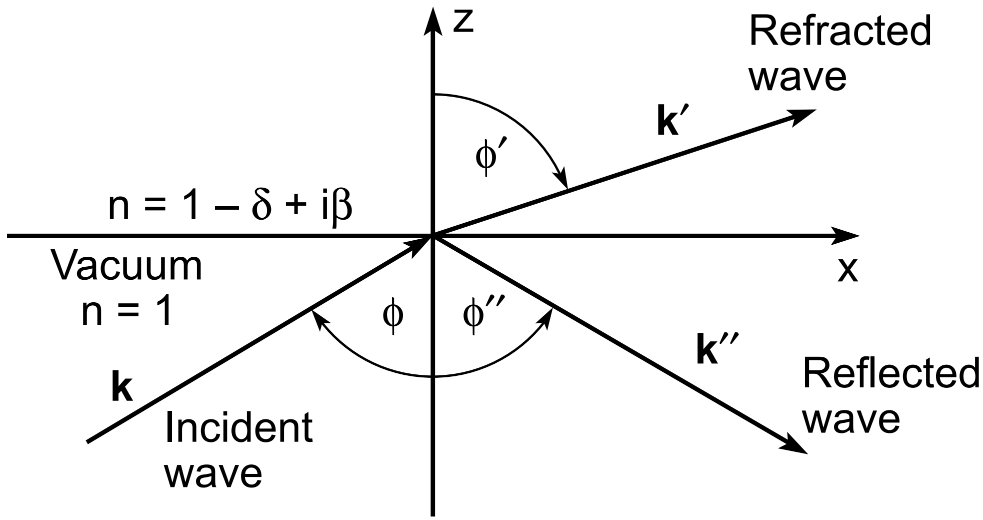
Reflection and Refraction at an Interface, Total Internal Reflection, Brewster's Angle



incident wave: $$\mathbf{E} = \mathbf{E}_0 e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})} \quad (3.30a)$$

refracted wave: $$\mathbf{E}' = \mathbf{E}_0' e^{-i(\omega t - \mathbf{k}'\cdot\mathbf{r})} \quad (3.30b)$$

reflected wave: $$\mathbf{E}'' = \mathbf{E}_0'' e^{-i(\omega t - \mathbf{k}''\cdot\mathbf{r})} \quad (3.30c)$$

(1) All waves have the same frequency, ω, and $|\mathbf{k}| = |\mathbf{k}''| = \dfrac{\omega}{c}$

(2) The refracted wave has phase velocity

$$V_\phi = \frac{\omega'}{k'} = \frac{c}{n} \text{ , thus } k' = |\mathbf{k}'| = \frac{\omega}{c}(1 - \delta + i\beta)$$

Boundary Conditions at an Interface

- **E** and **H** components parallel to the interface must be continuous

$$\mathbf{z}_0 \times (\mathbf{E}_0 + \mathbf{E}_0'') = \mathbf{z}_0 \times \mathbf{E}_0' \qquad (3.32a)$$

$$\mathbf{z}_0 \times (\mathbf{H}_0 + \mathbf{H}_0'') = \mathbf{z}_0 \times \mathbf{H}_0' \qquad (3.32b)$$

- **D** and **B** components perpendicular to the interface must be continuous

$$\mathbf{z}_0 \cdot (\mathbf{D}_0 + \mathbf{D}_0'') = \mathbf{z}_0 \cdot \mathbf{D}_0' \qquad (3.32c)$$

$$\mathbf{z}_0 \cdot (\mathbf{B}_0 + \mathbf{B}_0'') = \mathbf{z}_0 \cdot \mathbf{B}_0' \qquad (3.32d)$$

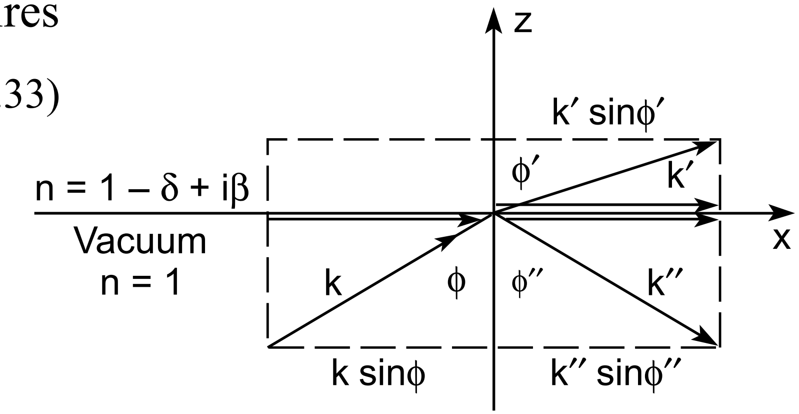
Spatial Continuity Along the Interface

Continuity of parallel field components requires

$$(\mathbf{k} \cdot \mathbf{x}_0 = \mathbf{k}' \cdot \mathbf{x}_0 = \mathbf{k}_0'' \cdot \mathbf{x}_0) \quad \text{at } z = 0 \qquad (3.33)$$

$$k_x = k_x' = k_x'' \qquad (3.34a)$$

$$k \sin\phi = k' \sin\phi' = k'' \sin\phi'' \qquad (3.34b)$$



Conclusions:

