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EE229B Project

Peak-to-Average Power Control in OFDM Systems

I. Introduction

High Peak-to-Average Power Ratio (PAPR) has been recognized as one of the major practical problem involving OFDM modulation. High PAPR results from the nature of the modulation itself where multiple subcarriers/sinusoids are added together to form the signal to be transmitted. When N sinusoids add, the peak magnitude would have a value of N, where the (rms) average might be quite low due to destructive interference between the sinusoids. High PAPR signals are usually undesirable for it usually strains the analog circuitry. High PAPR signals would require a large range of dynamic linearity from the analog circuits, which usually results in expensive devices, and higher power consumption/lower efficiency (for example, power amplifier has to operate with larger backoff to maintain linearity).

It's quite straightforward to observe that in an OFDM systems, some input sequences would result in higher PAPR than others. For example, an input sequence that requires *all* subcarriers to transmit their maximum amplitudes would certainly result in a high output PAPR. Thus by limiting the possible input sequences to a smallest subset, it should be possible to obtain output signals with a guaranteed low output PAPR. At the same time, by limiting input sequences to a smaller subset we also provide the system with more resiliency in the presence of noise – this idea is central in the area of error-control codes. Thus the question is can we set up a framework such that we can jointly optimize the selection of codewords based on *both* their PAPR and minimum distance performance.

In this project we will conduct a literature survey of the various techniques proposed to establish PAPR control in OFDM modulation. The emphasis will certainly be on those techniques which uses codes designed to have both low PAPR along with good error-correction capabilities. We will also briefly mention other methods that have been proposed in the literature. These methods also indirectly take advantage of the redundancy afforded by error-control coding performed on the input data sequences – in that the PAPR reduction system is allowed to alter the transmitted signal a little bit (introduce bit error) with the hope of optimizing for PAPR.

In most of the main results presented here, the goal is to convey the main fundamental ideas and intuitive understanding of the concept introduced using simple examples, rather than dwell into the details of the more involved results. This is done primarily to minimize the amount of terminology and notation that have to be introduced in this short report but also to give a complete overview of the techniques known today for PAPR reduction.

II. OFDM Modulation and Notation

It is useful to briefly review the OFDM modulation as a way to set up notation for the succeeding sections.

For an n-subcarrier OFDM signal with carrier frequencies $f_o + j.f_s$, with $0 \le j < n$, we have n carrier sinusoids of the form $e^{2\pi i(f_o + jf_s)t}$. Suppose at a particular symbol period, a length-n information vector $m = [m_0 m_1 .....m_{n-1}]$ is to be transmitted. Then the resulting OFDM modulated signal can be written as

$$S_{(m)}(t) = \sum_{j=0}^{n-1} m_j e^{2\pi i(f_o + jf_s)t}$$

Each of the message symbol m_j can be a general complex number. However in all the papers reviewed here a PSK modulation is assumed. This is done to ease the mathematical formulation of the problem and also the derivation of the results, as in PSK modulation all the subcarriers would have average envelope power of n.

The instantaneous power of the signal $S_{(m)}(t)$ can be written as :

$$P_{(m)}(t) = \left|\Re(S_{(m)}(t))\right|^2$$

Whereas the *envelope* power is defined as :

$$P_{env(m)}(t) = \left|S_{(m)}(t)\right|^2$$

We can easily see that

$$P_{(m)}(t) \le P_{env(m)}(t)$$

The Peak-to-Average Power Ratio (PAPR) can be defined as :

$$PAPR(m) = \frac{1}{n} \sup_{0 \le f_s t < 1} P_{(m)}(t)$$

Whereas the Peak-to-Mean Envelope Power Ratio (PMEPR) can be defined as :

$$PMEPR(m) = \frac{1}{n} \sup_{0 \le f_s t < 1} P_{env(m)}(t)$$

In the literature the term PMEPR and PAPR is sometimes used interchangeably. It is more convenient to work with PMEPR than with PAPR, thus in most of the theoretical literature PMEPR is used. And since PMEPR upper-bounded PAPR, the use of PMEPR can be well justified.

