Solutions to LESSON 2 Self-Study Problems

1) The nominal part $M$ maps $[w, r]^\top$ to $[v, y]^\top$ by:

$$M = \begin{bmatrix} -\frac{GC}{1+GC} & \frac{C}{1+GC} \\ \frac{G}{1+GC} & \frac{GC}{1+GC} \end{bmatrix}$$

2) The general formula for $F_U(M, \Delta)$ is

$$F_U(M, \Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$$

Substitute $M_{11} = A, M_{12} = B, M_{21} = C, M_{22} = D$, and $\Delta = \frac{1}{s}$ to show that

$$F_U(M, \Delta) = C(sI - A)^{-1}B + D = G(s)$$

3) The weight is $W(s) = \frac{2s+0.1}{s+1}$. This has a DC gain of $W(0) = 0.1$ implying $|\Delta(0)| \leq 0.1$. As $\omega \to \infty$, we have $|W(j\omega)| \to 2$. This represents larger uncertainty at higher frequencies with a bound of 2.

4) False. The IQC with $J = diag(1, -1)$ and $\Psi = I$ implies that $||w||_2 \leq ||v||_2$ for all $v$ $L_2$, $w = \Delta(v)$. The system $-\Delta$ satisfies the same IQC with $J = diag(1, -1)$ and $\Psi = I$. However, $-\Delta$ does not satisfy the IQC with $J = diag(-1,1)$ and $\Psi = I$, in general.

4) True. This can be verified directly from the definition and the given assumptions.