Since k = k″ (both in vacuum)

$$\sin\phi = \sin\phi'' \qquad (3.35a)$$

$$\therefore \quad \boxed{\phi = \phi''} \qquad (3.35b)$$

The angle of incidence equals the angle of reflection

$$k \sin\phi = k' \sin\phi' \qquad (3.36)$$

$$k = \frac{\omega}{c} \quad \text{and } k' = \frac{\omega'}{c/n} = \frac{n\omega}{c}$$

$$\sin\phi = n \sin\phi'$$

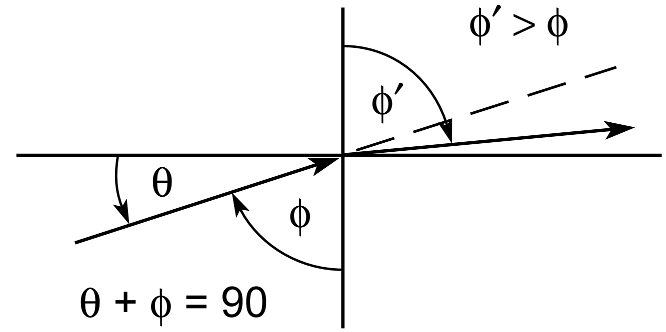
$$\boxed{\sin\phi' = \frac{\sin\phi}{n}} \qquad (3.38)$$

Snell's Law, which describes refractive turning, for complex n.

Total External Reflection of Soft X-Rays and EUV Radiation

Snell's law for a refractive index of $n \simeq 1 - \delta$, assuming that $\beta \to 0$

$$\sin\phi' = \frac{\sin\phi}{1-\delta} \qquad (3.39)$$



Consider the limit when $\phi' \to \frac{\pi}{2}$

$$1 = \frac{\sin\phi_c}{1-\delta}$$

$$\sin\phi_c = 1 - \delta \qquad (3.40)$$

$$\sin(90^\circ - \theta_c) = 1 - \delta$$

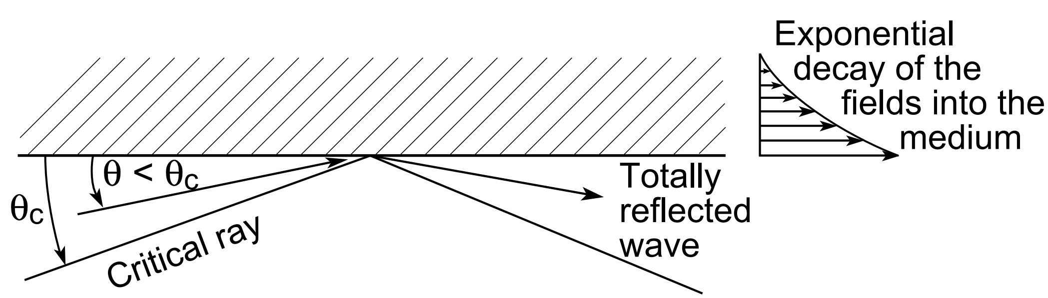
$$\cos\theta_c = 1 - \delta$$

$$1 - \frac{\theta_c^2}{2} + \cdots = 1 - \delta$$

$$\boxed{\theta_c = \sqrt{2\delta}} \qquad (3.41)$$

The critical angle for total external reflection.

