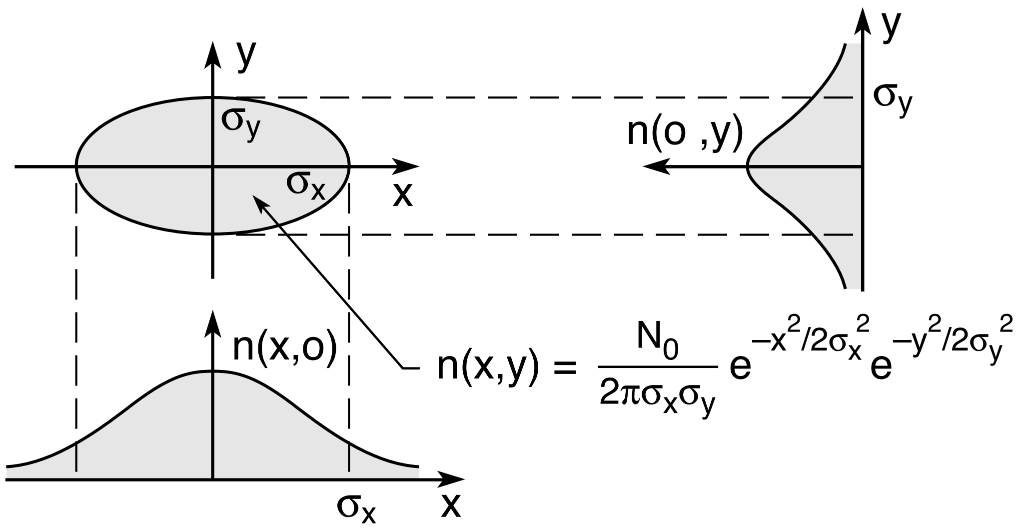
David Attwood

University of California, Berkeley

(http://www.coe.berkeley.edu/AST/srms)

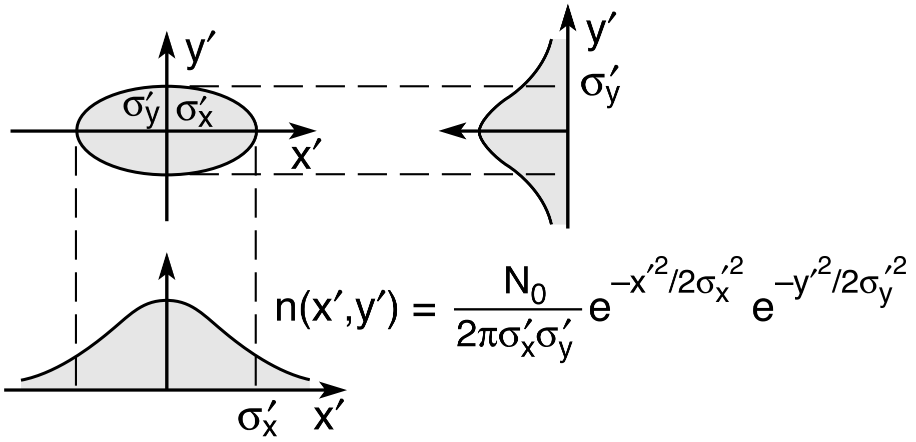
Finite Electron Beam Size and Divergence Affect Undulator Radiation

Beam size (σ)

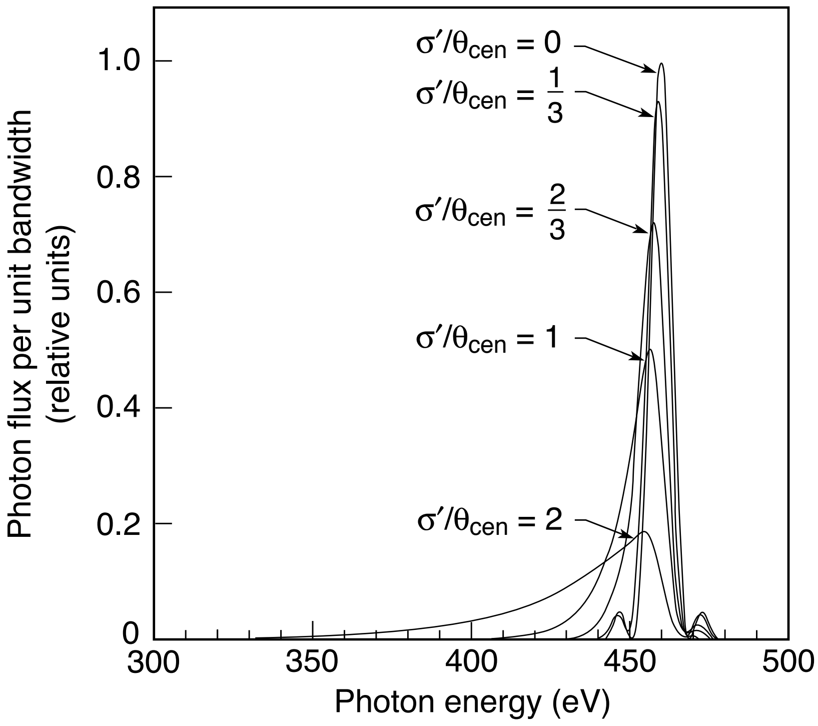


$$n(x,y) = \frac{N_0}{2\pi\sigma_x\sigma_y} e^{-x^2/2\sigma_x^2} e^{-y^2/2\sigma_y^2}$$

Beam angular divergence (σ′)



$$n(x',y') = \frac{N_0}{2\pi\sigma'_x\sigma'_y} e^{-x'^2/2\sigma_x'^2} e^{-y'^2/2\sigma_y'^2}$$



Preserving the spectral line shape of undulator radiation requires

$$\sigma'^2 \ll \theta_{\text{cen}}^2 \tag{5.55b}$$

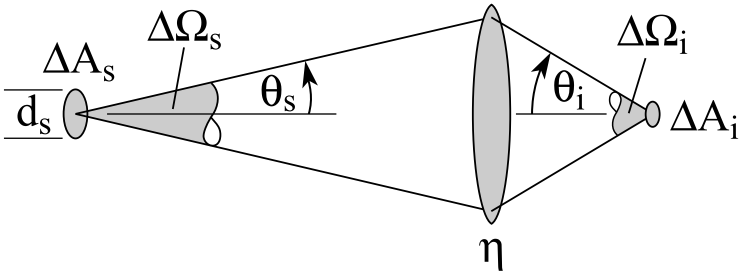
Define effective, or total central cone half-angles

$$\theta_{Tx} = \sqrt{\theta_{\text{cen}}^2 + \sigma_x'^2} \quad \text{and} \quad \theta_{Ty} = \sqrt{\theta_{\text{cen}}^2 + \sigma_y'^2} \tag{5.56}$$

Brightness and Spectral Brightness

Brightness is defined as radiated power per unit area and per unit solid angle at the source:

$$\boxed{B = \frac{\Delta P}{\Delta A \cdot \Delta\Omega}} \tag{5.57}$$

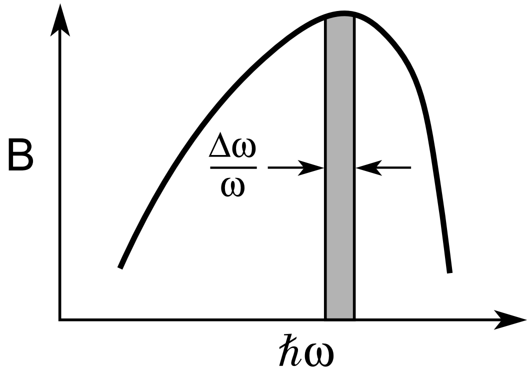


Brightness is a conserved quantity in perfect optical systems, and thus is useful in designing beamlines and synchrotron radiation experiments which involve focusing to small areas.