If error-control coding is to be used, the length n vector $\mathbf{c}$ from a code $\mathcal{C}$ is directly modulated - one coordinate of the vector $\mathbf{c}$, to one subcarrier in the OFDM modulation

III. Practical Methods of PAPR Reduction in the literature

Several approaches to reduce PAPR have been suggested in the literature. The simplest, and somewhat crude approach involves clipping the signal. High signal peaks, although possible, has a relatively low probability of occurring. As such, clipping these few peaks would not significantly impair the performance (in terms of BER) of the system. However clipping is a highly nonlinear operation and it has an adverse impact towards the spectral purity of this signal (spectral regrowth problem). For that reason this approach is usually not suitable for wireless application where spectral emission is tightly controlled. The paper by Li and Cimini [1] is just one example of such approach. In this paper the spectral regrowth problem is somewhat alleviated by a post-filtering operation to remove the out-of-band harmonics caused by the clipping operation. However by filtering the clipped signal, there is a possibility of peaks regrowing in the time domain. This fact really underlies the fundamental problem of signal clipping approach in PAPR reduction – it is still a relatively zero-sum game between achieving good spectral purity and low PAPR.

The method of tone injection reserves a tone (in the OFDM modulation) to be injected to the system to cancel out high peaks. The difficult part in this method is how to select the optimal input sequences to be modulated on this reserved tone to maximally reduce the PAPR. Tellado and Cioffi[10] proposed this method and uses LP optimization to get the optimal input sequence.

While both approaches mentioned above are viable options for systems, it does still incur some costs. Clipping would inevitably introduce error, effectively reducing the resiliency of the system in the presence of noise. In the tone injection method, a dedicated tone has to be reserved. In both these approaches, ultimately PAPR reduction would reduce the capacity of the system.

One PAPR reduction scheme that does not constitute a constant reduction of rate is the phase-shift method. The idea is quite simple, it is based on the observations that peaks in OFDM systems occur because of the particular phase alignment between the subcarriers. Thus there is a potential that by introducing a constant phase shifts on each of the subcarriers, a reduction in PAPR can be achieved. This constant phase shifts can be transmitted as a one time side information message – a rate reduction of which can then be amortized over many information blocks. The original idea was proposed by Eetvelt, et al. [8], in which a selected scrambling function is used to generate the phase shifts with PAPR optimization in mind. Although a reduction in PAPR was observed in this approach, no theoretical intuition was developed as to how to optimally introduce the right phase shifts on each of the subcarriers to gain the maximum reduction in PAPR. In general the relation between the phases of the subcarriers and the resulting PAPR is quite complicated – thus making optimization on a system with large number of subcarriers n very difficult.

Tarokh and Jafarkhani in [9] formalizes the problem of phase optimization, and established a sound optimization framework. They formulated the problem as follow : Given a code $\mathcal{C}$, select a constant phase shift vector $\Phi = [\phi_0 \phi_1 .... \phi_{n-1}]$ to be applied to the (n-1) subcarriers so as to give rise to a minimum average output PAPR under OFDM modulation *if* all codewords $\mathbf{c}$ in $\mathcal{C}$ is equally likely. We reproduce some of the interesting result from that paper below.

Instead of defining a PAPR for a particular message vector, we can define PAPR for a particular code as follows :

$$PAPR(\mathcal{C}) = \max_{c \in C} PAPR(\mathbf{c})$$

Or simply the PAPR of the code is the maximum PAPR over all possible codewords in the code.