Glancing incidence ($\theta < \theta_c$) and total external reflection



Total External Reflection (continued)

$$\boxed{\theta_c = \sqrt{2\delta}} \qquad (3.41)$$

$$\delta = \frac{n_a r_e \lambda^2 f_1^0(\lambda)}{2\pi}$$

$$\theta_c = \sqrt{2\delta} = \sqrt{\frac{n_a r_e \lambda^2 f_1^0(\lambda)}{\pi}} \qquad (3.42a)$$

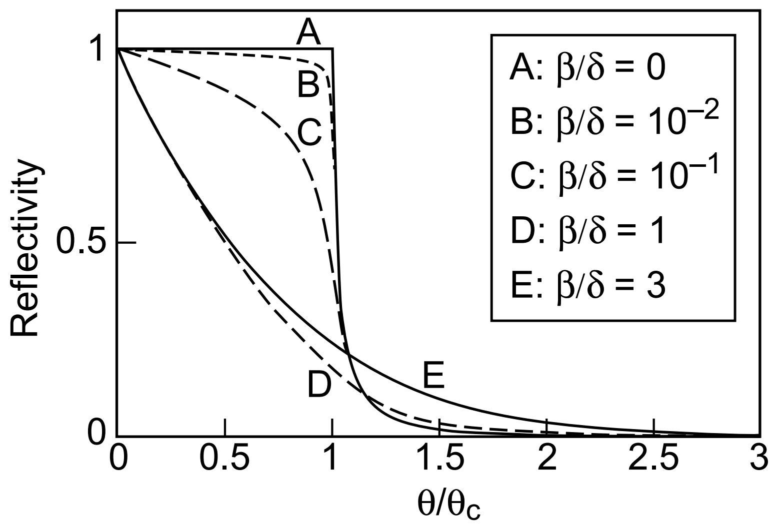
The atomic density n_a, varies slowly among the natural elements, thus to first order

$$\theta_c \propto \lambda\sqrt{Z} \qquad (3.42b)$$

where f_1^0 is approximated by Z. Note that f_1^0 is a complicated function of wavelength (photon energy) for each element.

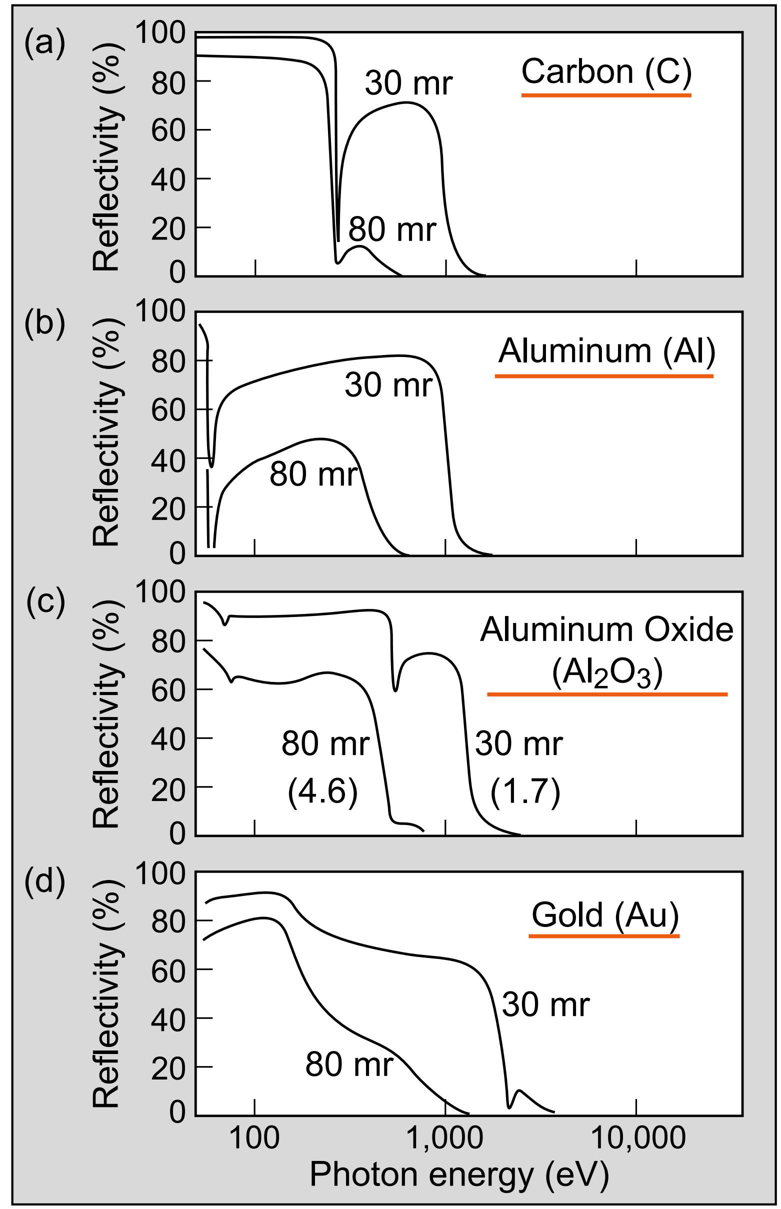
Total External Reflection with Finite b

Glancing incidence reflection as a function of β/δ



- finite β/δ rounds the sharp angular dependence
- cutoff angle and absorption edges can enhance the sharpness
- note the effects of oxide layers and surface contamination