Perfect optical system:
$\Delta A_s \cdot \Delta\Omega_s = \Delta A_i \cdot \Delta\Omega_i$; $\eta = 100\%$

Spectral brightness is that portion of the brightness lying within a relative spectral bandwidth $\Delta\omega/\omega$:

$$\boxed{B_{\Delta\omega/\omega} = \frac{\Delta P}{\Delta A \cdot \Delta\Omega \cdot \Delta\omega/\omega}} \tag{5.58}$$



Spectral Brightness of Undulator Radiation

The Synchrotron radiation community prefers to express spectral brightness in units of photons/sec, rather than power, and has standardized on a relative spectral bandwidth of $\Delta\omega/\omega = 10^{-3}$, or 0.1% BW. To obtain a relationship for spectral brightness of undulator radiation we can use our expression for $\bar{P}_{\text{cen}}$, radiated into a solid angle $\Delta\Omega = \pi\theta_{\text{cen}}^2 = \pi\theta_{Tx}\theta_{Ty}$, from an elliptically shaped source area of $\Delta A = \pi\sigma_x\sigma_y$, and within a relative spectral bandwidth $\Delta\omega/\omega = 1/N$. Defining the photon flux in the central radiation cone as

$$\bar{F}_{\text{cen}} = \frac{\bar{P}_{\text{cen}}}{\hbar\omega/\text{photon}} \tag{5.59}$$

$$\bar{B}_{\Delta\omega/\omega} = \frac{\bar{F}_{\text{cen}}}{\Delta A \cdot \Delta\Omega \cdot N^{-1}} = \frac{\bar{F}_{\text{cen}} \cdot (N/1000)}{\Delta A \cdot \Delta\Omega \cdot (0.1\%\text{BW})} \tag{5.60}$$

on-axis

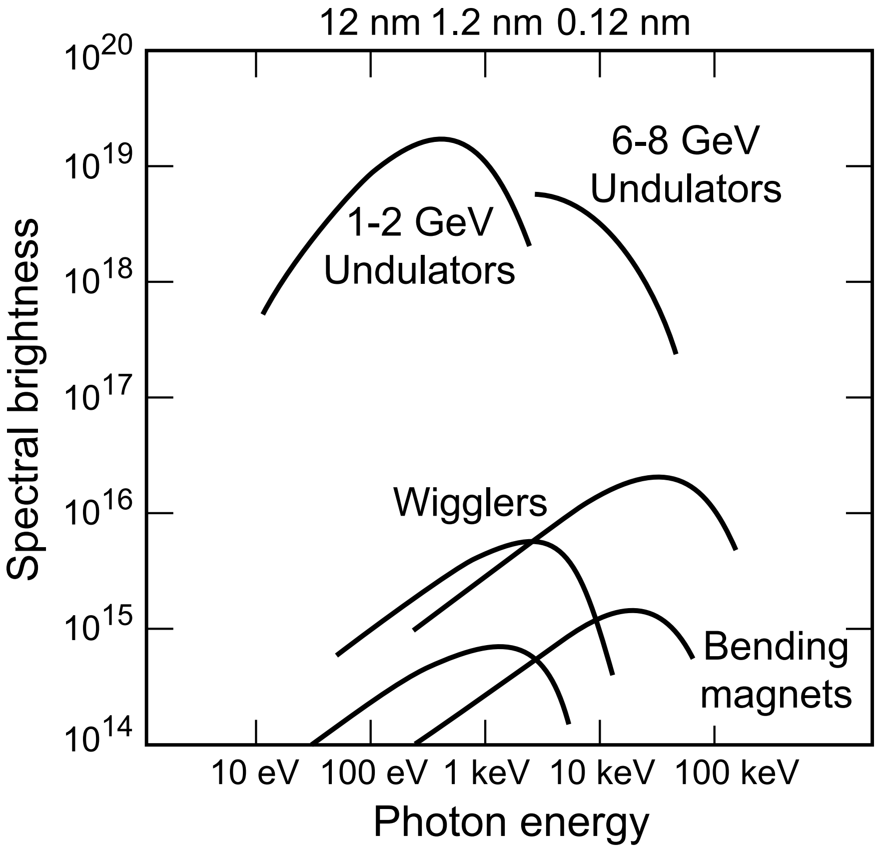
$$\bar{B}_{\Delta\omega/\omega}(0) = \frac{\bar{F}_{\text{cen}} \cdot (N/1000)}{2\pi^2\sigma_x\sigma_y\theta_{Tx}\theta_{Ty}(0.1\%\text{BW})} \tag{5.64}$$

or

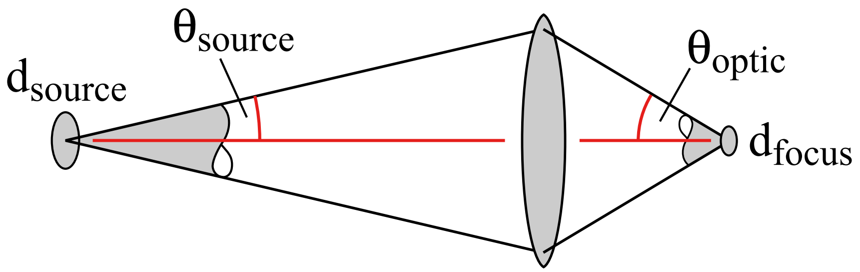
$$\bar{B}_{\Delta\omega/\omega}(0) = \frac{7.25 \times 10^6 \gamma^2 N^2 I(\text{A})}{\sigma_x(\text{mm})\sigma_y(\text{mm})\left(1 + \frac{\sigma_x'^2}{\theta_{\text{cen}}^2}\right)^{1/2}\left(1 + \frac{\sigma_y'^2}{\theta_{\text{cen}}^2}\right)^{1/2}} \cdot \frac{K^2 f(K)}{\left(1 + K^2/2\right)^2} \; \frac{\text{photons/s}}{\text{mm}^2\text{mrad}^2(0.1\%\text{BW})} \tag{5.65}$$

Assumes $\sigma'^2 \ll \theta_{\text{cen}}^2$. Note the N^2 factor.

Spectral Brightness is Useful for Experiments that Involve Spatially Resolved Studies



- Brightness is conserved (in lossless optical systems)

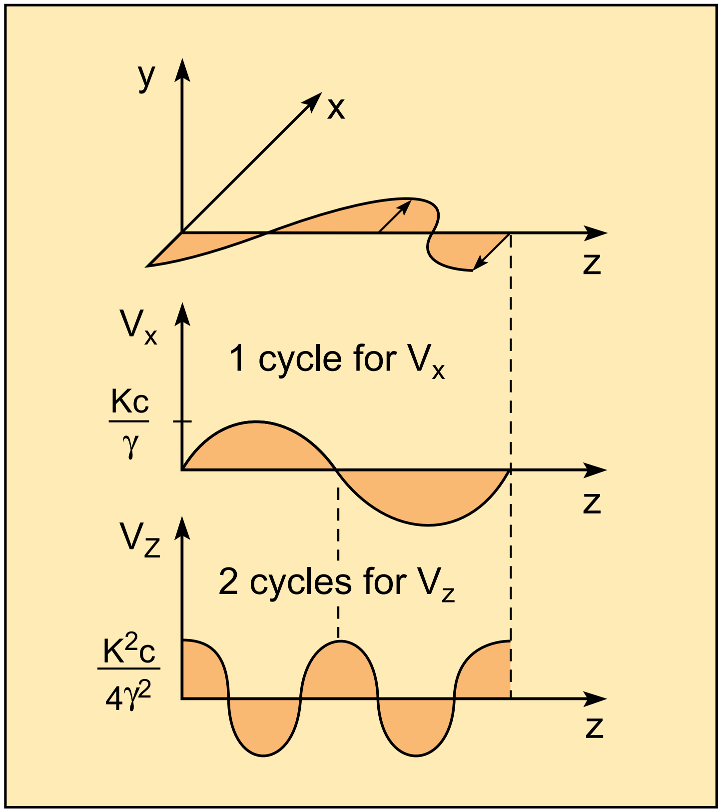


$$d_{\text{source}} \cdot \theta_{\text{source}} = \underset{\text{Smaller after focus}}{\underset{\uparrow}{d_{\text{focus}}}} \cdot \underset{\text{Large in a focusing optic}}{\underset{\uparrow}{\theta_{\text{optic}}}}$$

