

## Lecture 4: Motion Planning - Part III

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## 4.1 Sampling-Based Planners

Idea: Represent  $C_{\text{free}}$  via a sampled “road map.”

### 4.1.1 Probabilistic Road Maps (PRM) [Kavraki '96]

Given:

- A collision checker  $\gamma$  such that

$$\gamma(q) = \begin{cases} 1 & \text{if } q \in C_{\text{obs}} \\ 0 & \text{else} \end{cases}$$

- A simple planner  $B_s$  such that

$$B_s(q_1, q_2) \rightarrow \begin{cases} \text{A path if it finds one quickly} \\ \text{Failure otherwise} \end{cases}$$

Note that  $B_s$  does not necessarily return a path if one exists. An example of  $B_s$  is collision-checking the straight line connecting  $q_1$  and  $q_2$ .

Algorithm:

1. Start with the set  $\{q_s, q_g\}$ .
2. Sample  $M$  milestones in  $C_{\text{free}}$  using rejection sampling to avoid points in  $C_{\text{obs}}$ .
3. Try to connect all milestones using  $B_s$ . Three such options for doing so are:
  - (a) Connect all pairs.
  - (b) Connect everything in an  $R$ -disc, connecting configurations within a distance of  $R$  (S-PRM).
  - (c) Connect the  $k$ -nearest neighbors of each configuration (k-PRM).
4. If  $q_s$  and  $q_g$  are in the same connected component, find the path using graph search. Otherwise, return to step 2 and sample more milestones.

See Figure 4.1 for a visual example of PRM.

Note: The original PRM algorithm was meant to be a multi-query planner, thus able to deal with multiple queries  $\{(q_s, q_g)^{(i)}\}_{i=1}^N$  rather than a single query  $(q_s, q_g)$ . Since  $C_{\text{obs}}$  usually remains the same, we can build a road map during pre-processing.

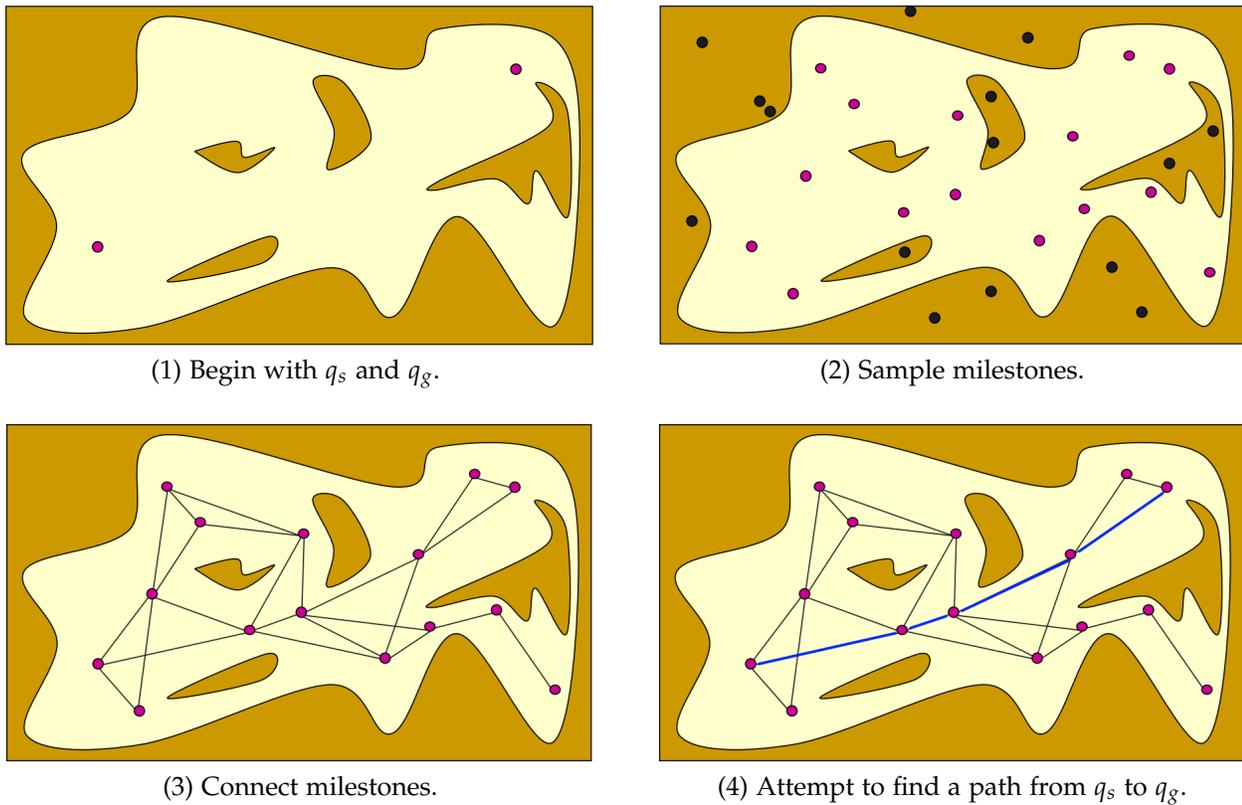


Figure 4.1: The PRM algorithm in action. The yellow region represents  $C_{\text{free}}$  while the tan region represents  $C_{\text{obs}}$ .

### 4.1.2 Rapidly-Exploring Random Trees (RRT) [Lavelle and Kuffner '01]

From a PRM, we can develop the Bi-directional Rapidly-Exploring Random Tree (Bi-RRT) using three ideas:

**Idea 1:**  $M = 1$ . Stop when  $q_s$  and  $q_g$  are in the same connected component.

**Idea 2:** Keep track of  $CC_s$  and  $CC_g$ , the connected components that contain  $q_s$  and  $q_g$ .

**Idea 3:** 1-PRM (a k-PRM with  $k=1$ ). Connect the newest milestone to the closest point in  $CC_s$  and  $CC_g$ .

#### 4.1.2.1 Pros and Cons

- (+) Fast and scalable. Works well even with high DOF.
- (+) Does not require overly restrictive assumptions.
- (+) Probabilistically complete (i.e.  $\lim_{n \rightarrow \infty} P(\text{sol} | \exists \text{sol}) = 1$ ). This is a weak pro as even a planner that randomly samples paths would be probabilistically complete.
- (-) Not optimal.
- (-) Requires a lot of post-processing.

### 4.1.3 Shortcutting

As a post-processing step, randomly sample two points to see if they can be connected. Skipping intermediate vertices can help smooth out the path.

### 4.1.4 Kinodynamic Planning

Kinodynamic motion planning is the problem of finding the optimal path for a robot with motion constraints such as velocities and accelerations from a starting configuration to a goal configuration. In addition to avoiding collisions, a kinodynamic RRT must also satisfy local differential constraints.

When planning in a control space instead of a configuration space, we can attempt multiple sequences of controls and keep the trajectory that results in the closest point to the goal. This allows us to avoid explicitly solving the 2-point boundary value problem. Note that we don't directly sample controls because that would lead to "hairballing."