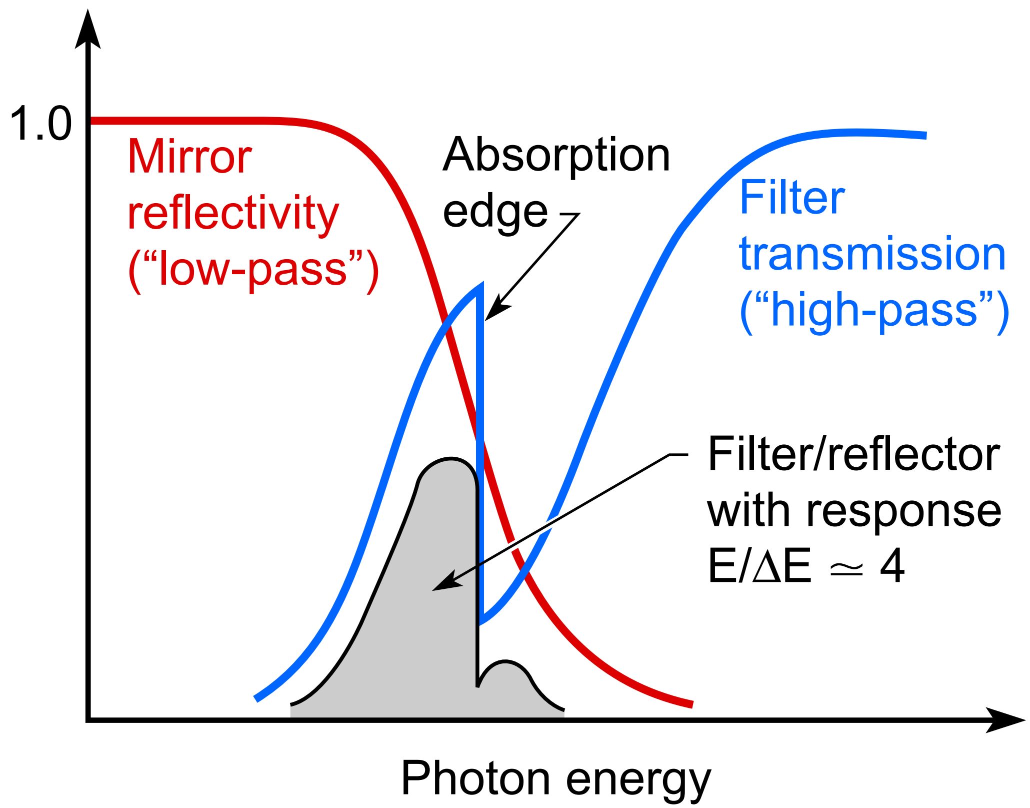
. . . for real materials



(Henke, Gullikson, Davis)

The Notch Filter

- Combines a glancing incidence mirror and a filter
- Modest resolution, $E/\Delta E \sim 3\text{-}5$
- Commonly used



Reflection at an Interface

E_0 perpendicular to the plane of incidence (s-polarization)

tangential electric fields continuous

$$E_0 + E_0'' = E_0' \tag{3.43}$$

tangential magnetic fields continuous

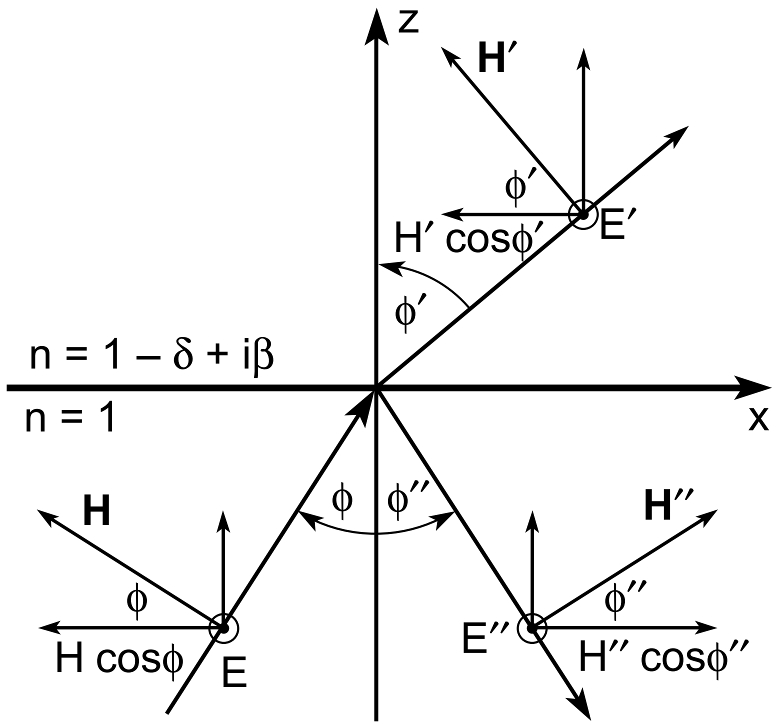
$$H_0 \cos\phi - H_0'' \cos\phi = H_0' \cos\phi' \tag{3.44}$$