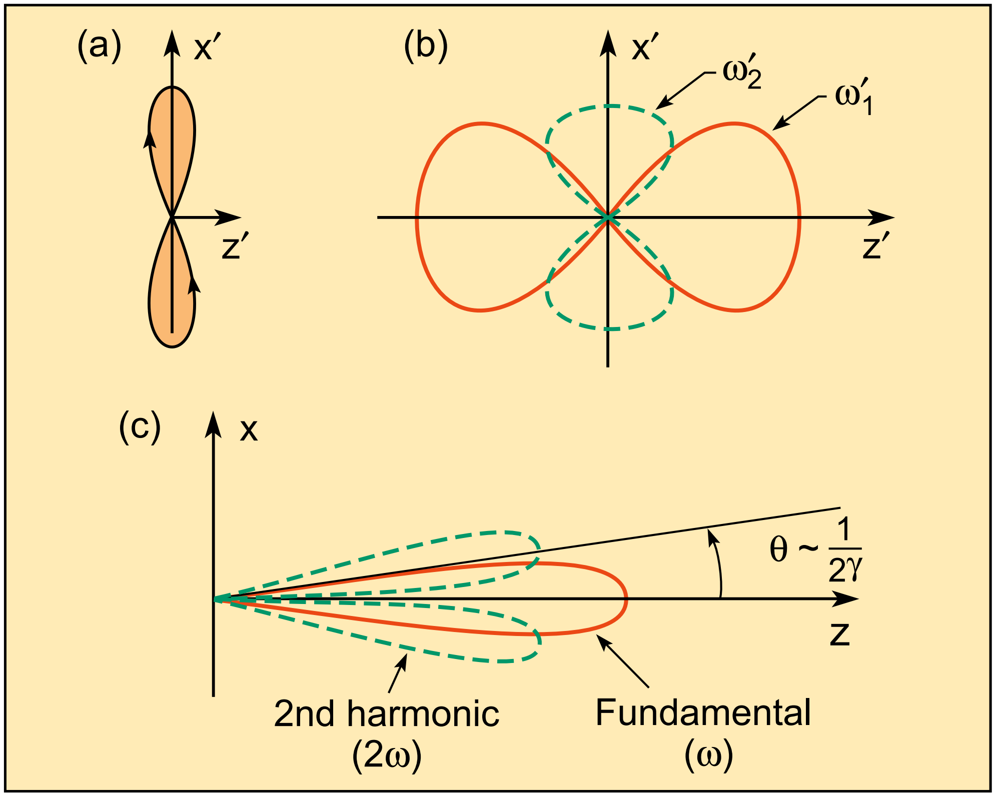
- Starting with many photons in a small source area and solid angle, permits high photon flux in an even smaller area

Comments on Undulator Harmonics

First and second harmonic motions



Radiation patterns in the electron and laboratory frames



$$\lambda_n = \frac{\lambda_u}{2\gamma^2 n}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right) \qquad (5.30)$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_n = \frac{1}{nN} \qquad (5.31)$$

Undulator Harmonics

Recall that the axial velocity has a double frequency component

$$\mathrm{v}_z = c\left[1 - \frac{1+K^2/2}{2\gamma^2} + \frac{K^2}{4\gamma^2}\cos(2k_u z)\right]$$

which in the frame of reference moving with the electrons, gives

$$z'(t') \simeq \frac{K^2}{8k'_u}\sin 2\omega'_u t' \qquad (5.70)$$

where $k'_u = \gamma^* k_u$ and $\omega'_u = \gamma^* \omega_u$. The transverse motion in this frame is

$$x'(t') \simeq -\frac{K}{k_u\gamma}\cos\omega_u\gamma^*\left(t' + \frac{z'}{c}\right)$$

To a higher degree of accuracy, we now keep the z'/c term

$$x'(t') \simeq -\frac{K}{k'_u}\cos\left(\omega'_u t' + \frac{K^2}{8}\sin 2\omega'_u t'\right) \qquad (5.71)$$

for small K

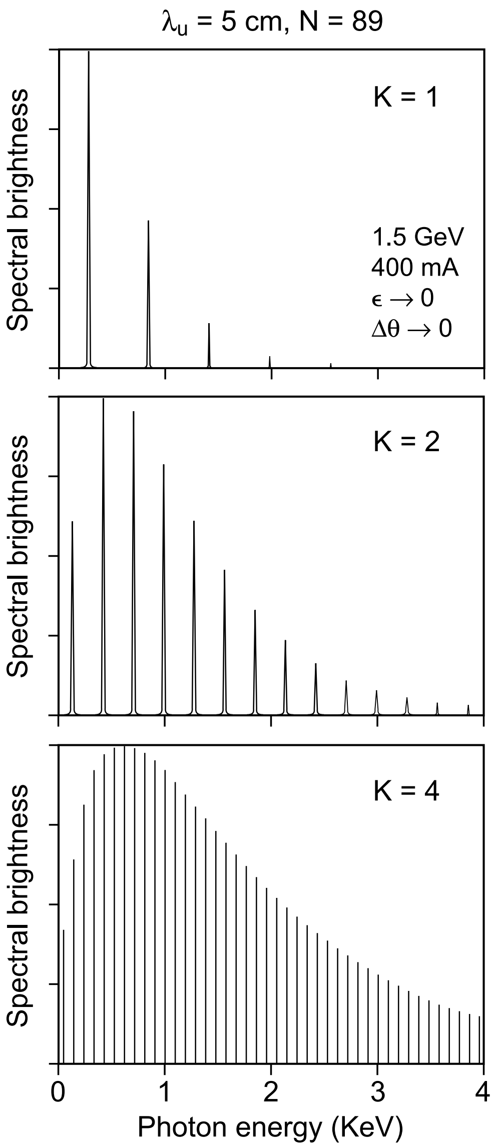
$$x'(t') \simeq -\frac{1}{k'_u}\left[\enclose{circle}{K}\cos\enclose{circle}{\omega'_u}t' + \frac{\enclose{circle}{K^3}}{16}\cos\enclose{circle}{3\omega'_u}t'\right] \qquad (5.72)$$

Taking second derivatives to find acceleration, and squaring $|a'(t')|^2$

$$\boxed{\frac{dP'}{d\Omega'} \propto n^4 K^{2n}}$$

Thus harmonics grow very rapidly for K > 1.

The Transition from Undulator Radiation ($K \leq 1$) to Wiggler Radiation ($K >> 1$)



(Courtesy of K.-J. Kim)

Undulator radiation ($K \lesssim 1$)