We can also take an alternative definition for $PAPR(\mathcal{C})$ as follows.
Define a function $g(t,\mathcal{C})$ as follows :

$$g(t,\mathcal{C}) = \max_{c \in C} \frac{1}{n} \left| \Re\left(\sum_{j=0}^{n-1} c_j e^{2\pi i (f_o + j f_s) t} \right) \right|^2$$

that is $g(t,\mathcal{C})$ is the maximum instantaneous PAPR at time t by selection of a codeword $\mathbf{c}$ from all the possible codewords in $\mathcal{C}$. Thus $PAPR(\mathcal{C})$ can be defined equivalently as

$$PAPR(\mathcal{C}) = \sup_{0 \le f_s t < 1} g(t,\mathcal{C})$$

The two definition is really exactly the same. But by taking this alternative view of PAPR definition, we can then reproduce the main result in [9].

Theorem 1 : *Define the vector* $\Omega(t)$ *as follows:*

$$\Omega(t) = \left[e^{2\pi i (f_o) t} e^{2\pi i (f_o + f_s) t} .....e^{2\pi i (f_o + (n-1) f_s) t} \right]$$

We can also define $-\Omega(t)$ *similarly as a vector whose* j^{th} *component is* $-e^{2\pi i (f_o + j f_s) t}$. *Then* $g(t,\mathcal{C})$ *is attained by the codeword* $\mathbf{c} \in \mathcal{C}$, *which is closest to either* $\Omega(t)$ *or* $-\Omega(t)$

This is quite a powerful result and tool in the development of good codes with low PAPR as it gives a rather geometric view on the relation between a particular code $\mathcal{C}$ and the resulting PAPR under OFDM modulation. This result is also used in [4] to prove achievable regions of the triples (R,d, PAPR($\mathcal{C}$)). Furthermore by establishing a relation between a particular codeword and modulated output with the measure of distance as a describing function, this result can be further strengthened to state that picking the (PAPR) optimal phase shifts vector $\Phi = [\phi_0 \phi_1 .... \phi_{n-1}]$ is equivalent to a minimum distance decoding of codes. A large remainder of the paper is dedicated to establish the formalism to justify this claim, for the most part it is not theoretically interesting nor related to our survey, as it simply establishes an efficient algorithm to optimize the vector Φ.

IV. Overview of Error-Control Codes with low PAPR in OFDM systems

The idea of combining error-correction capabilities while optimizing for output PAPR at the same time was first published in a short paper by Wilkinson and Jones [2], [3]. However in both papers no clear method of designing the codes were suggested. Instead an exhaustive search was performed to obtain the best codes. This exhaustive search limits the viability of the approach to only a set of short codes. And moreover due to the nature of the exhaustive search, the resulting codes do not posses a good structure and thus are rather hard to encode and decode. Wulich further proposed a ¾ cyclic code which results in 3 dB PAPR reduction. However upon further examination this cyclic coding scheme is no more than a tone injection scheme, with marginal and non-scalable error-correction capabilities. Paterson and Tarokh [4] managed to derive some bounds on the achievable triples (R,d, PAPR($\mathcal{C}$)), where R is the rate, d is the minimum Euclidian distance between codewords. They however failed to describe any practical construction method that achieve good asymptotic performance.

In 1999 Davis and Jedwab [6] published a paper which for the first time managed to build on previous results and came up with a systematic method of constructing codes with low and bounded PAPR while offering a guaranteed and scalable error-correction capabilities. This paper is then followed by a generalizing work by Peterson, which also uses a similar approach to constructing codes with good PAPR as well as error-correction capabilities. Both papers construct codes from Golay complementary sequences which can be easily shown to have a low peak-to-average ratio. As it turned out Reed-Muller codes can be subdivided in cosets in which all codewords in that particular coset are Golay complimentary sequences. These two papers presented what perhaps is currently the most interesting and theoretically sound approach to systematically constructing codes with good PAPR and error-correction capability.