$$\mathbf{H}(\mathbf{r}, t) = n\sqrt{\frac{\epsilon_0}{\mu_0}}\mathbf{k}_0 \times \mathbf{E}(\mathbf{r}, t) \;\Rightarrow\; H = n\sqrt{\frac{\epsilon_0}{\mu_0}}E$$

$$\sqrt{\frac{\epsilon_0}{\mu_0}}E_0 \cos\phi - \sqrt{\frac{\epsilon_0}{\mu_0}}E_0'' \cos\phi = n\sqrt{\frac{\epsilon_0}{\mu_0}}E_0' \cos\phi'$$

$$(E_0 - E_0'')\cos\phi = nE_0' \cos\phi' \tag{3.45}$$

Snell's Law: $\sin\phi' = \dfrac{\sin\phi}{n}$



Three equations in three unknowns

(E_0', E_0'', ϕ') (for given E_0 and ϕ)

Reflection at an Interface (continued)

E_0 perpendicular to the plane of incidence (s-polarization)

$$\frac{E_0'}{E_0} = \frac{2\cos\phi}{\cos\phi + \sqrt{n^2 - \sin^2\phi}} \tag{3.47}$$

$$\frac{E_0''}{E_0} = \frac{\cos\phi - \sqrt{n^2 - \sin^2\phi}}{\cos\phi + \sqrt{n^2 - \sin^2\phi}} \tag{3.46}$$

The reflectivity R is then

$$R = \frac{\bar{I}''}{\bar{I}_0} = \frac{|\bar{\mathbf{S}}''|}{|\bar{\mathbf{S}}|} = \frac{\frac{1}{2}\mathrm{Re}(\mathbf{E}_0'' \times \mathbf{H}_0''^{*})}{\frac{1}{2}\mathrm{Re}(\mathbf{E}_0 \times \mathbf{H}_0^{*})} \tag{3.48}$$

With n = 1 for both incident and reflected waves,

$$R = \frac{|E_0''|^2}{|E_0|^2}$$

Which with Eq. (3.46) becomes, for the case of perpendicular (s) polarization

$$R_s = \frac{\left|\cos\phi - \sqrt{n^2 - \sin^2\phi}\right|^2}{\left|\cos\phi + \sqrt{n^2 - \sin^2\phi}\right|^2} \tag{3.49}$$

Normal Incidence Reflection at an Interface

Normal incidence ($\phi = 0$)

$$R_s = \frac{\left|\cos\phi - \sqrt{n^2 - \sin^2\phi}\right|^2}{\left|\cos\phi + \sqrt{n^2 - \sin^2\phi}\right|^2} \qquad (3.49)$$

$$R_{s,\perp} = \frac{|1-n|^2}{|1+n|^2} = \frac{(1-n)(1-n^*)}{(1+n)(1+n^*)}$$

For $n = 1 - \delta + i\beta$

$$R_{s,\perp} = \frac{(\delta - i\beta)(\delta + i\beta)}{(2 - \delta + i\beta)(2 - \delta - i\beta)} = \frac{\delta^2 + \beta^2}{(2-\delta)^2 + \beta^2}$$

Which for $\delta << 1$ and $\beta << 1$ gives the reflectivity for x-ray and EUV radiation at normal incidence ($\phi = 0$) as

$$\boxed{R_{s,\perp} \simeq \frac{\delta^2 + \beta^2}{4}} \qquad (3.50)$$

Example: Nickel @ 300 eV (4.13 nm)
From table C.1, p. 433
$f_1^0 = 17.8$ $\quad f_2^0 = 7.70$
$\delta = 0.0124$ $\quad \beta = 0.00538$

$\Rightarrow R_\perp = 4.58 \times 10^{-5}$

Glancing Incidence Reflection (s-polarization)

$$R_s = \frac{\left|\cos\phi - \sqrt{n^2 - \sin^2\phi}\right|^2}{\left|\cos\phi + \sqrt{n^2 - \sin^2\phi}\right|^2} \qquad (3.49)$$

