Self-Study Problems

1) **Antelopes, hyenas, and lions:**

   Consider the Lotka-Volterra model for three species, where species 2 and 3 both prey on species 1:

   $$e_{12} < 0 \quad e_{13} < 0 \quad e_{21} > 0 \quad e_{31} > 0$$

   but they are neutral to each other:

   $$e_{23} = e_{32} = 0$$

   Recall also that the diagonal entries of E are zero. Investigate whether a diagonal $P \succ 0$ exists such that

   $$P(E - \Gamma) + (E - \Gamma)^\top P \preceq 0$$

   where $\Gamma \succeq 0$ is a diagonal matrix of parameters appearing in the model. Your answer should not depend on specific values of E and Γ, but only their sparsity and sign structure.

Self-Study Problems

2) A cyclic interconnection

Consider the system

$$\dot{x}_1 = f(x_1) - h(x_3)$$
$$\dot{x}_2 = f(x_2) - h(x_1)$$
$$\dot{x}_2 = f(x_2) - h(x_2)$$

where f is a strictly decreasing function and h is an increasing function. If f is onto, there exists a unique equilibrium.

a) Decompose this system into a cyclic interconnection of three first order subsystems.

b) Provide a condition on f and h such that the equilibrium is guaranteed to be stable (without knowledge of where the equilibrium is).