V. Achievable region of triples (R,d,PAPR($\mathcal{C}$))

Paterson and Tarokh in [4] derived achievable region for the triples (R,d,PAPR) for a particular code. The main result of that paper is presented below :

Lemma 2: *Let $\mathcal{C}$ of length n, rate R and minimum Euclidian distance d. Let d_* denote the minimum Euclidian distance between the codewords of C and the points of* $\Omega(t) \cap -\Omega(t)$ *for $0 \leq f_s t < 1$. Then $d_* \leq \sqrt{2n}$ and PAPR($\mathcal{C}$)$\geq n(1-\delta)^2$ where $\delta = d_*^2 / \sqrt{2n}$*

As mentioned before the derivation of the above lemma is based on the geometric view first introduced in [9]. Although the bound above is rather vague for practical calculations of code performance, it states mathematically a fact that is hardly surprising. It states that there is at tradeoff between rate, minimum Euclidian distance in a code and the resulting PAPR of the code. Intuitively speaking by enforcing a bound on output PAPR, we already inherently reduced the ‘volume’ of the subspace for possible codewords, so it’s not surprising that from this limited subspace the minimum Euclidian distance between possible codewords can only be smaller.

Thus the next interesting question to ask is given a set of rate, and minimum Euclidian distance requirement, what is the achievable PAPR. This question is answered in [4] through the following theorem :

Theorem 3 : *Let R≥0 and Δ≥0 such that* $2^R\sqrt{2\Delta(1-\frac{\Delta}{2})}<1$*, then for all sufficiently large n, there exists a code* $\mathcal{C}$ *of length n, rate R and minimum Euclidian distance d =* $\sqrt{2\Delta n}$ *with PAPR(*$\mathcal{C}$*)≤8.log n*

The above theorems are the only bound on the achievable region on the triple (R,d,PAPR($\mathcal{C}$)) known today.

VI. Golay Complimentary Sequences and Reed-Muller Codes

In this section we will concentrate on two papers by Jones and Wilkinson [3] and another by Paterson [4]. Both papers are very tightly related, with the original idea proposed by Jones and Wilkinson, and then further generalized by Paterson.

In a nutshell both papers construct codes using a particular conditioning (Golay complimentary sequence) to get a subspace containing only codewords with low PAPR under OFDM modulation. It turns out that this conditioning shares many of the structures and properties of Reed-Muller codes. The key contribution is that they establish this crucial link between the conditioning for low PAPR via Golay complimentary sequences, and the structure of Reed Muller codes – thus effectively giving them a powerful tool to build on the foundation of Reed-Muller code to generate code with good error correction capabilities and low PAPR.

To start let's define what we mean by Golay complementary pair.
Let **a** = $[a_0a_1\ldots a_{n-1}]$ and **b** = $[b_0b_1\ldots b_{n-1}]$ be two length n complex vectors. The function C(**a**,**b**)(ℓ) as the cross correlation between the two vectors at time ℓ, i.e. ;

$$C(\mathbf{a},\mathbf{b})(\ell) = \sum a_{i+\ell}b^*_{\ell}$$

Where the summation is defined over the appropriate range covering both vectors **a** and **b**. We also define an autocorrelation function A(**a**)(ℓ) of a vector **a** simply as C(**a**,**a**)(ℓ) or as correlation of the vector with itself. Both definition of correlation and autocorrelation applies to complex vectors and vectors over GF(q).

Definition 1 : *A set of N length n vectors* $\mathbf{a}^0, \mathbf{a}^1, \ldots \mathbf{a}^{N-1}$ *is said to be a* <u>*Golay complimentary set*</u> *if :*

$$A(\mathbf{a}^0)(\ell) + A(\mathbf{a}^1)(\ell) + \ldots + A(\mathbf{a}^{N-1})(\ell) = 0 \qquad \text{for } \ell \neq 0$$

A Golay complimentary set of size N=2 is called a Golay Complimentary Pair. A sequence in any such pair is called a Golay Complimentary Sequence.