For $\quad \theta = 90^\circ - \phi \leq \theta_c$

where $\quad \theta_c = \sqrt{2\delta} \ll 1$

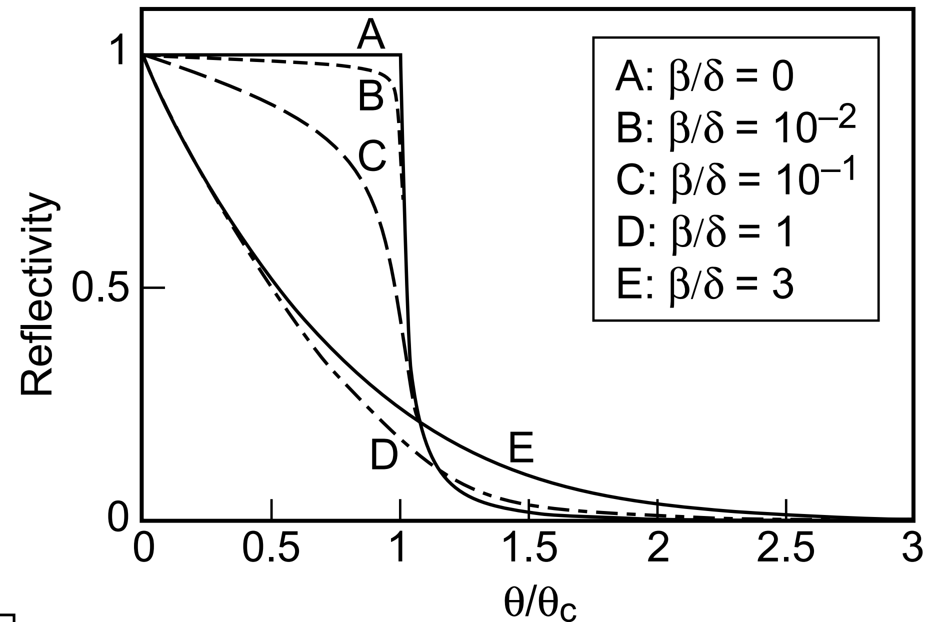
$$\cos\phi = \sin\theta \simeq \theta$$

$$\sin^2\phi = 1 - \cos^2\phi = 1 - \sin^2\theta \simeq 1 - \theta^2$$

For $n = 1 - \delta + i\beta$

$$n^2 = (1-\delta)^2 + 2i\beta(1-\delta) - \beta^2$$

$$R_{s,\theta} = \frac{\left|\theta - \sqrt{(\theta^2 - \theta_c^2) + 2i\beta}\right|^2}{\left|\theta + \sqrt{(\theta^2 - \theta_c^2) + 2i\beta}\right|^2} \qquad (\theta \ll 1)$$



E. Nähring, "Die Totalreflexion der Röntgenstrahlen", Physik. Zeitstr. XXXI, 799 (Sept. 1930).

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Reflection at an Interface

E_0 perpendicular to the plane of incidence (p-polarization)

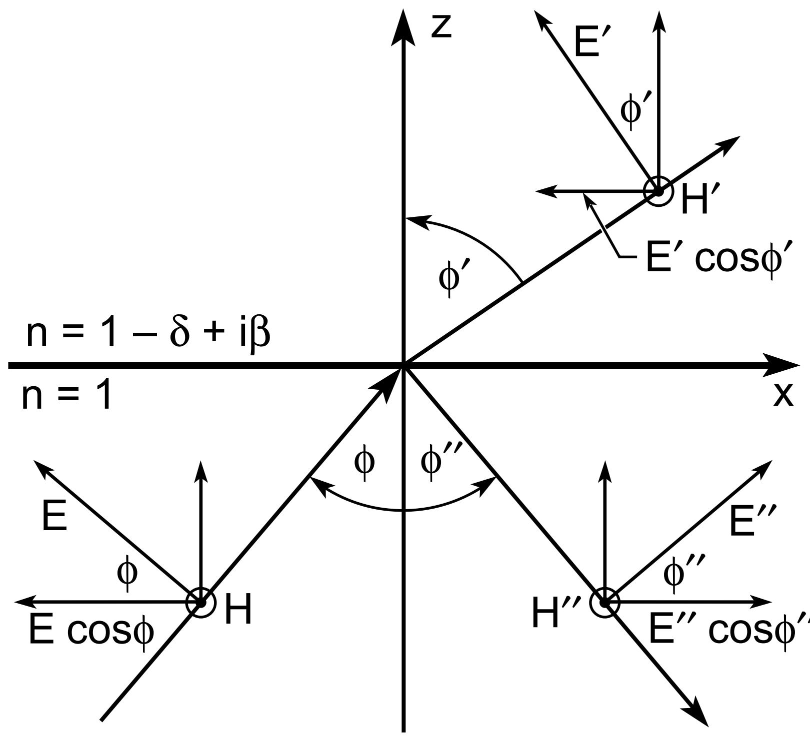
$$\frac{E_0''}{E_0} = \frac{n^2\cos\phi - \sqrt{n^2 - \sin^2\phi}}{n^2\cos\phi + \sqrt{n^2 - \sin^2\phi}} \quad (3.54)$$