- Narrow spectral lines
- High spectral brightness
- Partial coherence

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

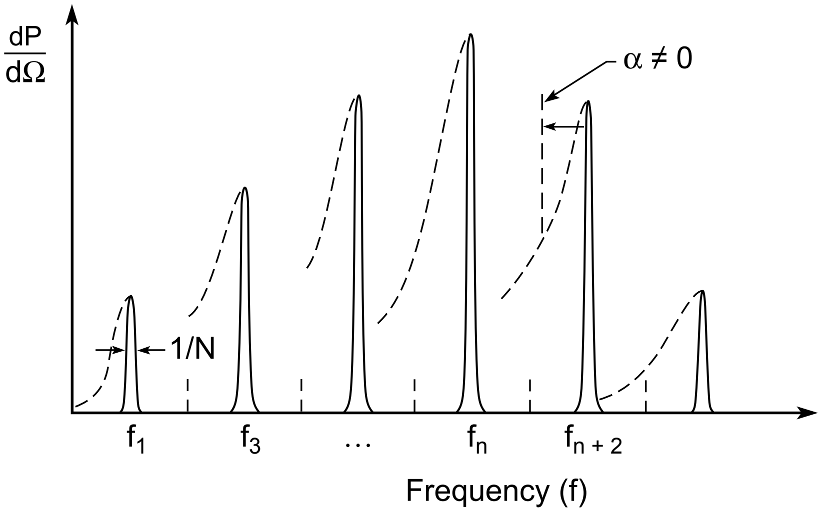
$$K = \frac{eB_o\lambda_u}{2\pi mc}$$

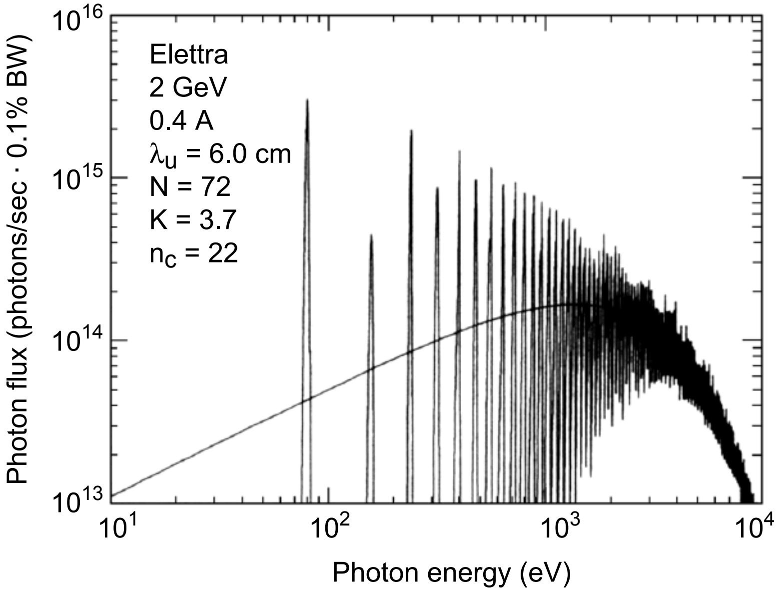
Wiggler radiation ($K >> 1$)

- Higher photon energies
- Spectral continuum
- Higher photon flux (2N)

$$\hbar\omega_c = \frac{3}{2}\,\frac{\hbar\gamma^2 eB_o}{m}$$

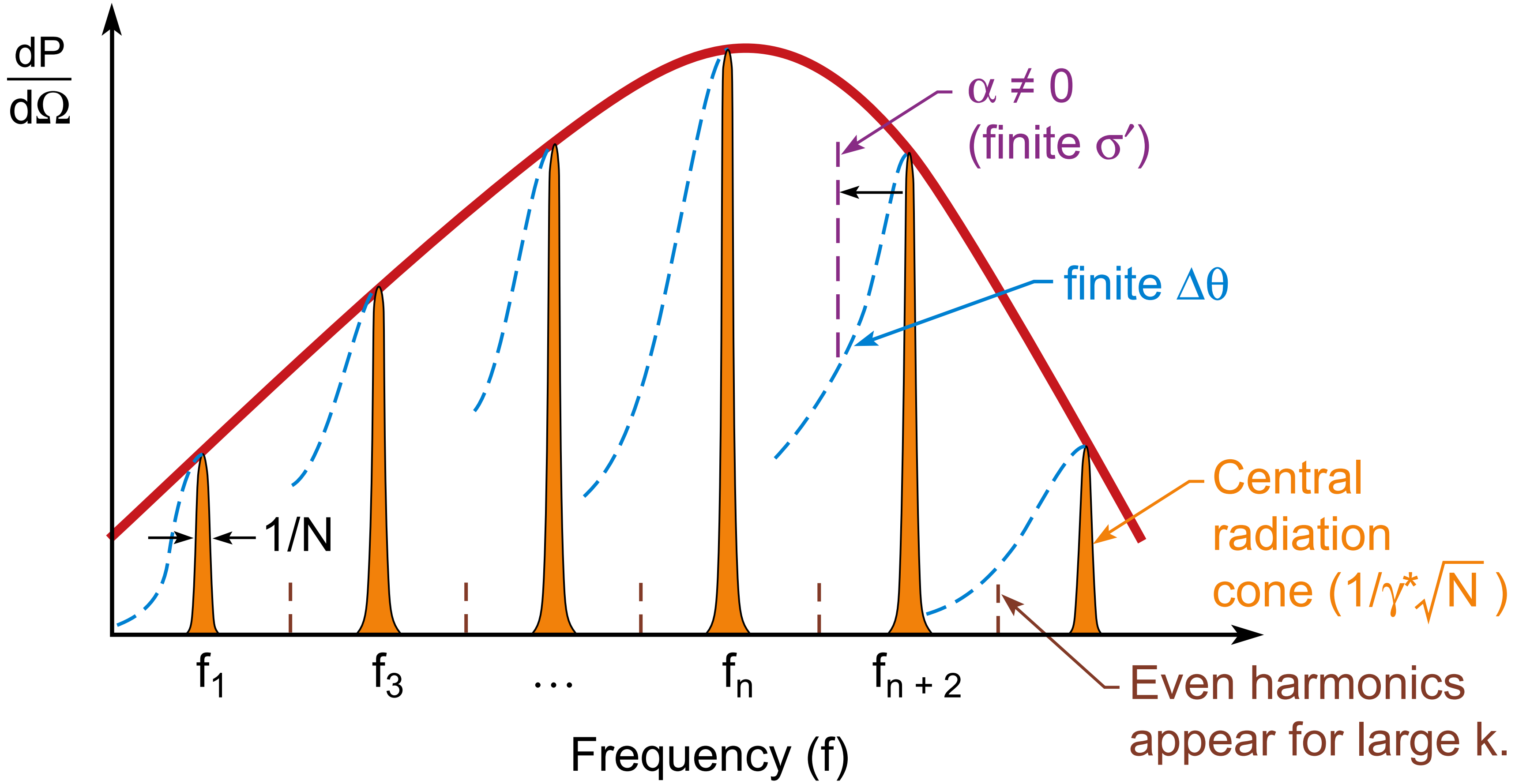
$$n_c = \frac{3K}{4}\left(1 + \frac{K^2}{2}\right)$$





(Courtesy of R.P. Walker and B. Diviacco)

For Very Large K >> 1, and Large Dq, a Continuum Emerges



$\alpha \neq 0$
(finite σ')

finite $\Delta\theta$

Central radiation cone $(1/\gamma^*\sqrt{N})$

Even harmonics appear for large k.

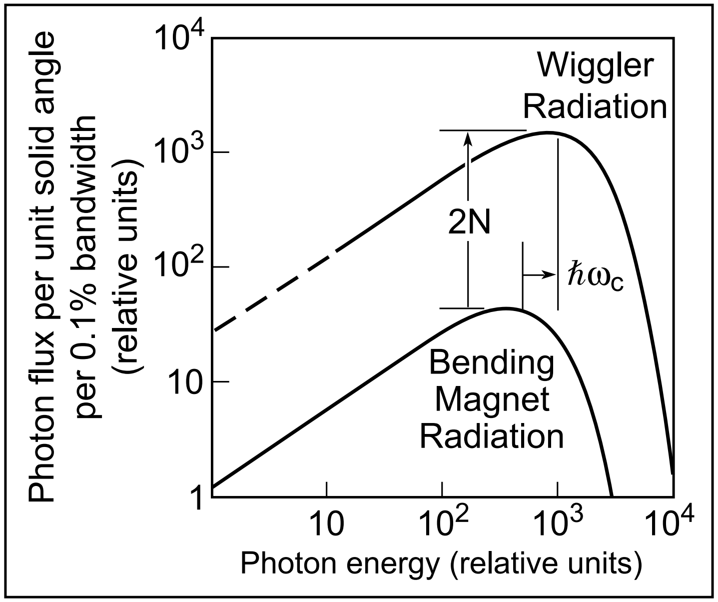
Wiggler Radiation

At very high K >> 1, the radiated energy appears in very high harmonics, and at rather large horizontal angles $\theta \simeq \pm K/\gamma$ (eq. 5.21). Because the emission angles are large, one tends to use larger collection angles, which tends to spectrally merge nearby harmonics. The result is a continuum at very high photon energies, similar to that of bending magnet radiation, but increased by 2N (the number of magnet pole pieces).