It can be easily shown that the instantaneous envelope power of an OFDM signal $S_{(a)}(t)$ can be defined using the notation of autocorrelation as

$$P_{env(a)}(t) = \left|S_{(a)}(t)\right|^2 = \sum_{\ell=1-n}^{n-1} A(a)(\ell)e^{2\pi i \ell f_s t}$$

$$= A(a)(0) + 2.\Re\left(\sum_{\ell=1}^{n-1} A(a)(\ell)e^{2\pi i \ell f_s t}\right)$$

The main connection between the Golay complementary pair and PAPR can be set forth by the following :

$$P_{env(a\ )}(t) + P_{env(a^1)}(t) = \sum_{\ell=1}^{n-1}\left[A(a^0)(\ell) + A(a^1)(\ell)\right]e^{2\pi i \ell f_s t}$$

$$= A(a^0)(0) + A(a^1)(0) = 2n$$

The above equation states that if you have a Golay complimentary pair of length-n, then the sum of the envelope power of the two modulated sequences will be equal to 2n. As a consequence we have a bound on the envelope power of a *single* Golay complimentary sequence, i.e. $P_{env(ao)} \leq 2n$. We can make similar generalization for a Golay complimentary set to come up with the following result

Corollary 4 : If a length-n sequence a^0 is part of a size N Golay complimentary set, it's envelope power is $\leq$ Nn

The following two theorems then established the connection between Golay complimentary sequences with RM codes.

Theorem 5 : *For any permutation π of the symbols {1,2,...,m} and for any c, $c_k \in GF(2)$, the sequence denoted by the Boolean function a :*

$$a(x_1, x_2, ..., x_m) = \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} c_k x_k + c$$

is a Golay sequence over GF(2) of length 2^m.

From the theorem above it's quite easy to make the connection to the following corolary

Corollary 6 : *Each of the m!/2 cosets of RM(1,m) in RM(2,m) having a coset representative of the form*

$$\sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$$

Comprises of 2^{m+1} binary Golay sequences of length 2^m, where π is a permutation of the symbols {1,2,...,m}

There's a lot of notation needed to be set up in the two papers to even state to the results above. In this survey paper, we'll try to bypass the notation through some examples below. Let x_i be a monomials as used in defining RM codes, i.e. for an RM(4,1) code, $x_1 \ldots x_4$ is as follows :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

π is a particular permutation of the m x's. Suppose m=4 as in the RM(1,4) code, we can take one possible permutation of the set {1,2,3,4} as {3,2,4,1}, then we can write theorem 5 specific to this example as :

$$a(x_1, x_2, x_3, x_4) = x_3 x_2 + x_2 x_4 + x_4 x_1 + \sum_{k=1}^{4} c_k x_k + c$$

Theorem 5 states that all sequences of the form above *are* Golay complimentary sequences. Of course the above is only one possible realization of the permutation.

The proof of the above results are actually quite simple (as first published by Davis and Jedwab), it simply involves mathematical manipulation of the stated claim and using the definition of Golay complimentary sequences.

The results above are actually the most basic result presented in [6]. It can be generalized to a larger set of codes. This is referred as generalized Reed Muller codes in the paper denoted $RM_2^h(r,m)$ and $ZRM_2^h(r,m)$ - where $RM_2^h(r,m)$ is a linear code over $GF(2^h)$ with generator matrix from the RM(r,m) code. $ZRM_2^h(r,m)$ is also a linear code over $GF(2^h)$ with a generator matrix *similar* to RM(r,m) except that the higher order monomials are multiplied by a factor 2^{h-1}. Thus for example the code $ZRM_4(2,4)$ is a linear code over GF(4) with generator matrix :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{matrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ 2x_1x_2 \\ 2x_1x_3 \\ 2x_1x_4 \\ 2x_2x_3 \\ 2x_2x_4 \\ 2x_3x_4 \end{matrix}$$