$$\frac{E_0'}{E_0} = \frac{2n\cos\phi}{n^2\cos\phi + \sqrt{n^2 - \sin^2\phi}} \quad (3.55)$$

The reflectivity for parallel (p) polarization is

$$R_p = \left|\frac{E_0''}{E_0}\right|^2 = \frac{\left|n^2\cos\phi - \sqrt{n^2 - \sin^2\phi}\right|^2}{\left|n^2\cos\phi + \sqrt{n^2 - \sin^2\phi}\right|^2} \quad (3.56)$$

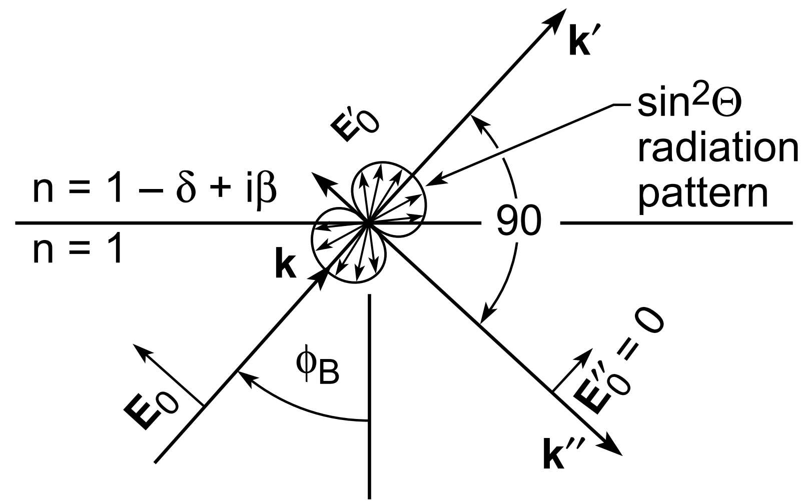
which is similar in form but slightly different from that for s-polarization. For $\phi = 0$ (normal incidence) the results are identical.



Brewster's Angle for X-Rays and EUV

For p-polarization

$$R_p = \left|\frac{E_0''}{E_0}\right|^2 = \frac{\left|n^2\cos\phi - \sqrt{n^2-\sin^2\phi}\right|^2}{\left|n^2\cos\phi + \sqrt{n^2-\sin^2\phi}\right|^2} \qquad (3.56)$$



There is a minimum in the reflectivity where the numerator satisfies

$$n^2\cos\phi_B = \sqrt{n^2-\sin^2\phi_B} \qquad (3.58)$$

Squaring both sides, collecting like terms involving ϕ_B, and factoring, one has

$$n^2(n^2-1) = (n^4-1)\sin^2\phi_B$$

or

$$\sin\phi_B = \frac{n}{\sqrt{n^2+1}}$$

the condition for a minimum in the reflectivity, for parallel polarized radiation, occurs at an angle given by