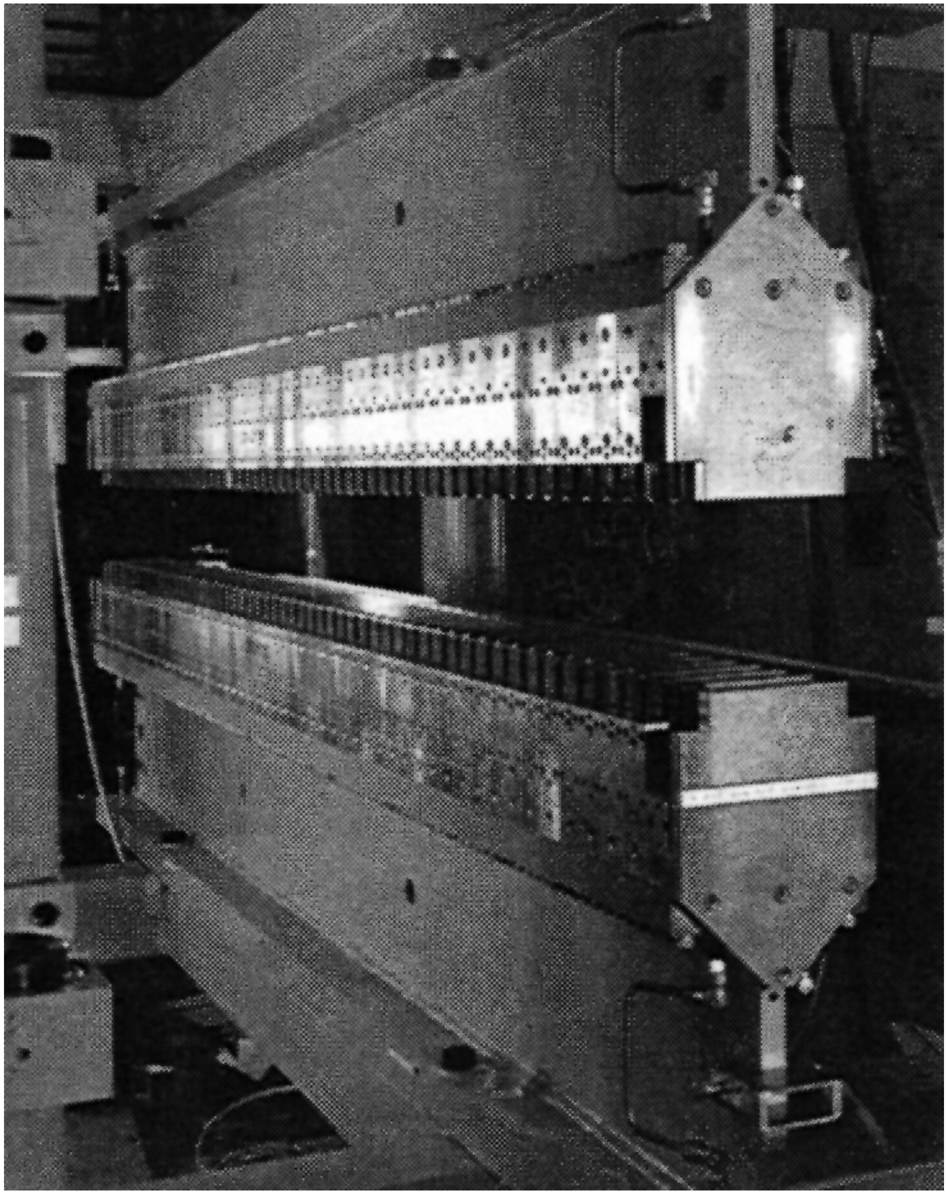
$$E_c = \hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m} \quad ; \quad n_c = \frac{3K}{4}\left(1+\frac{K^2}{2}\right) \tag{5.7a \& 82}$$

$$\left.\frac{d^2F}{d\theta\, d\Psi\, d\omega/\omega}\right|_0 = 2.65\times10^{13} N E_e^2(\mathrm{GeV}) I(\mathrm{A}) H_2(E/E_c) \frac{\mathrm{photons/s}}{\mathrm{mrad}^2(0.1\%\mathrm{BW})} \tag{5.86}$$

$$\frac{d^2F}{d\theta\, d\omega/\omega} = 4.92\times10^{13} N E_e(\mathrm{GeV}) I(\mathrm{A}) G_1(E/E_c) \frac{\mathrm{photons/s}}{\mathrm{mrad}\cdot(0.1\%\mathrm{BW})} \tag{5.87}$$



Stanford Permanent Magnet Wiggler



LBNL/EXXON/SSRL (1982), SSRL Beamline VI
55 pole ($N = 27.5$), $\lambda_w = 7$ cm

Typical Parameters for Synchrotron Radiation

Facility	ALS	ELETTRA	Australian Synchrotron	APS
Electron energy	1.90 GeV	2.0 GeV	3.0 GeV	7.00 GeV
γ	3720	3910	5871	13,700
Current (mA)	400	300	200	100
Circumference (m)	197	259	216	1100
RF frequency (MHz)	500	500	500	352
Pulse duration (FWHM) (ps)	35-70	37	~100	100
Bending Magnet Radiation:				
Bending magnet field (T)	1.27	1.2	1.31	0.599
Critical photon energy (keV)	3.05	3.2	7.84	19.5
Critical photon wavelength	0.407 nm	0.39 nm	1.58 Å	0.636 Å
Bending magnet sources	24	12	28	35
Undulator Radiation:				
Number of straight sections	12	12	14	40
Undulator period (typical) (cm)	5.00	5.6	22.0	3.30
Number of periods	89	81	80	72
Photon energy ($K = 1, n = 1$)	457 eV	452 eV	2.59 keV	9.40 keV
Photon wavelength ($K = 1, n = 1$)	2.71 nm	2.74 nm	0.478 nm	1.32 Å
Tuning range ($n = 1$)	230-620 eV	2.0-6.7 nm	0.319-0.835 nm	3.5-12 keV
Tuning range ($n = 3$)	690-1800 eV	0.68-2.2 nm	0.106-0.278 nm	10-38 keV
Central cone half-angle ($K = 1$)	35 μrad	35 μrad	23 μrad	11 μrad
Power in central cone ($K = 1, n = 1$) (W)	2.3	1.7	6.6	12
Flux in central cone (photons/s)	3.1×10^{16}	2.3×10^{16}	1.6×10^{16}	7.9×10^{15}
σ_x, σ_y (μm)	260, 16	255, 23	320, 16	320, 50
σ'_x, σ'_y (μrad)	23, 3.9	31, 9	34, 6	23, 7
Brightness ($K = 1, n = 1$)[a] $[(\text{photons/s})/\text{mm}^2 \cdot \text{mrad}^2 \cdot (0.1\%\text{BW})]$	2.3×10^{19}	9.9×10^{18}	1.3×10^{19}	5.9×10^{18}
Total power ($K = 1$, all n, all θ) (W)	83	126	476	350
Other undulator periods (cm)	3.65, 8.00, 10.0	8.0, 12.5	6.8, 18.3	2.70, 5.50, 12.8
Wiggler Radiation:				
Wiggler period (typical) (cm)	16.0	14.0	6.1	8.5
Number of periods	19	30	30	28
Magnetic field (maximum) (T)	2.1	1.5	1.9	1.0
K (maximum)	32	19.6	12	7.9
Critical photon energy (keV)	5.1	4.0	11.4 keV	33
Critical photon wavelength	0.24 nm	0.31 nm	0.11 nm	0.38 Å
Total power (max. K) (kW)	13	7.2	9.3	7.4

[a] Using Eq. (5.65). See comments following Eq. (5.64) for the case where $\sigma'_{x,y} \simeq \theta_{cen}$.