The results of theorem 5 and corollary 6 can be generalized for the above codes. We state the following result as an extension of corollary 6 :

Theorem 7 : *Each of the m!/2 cosets of $RM_2^h(1,m)$ in $ZRM_2^h(2,m)$ having a coset representative of the form*

$$2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$$

Comprises of $2^{h(m+1)}$ binary Golay sequences of length 2^m over $GF(2^h)$, where π is a permutation of the symbols {1,2,…,m}, h>1

The distance properties of this code is also proven in the paper :

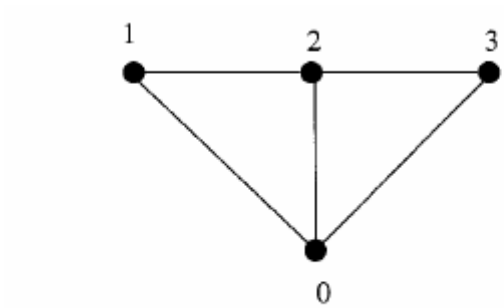
Theorem 8 : *For 0≤r≤m, the following is true :*

	$RM_{2^h}(r, m)$ $(h \geq 1)$	$ZRM_{2^h}(r, m)$ $(h > 1)$
minimum Hamming distance	2^{m-r}	2^{m-r}
minimum Lee distance	2^{m-r}	2^{m-r+1}

The combination of corollary 4, theorem 7 and theorem 8 gives the complete picture on the performance of these generalized RM code in terms of both their error-correction capabilities as well as their PAPR performance (stated as envelope power). The paper by Davis and Jedwab went further to describe a practical encoding method in which the transmitted codewords are from the $RM_2^h(1,m)$ coset with the coset representative as a side information sent to the receiver.

For the most part the paper by Paterson gives the same result with different proofs and notation - which can be somewhat more intuitive. Paterson put the result in theorem 5 in the following form : $f = Q + \sum_{i=1}^{m-1} g_i x_i + g'$, where Q is a quadratic function in $x_1, x_2, \ldots, x_m$.

Paterson describes the resulting Golay sequences as an evaluation of the function *f* if $x_i = c$ with some *c* in GF(q). This function can also be described graphically, in that the resulting Golay sequences, i.e. the particular realization of the permutation of the *x*'s is represented as a path on the graph that describes the quadratic function Q.



The graph of the quadratic form $x_0x_1 + x_0x_2 + x_0x_3 + x_1x_2 + x_2x_3$.

To illustrate the comment above, the above is an example of the graph Q for the case of m = 4, whose quadratic function Q is given as $x_0x_1 + x_0x_2 + x_0x_3 + x_1x_2 + x_2x_3$. Thus a particular permutation can be achieved by evaluating $x_0=0$ in which case the quadratic degenerate to $x_1 + x_2 + x_3 + x_1x_2 + x_2x_3$. Notice that in the graph $x_1.x_2$ and x_2x_3 produces a path on the graph above.

Again for the most part, the results are similar if not exactly the same. However by looking at the Golay sequences as represented by the graphs and evaluation of the quadratic functions, it lends more intuition on the construction of low PAPR RM codes.

Another contribution of the paper is that they managed to get a tighter bound on the PAPR of the resulting RM codes than in the Davis and Jedwab paper.

Theorem 9 : *Suppose m is odd and let*

$$Q(x_1, x_2, ..., x_m) = \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$$

Where *π is a permutation of {0,1,…,m-1}. Then the PMEPR of the coset $Q+RM_2(1,m)$ is equal to 2.*

We reproduce above the GF(2) version of the PMEPR bound presented in [7]. It's actually does not tell much more than what we already know from Davis and Jedwab. The siginificant contribution of this paper is the bound for the GF(q) case (i.e. with $ZRM_q(2,m)$ and its $RM_q(1,m)$ cosets), which we reproduce below.