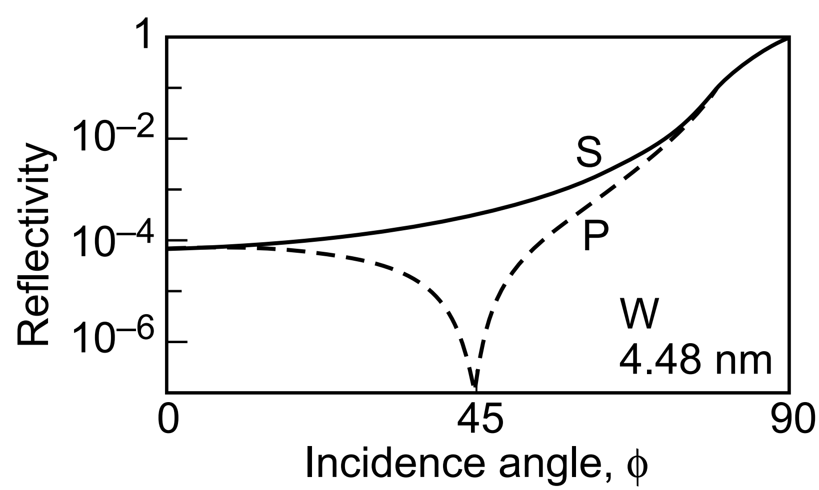
$$\tan\phi_B = n \qquad (3.59)$$

For complex n, Brewster's minimum occurs at

$$\tan\phi_B = 1-\delta$$

or

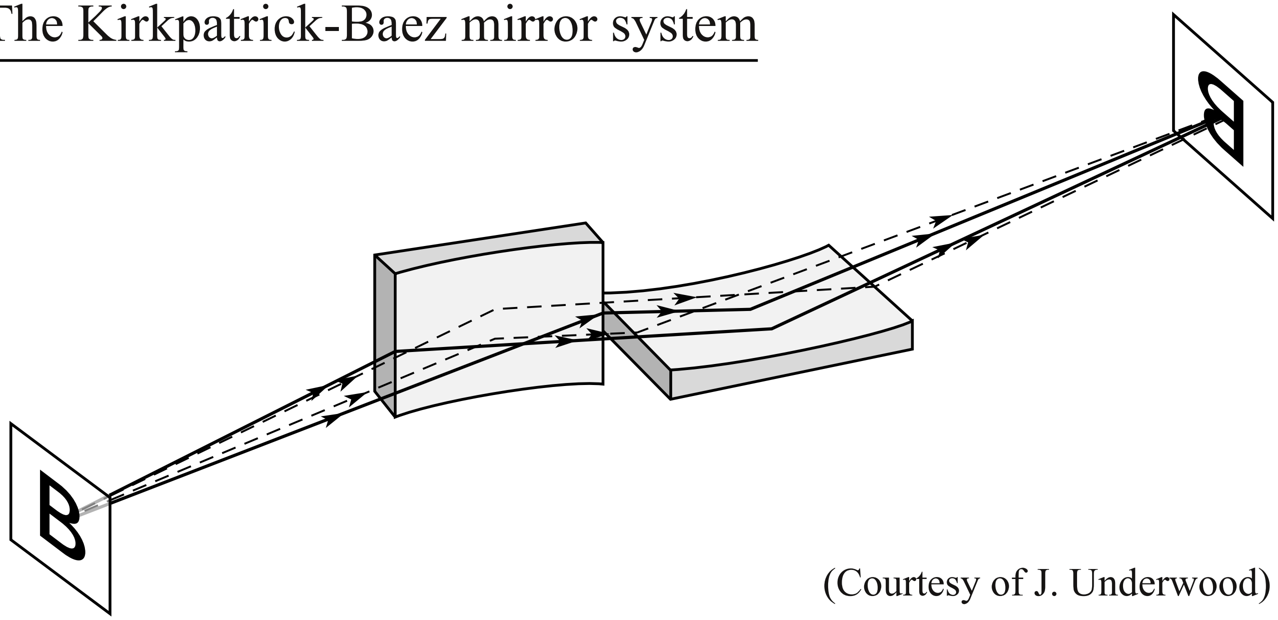
$$\boxed{\phi_B \simeq \frac{\pi}{4} - \frac{\delta}{2}} \qquad (3.60)$$



(Courtesy of J. Underwood)

Focusing with Curved, Glancing Incidence Optics

The Kirkpatrick-Baez mirror system



(Courtesy of J. Underwood)

- Two crossed cylinders (or spheres)
- Astigmatism cancels
- Fusion diagnostics
- Common use in synchrotron radiation beamlines
- See hard x-ray microprobe, chapter 4, figure 4.14

Determining f_1^0 and f_2^0

- f_2^0 easily measured by absorption
- f_1^0 difficult in SXR/EUV region
- Common to use Kramers-Kronig relations

$$f_1^0(\omega) = Z - \frac{2}{\pi}\mathcal{P}_C \int_0^\infty \frac{u f_2^0(u)}{u^2 - \omega^2}\, du \qquad (3.85a)$$

$$f_2^0 = \frac{2\omega}{\pi}\mathcal{P}_C \int_0^\infty \frac{f_1^0(u) - Z}{u^2 - \omega^2}\, du \qquad (3.85b)$$

as in the Henke & Gullikson tables (pp. 428-436)

- Possible to use reflection from clean surfaces; Soufli & Gullikson
- With diffractive beam splitter can use a phase-shifting interferometer; Chang et al.
- Bi-mirror technique of Joyeux, Polack and Phalippou (Orsay, France)