Typical Parameters for Synchrotron Radiation

Facility	ALS	MAX II	BESSY II	APS	ESRF
Electron energy	1.90 GeV	1.50 GeV	1.70 GeV	7.00 GeV	6.04 GeV
γ	3720	2940	3330	13,700	11,800
Current (mA)	400	250	200	100	200
Circumference (m)	197	90	240	1100	884
RF frequency (MHz)	500	500	500	352	352
Pulse duration (FWHM) (ps)	35-70	200	20-50	100	70
Bending Magnet Radiation:					
Bending magnet field (T)	1.27	1.48	1.30	0.599	0.806
Critical photon energy (keV)	3.05	2.21	2.50	19.5	19.6
Critical photon wavelength	0.407 nm	0.560 nm	0.50 nm	0.636 Å	0.634 Å
Bending magnet sources	24	20	32	35	32
Undulator Radiation:					
Number of straight sections	12	10	16	40	32
Undulator period (typical) (cm)	5.00	5.20	4.90	3.30	4.20
Number of periods	89	49	84	72	38
Photon energy ($K = 1$, $n = 1$)	457 eV	274 eV	373 eV	9.40 keV	5.50 keV
Photon wavelength ($K = 1$, $n = 1$)	2.71 nm	4.53 nm	3.32 nm	1.32 Å	0.225 nm
Tuning range ($n = 1$)	230-620 eV	130-410 eV	140-500 eV	3.5-12 keV	2.6-7.3 keV
Tuning range ($n = 3$)	690-1800 eV	400-1200 eV	410-1100 eV	10-38 keV	7.7-22 keV
Central cone half-angle ($K = 1$)	35 μrad	59 μrad	33 μrad	11 μrad	17 μrad
Power in central cone ($K = 1$, $n = 1$) (W)	2.3	0.88	0.95	12	14
Flux in central cone (photons/s)	3.1×10^{16}	2.0×10^{16}	1.6×10^{16}	7.9×10^{15}	1.6×10^{16}
σ_x, σ_y (μm)	260, 16	300, 45	314, 24	320, 50	395, 9.9
σ'_x, σ'_y (μrad)	23, 3.9	26, 20	18, 12	23, 7	11, 3.9
Brightness ($K = 1$, $n = 1$)[a] [(photons/s)/mm^2 · mrad^2 · (0.1%BW)]	2.3×10^{19}	7.8×10^{17}	4.6×10^{18}	5.9×10^{18}	5.1×10^{18}
Total power ($K = 1$, all n, all θ) (W)	83	17	32	350	480
Other undulator periods (cm)	3.65, 8.00, 10.0	5.88, 6.60	4.1, 5.6, 12.5	2.70, 5.50, 12.8	2.3, 3.2, 5.2, 8.5
Wiggler Radiation:					
Wiggler period (typical) (cm)	16.0	17.4	12.5	8.5	8.0
Number of periods	19	13	32	28	20
Magnetic field (maximum) (T)	2.1	1.80	1.15	1.0	0.81
K (maximum)	32	29.3	12.8	7.9	6.0
Critical photon energy (keV)	5.1	2.69	2.11	33	20
Critical photon wavelength	0.24 nm	0.46 nm	0.59 nm	0.38 Å	0.62 Å
Total power (max. K) (kW)	13	5.9	1.8	7.4	4.8

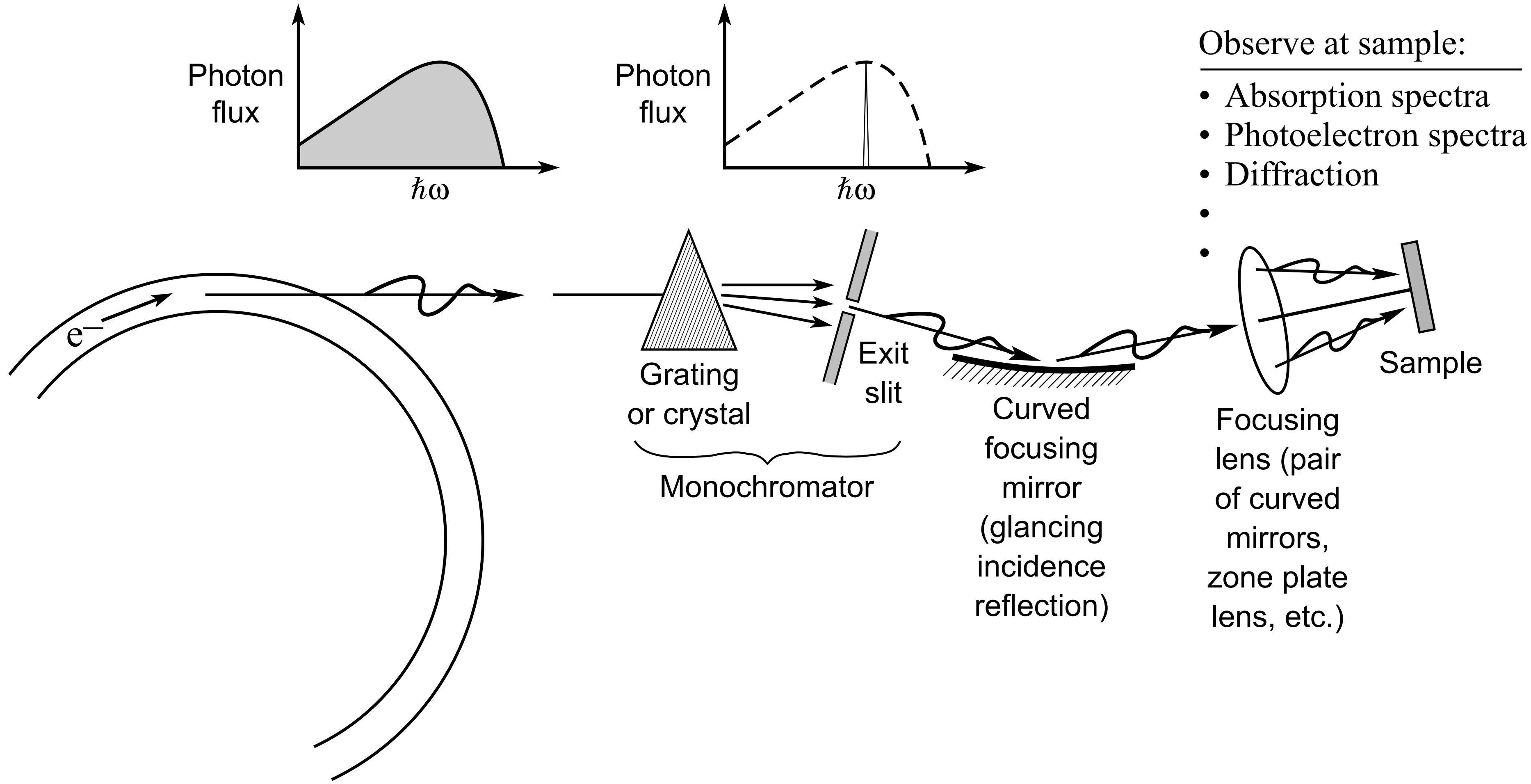
[a] Using Eq. (5.65). See comments following Eq. (5.64) for the case where $\sigma'_{x,\,y} \simeq \theta_{cen}$.

Typical Parameters for Synchrotron Radiation

Facility	ALS	New Subaru	APS	SP-8
Electron energy	1.90 GeV	1.00 GeV	7.00 GeV	8.00 GeV
γ	3720	1957	13,700	15,700
Current (mA)	400	100	100	100
Circumference (m)	197	119	1100	1440
RF frequency (MHz)	500	500	352	509
Pulse duration (FWHM) (ps)	35-70	26	100	120
Bending Magnet Radiation:				
Bending magnet field (T)	1.27	1.03	0.599	0.679
Critical photon energy (keV)	3.05	0.685	19.5	28.9
Critical photon wavelength	0.407 nm	1.81 nm	0.636 Å	0.429 Å
Bending magnet sources	24	4	35	23
Undulator Radiation:				
Number of straight sections	12	4	40	48
Undulator period (typical) (cm)	5.00	5.40	3.30	3.20
Number of periods	89	200	72	140
Photon energy ($K = 1$, $n = 1$)	457 eV	117 eV	9.40 keV	12.7 keV
Photon wavelength ($K = 1$, $n = 1$)	2.71 nm	10.6 nm	1.32 Å	0.979 Å
Tuning range ($n = 1$)	230-620 eV	43-170 eV	3.5-12 keV	4.7-19 keV
Tuning range ($n = 3$)	690-1800 eV	130-500 eV	10-38 keV	16-51 keV
Central cone half-angle ($K = 1$)	35 μrad	44 μrad	11 μrad	6.6 μrad
Power in central cone ($K = 1$, $n = 1$) (W)	2.3	0.15	12	16
Flux in central cone (photons/s)	3.1×10^{16}	7.9×10^{15}	7.9×10^{15}	7.9×10^{15}
σ_x, σ_y (μm)	260, 16	450, 220	320, 50	380, 6.8
σ'_x, σ'_y (μrad)	23, 3.9	89, 18	23, 7	16, 1.8
Brightness ($K = 1$, $n = 1$)^a $[(\text{photons/s})/\text{mm}^2 \cdot \text{mrad}^2 \cdot (0.1\%\text{BW})]$	2.3×10^{19}	1.7×10^{17}	5.9×10^{18}	1.8×10^{20}
Total power ($K = 1$, all n, all θ) (W)	83	27	350	2,000
Other undulator periods (cm)	3.65, 8.00, 10.0	7.60	2.70, 5.50, 12.8	2.4, 10.0, 3.7, 12.0
Wiggler Radiation:				
Wiggler period (typical) (cm)	16.0		8.5	12.0
Number of periods	19		28	37
Magnetic field (maximum) (T)	2.1		1.0	1.0
K (maximum)	32		7.9	11
Critical photon energy (keV)	5.1		33	43
Critical photon wavelength	0.24 nm		0.38 Å	0.29 Å
Total power (max. K) (kW)	13		7.4	18

[a] Using Eq. (5.65). See comments following Eq. (5.64) for the case where $\sigma'_{x,y} \simeq \theta_{cen}$.

