Solutions to LESSON 3 Self-Study Problems

1) First note that

$$E - \Gamma = \begin{bmatrix} -\gamma_1 & e_{12} & e_{13} \\ e_{21} & -\gamma_2 & 0 \\ e_{31} & 0 & -\gamma_3 \end{bmatrix}$$

Next take

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{e_{12}}{e_{21}} & 0 \\ 0 & 0 & -\frac{e_{13}}{e_{31}} \end{bmatrix}$$

whose diagonal entries are positive because

$e_{12} < 0 \quad e_{13} < 0 \quad e_{21} > 0 \quad e_{31} > 0$

It follows that

$$P(E-\Gamma) + (E-\Gamma)^\top P = \begin{bmatrix} -2\gamma_1 & 0 & 0 \\ 0 & 2\frac{e_{12}}{e_{21}}\gamma_2 & 0 \\ 0 & 0 & 2\frac{e_{13}}{e_{31}}\gamma_3 \end{bmatrix}$$

whose diagonal entries are nonpositive because

$\gamma_i \geq 0, i = 1, 2, 3$

2) a) Take the subsystems to be

$\dot{x}_i = f(x_i) + u_i, \; y_i = h(x_i) \; i = 1, 2, 3$

which recover the system model when interconnected according to

$u_1 = -y_3, u_2 = -y_1, u_3 = -y_2$

b) As we saw in Lesson 3, if

$f + \varepsilon h$

Is nonincreasing, then each subsystem is equilibrium independent passive with

$$X = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -\varepsilon \end{bmatrix}$$

Then the secant condition for stability of cyclic interconnections dictates

$\varepsilon^{-3} \leq \sec(\pi/3)^3$

That is,

$\varepsilon \geq \cos(\pi/3) = 0.5$

Thus, the condition on f, h is that

$f + 0.5h$

be a nonincreasing function.