Theorem 10 : *Suppose $Q : \{0,1\}^m \rightarrow GF(q)$ is a quadratic form in variables $x_1, x_2, \ldots, x_m$. Suppose further that G(Q) contains a set of k distinct vertices with the property that deleting those k vertices and all their edges results in a path. Then every word in the coset $Q+RM_q(1,m)$ has PMEPR at most 2^{k+1}.*

The theorem above is rather hard to understand without dwelling too much on the details of Paterson's paper. In a sentence, the above bound states that the PMEPR depends on the number of second order monomials on the general RM code that constitutes as a coset representative on the $RM_q(1,m)$ code. The number k is the number of edges that can be deleted from the graph Q, while still keeping an intact path on the graph.

The significance of the generalized RM code proposed on the two papers above is the very fact that the resulting code has a well-known structure. The connection between PMEPR and RM codes through the Golay complimentary sequences is key in producing the results.

VII. Conclusion and Comments

In this project we conducted a survey of the currently available methods in Peak-to-Average Power Ratio reduction in OFDM systems. The emphasis is on methods and techniques which jointly optimize code performance for low PAPR as well as the minimum distance of the code. The first step in designing codes with good PAPR is to understand the relation between a particular codeword in a code to its resulting PAPR under OFDM modulation. Building on top of that we would like to understand the PAPR performance of the code – defined as the *maximum* worst case PAPR over all the codewords in the code. This relation between a code and its PAPR was developed by Tarokh in [9]; originally in the context of phase optimization to improve PAPR performance. With this perspective Paterson and Tarokh established some fundamental

limits on the joint optimization of codes with good PAPR and minimum Euclidian distance They mathematically formalized a notion that is hardly surprising - that by limiting the selection of possible code to only those with low PAPR, you are already inherently limiting the ‘volume’ of space available. Therefore the minimum Euclidian distance of a code selected from this subspace can only be smaller if the bound on the code’s PAPR is lowered.

In the last section we looked at two papers which established a connection between Golay complimentary sequences with a generalized version of RM codes. It’s quite easy to show that Golay complimentary sequences have a bounded PAPR, which depends on the size of this set. The next step is to take advantage of this knowledge and build codes whose codewords constitutes of Golay complimentary sequences. This is the second contribution of the two papers, whereas they make the connection between Golay complimentary sequences and a generalized version of RM code.

As of today the problem of PAPR control has rarely been addressed in deployed standards and products. The practical solution has been to design the hardware for close to the worst case conditions (inefficient). By employing a systems approach to the problem there is a potential to increase the performance and efficiency of OFDM systems. In this project we conducted a survey of the available approaches and techniques for PAPR reduction as of today. While the idea of limiting the possible transmitted sequences to a smaller subset is shared for both goals of obtaining a low PAPR output as well as error correction, in most cases these two goals are not mutually independent, and thus at the end of the day, to get a larger min. Euclidian distance, a higher PAPR would result, and vice versa. The RM code proposed by Davis and Jedwab does posses a good structure and the explanation provided by Paterson does give a good insight on how to design good codes with low PAPR using these Golay complimentary sequences. Despite those facts this RM code is still somewhat limited in that it probably does not perform as well compared to other more sophisticated modern error-control codes today.

There are a few interesting avenues to pursue based on our survey. The first one is to evaluate the performance of the RM code developed by Jedwab and Davis under the framework of the achievable region of the triples (R,d, PAPR($\mathcal{C}$)). Note that there is no solid proof that the bound derived by Tarokh is necessarily a tight bound. But it would be interesting to apply the geometrical point of view developed by Tarokh [9] to the construction of RM codes, as so far these are two distinct approaches.

The second interesting extension to the results above is to relax the assumptions on PSK modulations. Furthermore it would be interesting to evaluate these PAPR-optimized codes in a simply multi-carrier system, where the subcarriers frequencies are not necessarily orthogonal nor contiguous.

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