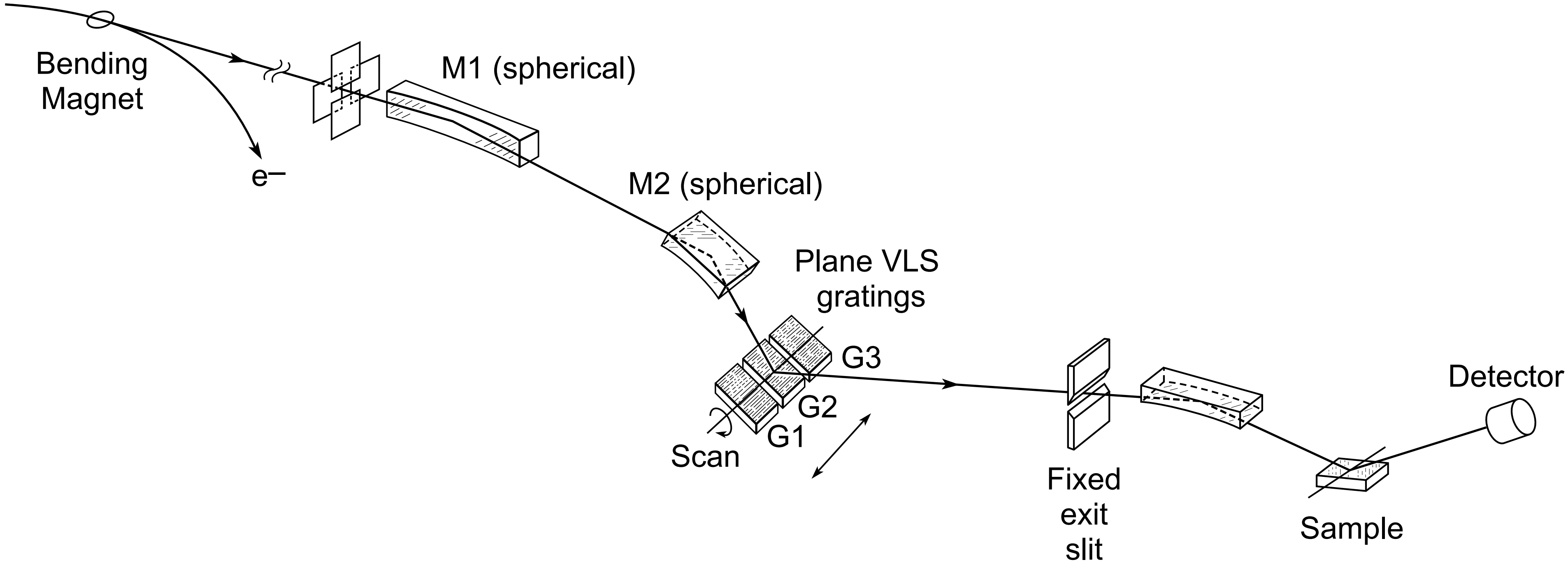
Beamlines are Used to Transport Photons to the Sample, and Take a Desired Spectral Slice



Observe at sample:

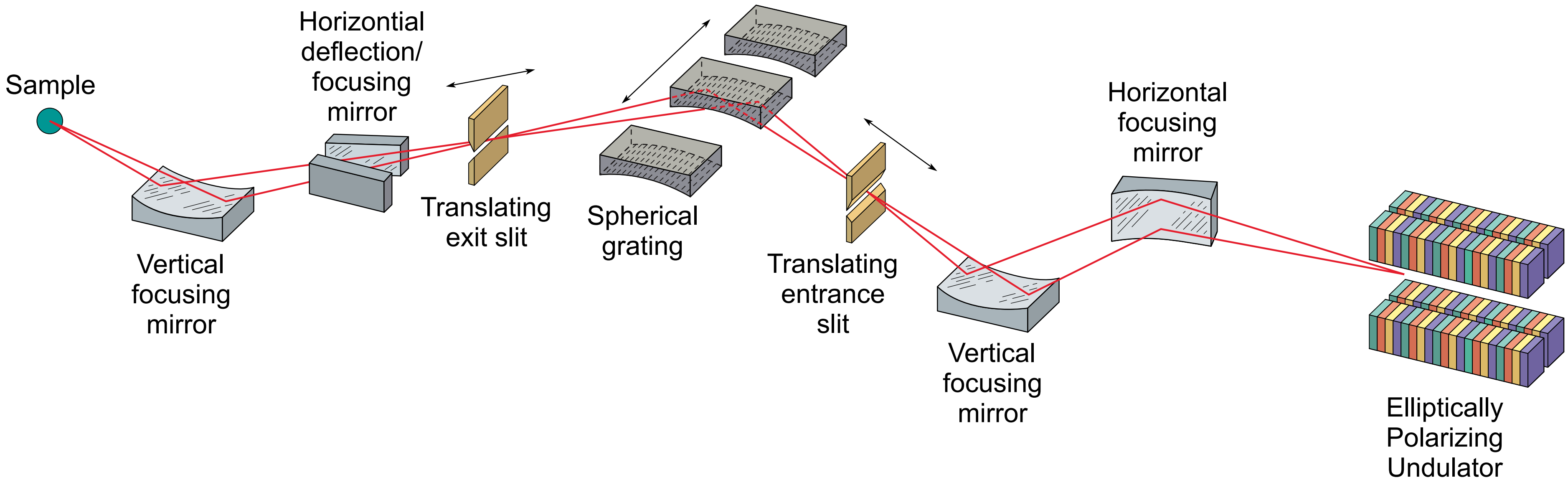
- Absorption spectra
- Photoelectron spectra
- Diffraction
-
-

A Typical Beamline: Monochromator Plus Focusing Optics to Deliver Radiation to the Sample



Courtesy of James Underwood (EUV Technology Inc.)

