International Graduate School on Control: Lesson 6

1. For this problem you can use the MATLAB file EDM.m or Python code EDM.py (see also Jupyter Notebook EDM.ipynb).

   a) Simulate the trajectories of the Rock Paper Scissors game:

$$F(x) = \begin{bmatrix} 0 & -l & w \\ w & 0 & -l \\ -l & w & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

   with equal weights $w = l = 1$, using the following learning rules: i) Imitation, ii) Brown-von Neumann-Nash, iii) MJ Smith.

   b) Repeat part (a) with unequal weights, for both cases $w > l$ and $w < l$.

2. The Heavy-ball algorithm is

$$x_{k+1} = x_k - \alpha \nabla f(x_k) + \beta(x_k - x_{k-1}),$$

   Consider the case where the step-size parameters are chosen as follows:

$$\alpha_0 := \frac{4}{\left(\sqrt{L} + \sqrt{m}\right)^2} \text{ and } \beta_0 := \left(\frac{\sqrt{\kappa} - 1}{\sqrt{\kappa} + 1}\right)^2,$$

   Analyze the convergence rate ρ for functions in $S(m, L)$ using the SDP condition presented in Lesson 6. Specifically, generate a plot of convergence rate vs. condition ratio $\kappa := \frac{L}{m}$ using both the sector QC and the off-by-1 ρ-Hard IQC from Lessard, Packard, Recht. Your results should match the black curve (labelled "LMI: Weighted Off-by-1") in Figure 1.

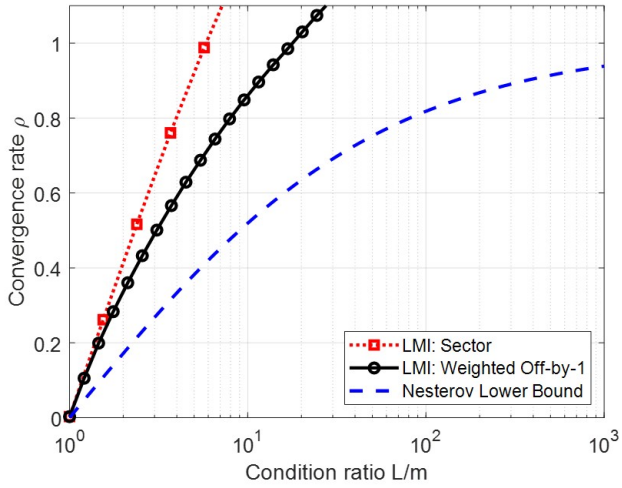

Figure 1: Heavy-ball convergence rate vs. condition ratio.