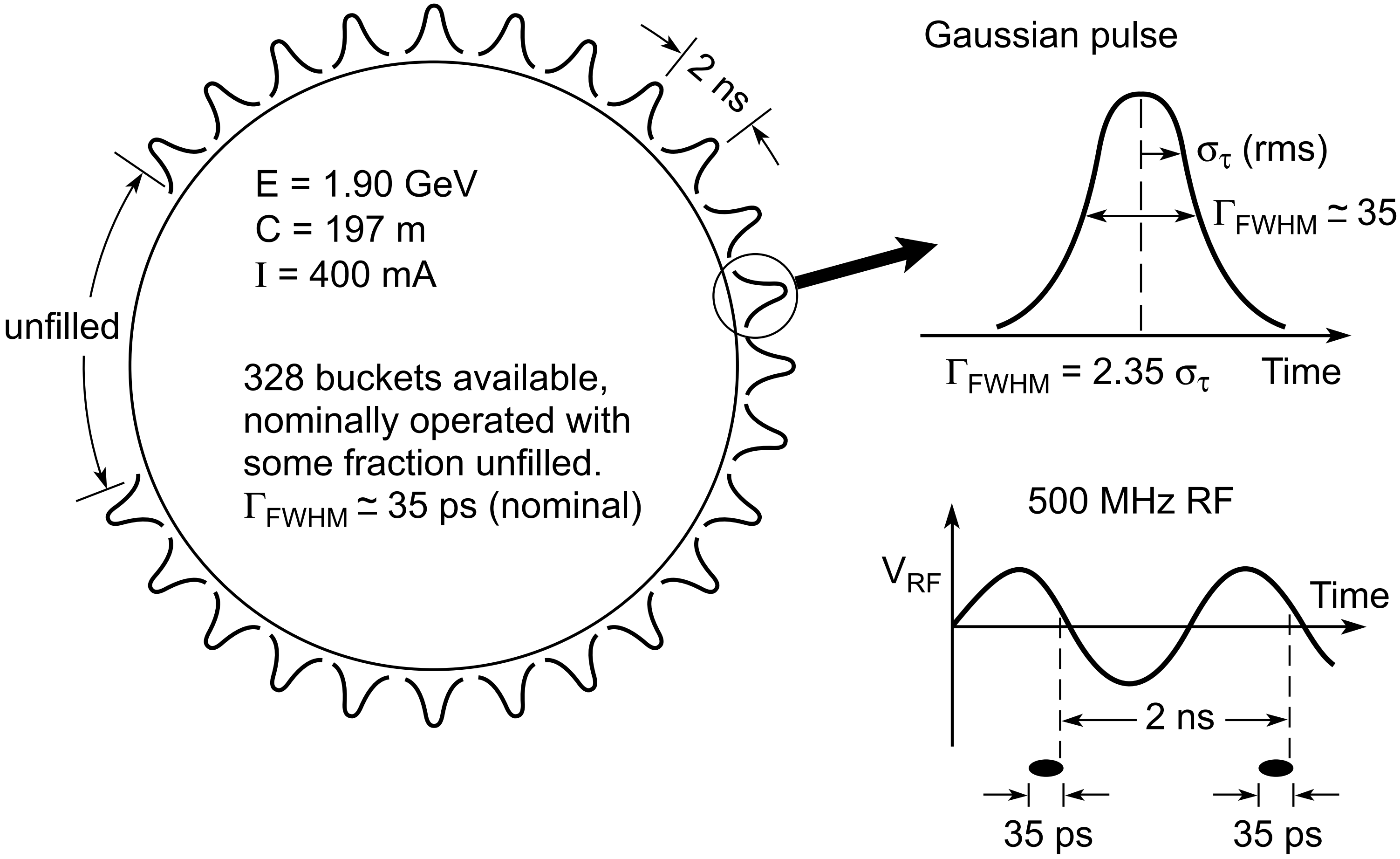
High Spectral Resolution (meV) Beamline



Beamline 7.0 at Berkeley's Advanced Light Source

Time Structure of Synchrotron Radiation

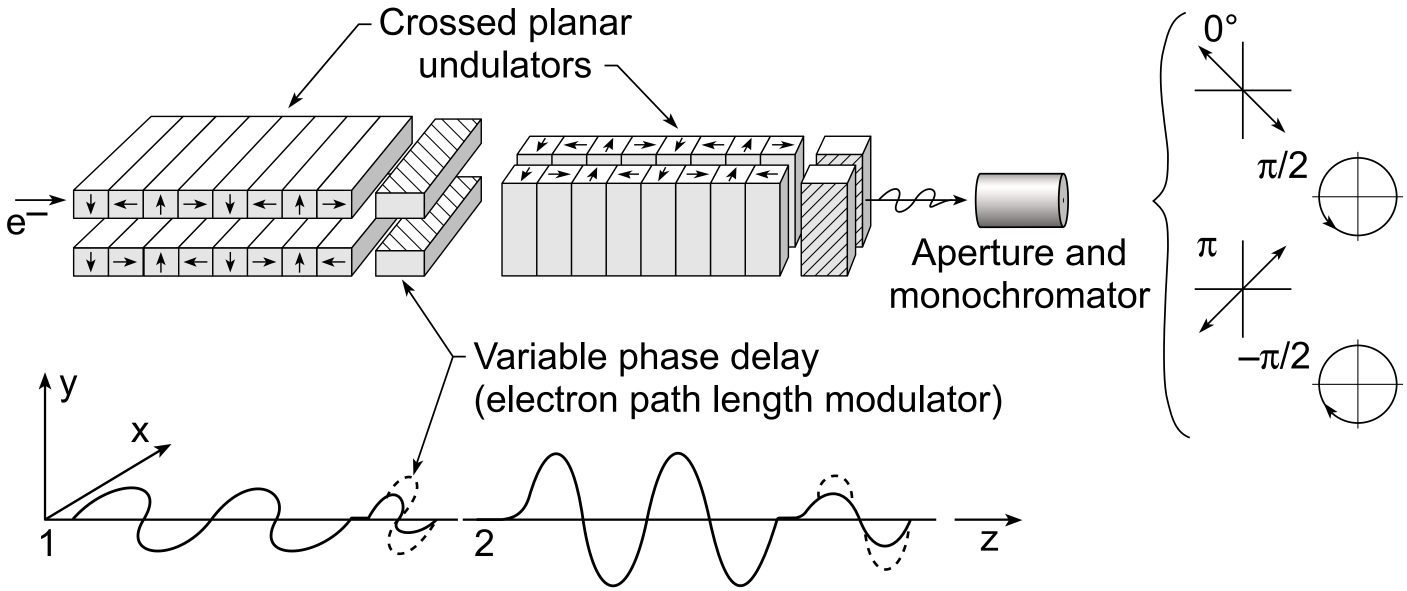
The axial electric field within the RF cavity, used to replenish lost (radiated) energy, forms a potential well “bucket” system that forces electrons into axial electron “bunches”. This leads to a time structure in the emitted radiation.



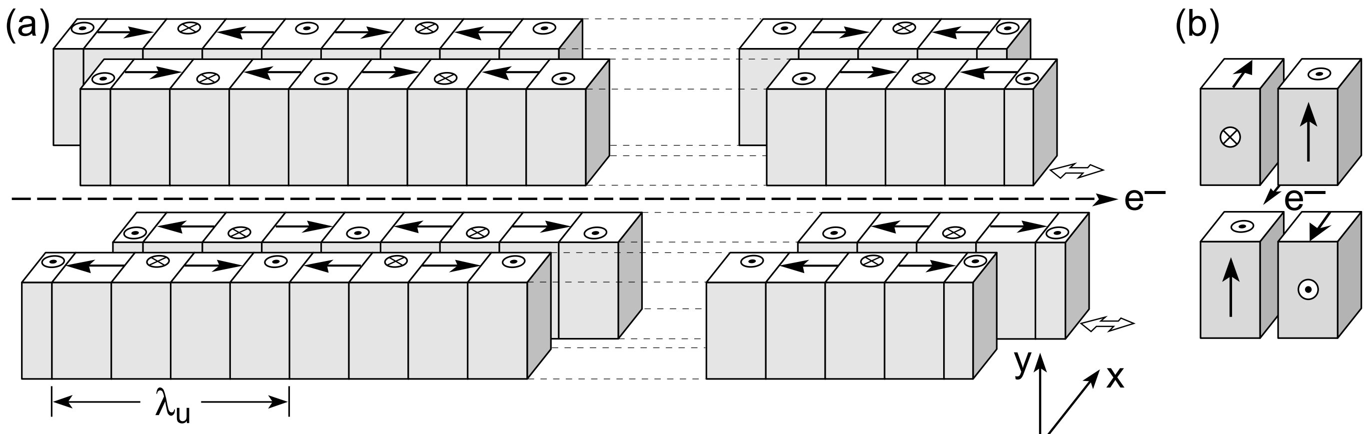
$E = 1.90$ GeV
$C = 197$ m
$I = 400$ mA

328 buckets available, nominally operated with some fraction unfilled.
$\Gamma_{FWHM} \simeq 35$ ps (nominal)

Variable Polarization Undulator Radiation



(Courtesy of Kwang-Je Kim)



(Following S. Sasaki)

What are the Relative Merits?

Bending magnet radiation	Wiggler radiation	Undulator radiation
• Broad spectrum	• Higher photon energies	• Brighter radiation
• Good photon flux	• More photon flux	• Smaller spot size
• No heat load	• Expensive magnet structure	• Partial coherence
• Less expensive	• Expensive cooled optics	• Expensive
• Easier access	• Less access	• Less access

References

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