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Optimal Routing Control: Game Theoretic Approach[1]

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Abstract

Communication networks shared by selfish users are considered and modeled as noncooperative *repeated* games. Each user is interested only in optimizing its own performance by controlling the routing of its load. We investigate the existence of a NEP that achieves the system-wide optimal cost. The existence of a NEP that not only achieves the system-wide optimal cost but also yields a cost for each user no greater than its stage game NEP cost is shown for two-node multiple link networks. It is shown that more general networks where all users have the same source-destination pair have a NEP that achieves the minimum total system cost under a mild technical condition. It is shown general networks with users having multiple source-destination pairs don't necessarily have such an NEP.

1 Introduction

Communication networking is one of the fastest growing areas of technology and has therefore received a lot of attention lately. Since problems in networking area involve the interaction of multiple agents, it is natural to take a game-theoretic approach to some of these problems. That is, we model the network as consisting of a collection of selfish users, each of which attempts to minimize its own cost. In the absence of collusion, this is a noncooperative game.

An interesting recent paper along these lines is the one of Orda, Rom, and Shimkin [7]. Here a routing problem is analyzed in a static setting, namely as a single stage game, and the existence of NEP's for this routing problem is proved for certain types of cost functions related to the latency on the links. Uniqueness of the NEP when the network consists of parallel links connecting a pair of nodes is also shown for these cost functions. In more general networks, uniqueness of NEP does not hold even for rather natural cost functions, and such examples are given in [7].

The agents in this routing game are naturally thought of as being the network access providers. While a network access provider might typically handle loads between several origin-destination pairs, we focus here on the situation where each of the competing users carries flow between a specific origin-destination pair. The users would typically interact with each other several times before the nature of the game changes significantly, which might happen, for instance, because of the addition of new network access providers, a change in the topology of the network, or a significant change in the net load being handled by a network access provider. Thus, it is interesting to approach the routing problem as a repeated game rather than just a single shot game. In a repeated game there is the possibility of strategies that result in NEP's which are more efficient than in the single shot game. In this paper we prove that in the parallel link case, it is possible to have NEP's where the agents are required to operate at the unique system-wide optimum point while each user's cost is still no greater than the stage game NEP cost. Such strategies are supported by credible threats or rewards that the users might make or offer to one another. Clearly any such strategy is not only a NEP but also a subgame-perfect NEP (SPNEP) because it can be shown that no user will be able to reduce its own cost in any subgame by deviating from such an equilibrium. In more general networks, it is much harder to determine if strategies exist in the repeated game that yield a cost for each user that is smaller than or equal to its cost in every stage game NEP. Much of the difficulty lies in the analysis of the stage game and the characterization of its NEP's. Nevertheless, in networks where every user has same source and destination nodes, we show under a mild technical condition that there exists a SPNEP that drives users to operate at a system-wide optimal point. When there are users with different source-destination pairs, we show that the existence of such a SPNEP cannot be guaranteed in general. A few examples of this sort will be given at the end of the paper.

2 A Network of Parallel Links

2.1 Model and Problem Formulation

We are given a network with a set $\mathcal{I} = \{ 1, 2, ..., I\}$, $I \geq 2$, of users that share a set of parallel communication

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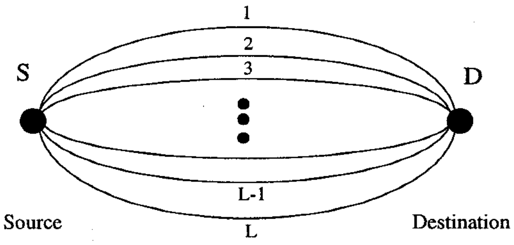

Figure 1: Parallel link network.

links $L = \{1, 2, ..., L\}$ interconnecting a common source to a common destination node. Without loss of generality, we assume that the links are ordered by decreasing capacity, i.e., $C_1 \geq C_2 \geq \cdots \geq C_L$. Each user $i \in I$ is assumed to be selfish in the sense that it attempts to minimize its own cost regardless of what other users are doing. User i has a demand, which is some ergodic process with average rate of r^i. Without loss of any generality, we will assume that users are ordered by decreasing order of average rate, $r^1 \geq r^2 \geq \cdots \geq r^I$.

We will first describe the stage game, which we assume is repeated many times. If the users are not sure when the game will end, we can model this by an infinitely repeated game with an appropriate discounting factor δ. In the stage game, each user splits its demand over the communication links, i.e., user i decides how much of its demand, f_l^i, it will send on link l. We must have $f_l^i \geq 0$ (nonnegativity constraint) and $\sum_{l \in L} f_l^i = r^i$ (demand constraint). Let f_l denote the total flow on link l, i.e., $f_l = \sum_{i \in I} f_l^i$, and $\mathbf{f}_l$ the vector of all user flows on link l, i.e., $\mathbf{f}_l = (f_l^1, f_l^2, ..., f_l^I)$. The flow configuration of user i is denoted by f^i, and the system flow configuration by $\mathbf{f} = (f^1, f^2, ..., f^I)$. A user flow configuration is said to be feasible if it satisfies the nonnegativity and demand constraints. We denote the set of all feasible f^i by $\mathbf{F^i}$. Similarly, a system flow configuration, $\mathbf{f}$, is feasible if every user flow configuration is feasible, and $\mathbf{F}$ denotes the set of all feasible $\mathbf{f}$'s.

In order to compare the performance of each user $i \in I$, we need to have a performance measure. This is given by a cost function $J^i(\mathrm{f})$ defined for each user i. The goal of each user is to minimize its cost by distributing its demand over the links. When the cost depends on the total flow carried on the links, the cost of user i depends not only on its flow configuration but also on those of other users. Since we are assuming that every user is selfish, the problem can be modeled as a noncooperative game where each user is trying to minimize its cost [7]. A natural question that arises in this type of setting is whether there is a Nash equilibrium of the game or not. In other words, we are interested in finding a system flow configuration such that no user finds it beneficial to change its own flow configuration assuming that no other users do. Mathematically a system flow configuration $\tilde{f} = (\tilde{f}^1, \tilde{f}^2, ..., \tilde{f}^I)$ is a Nash Equilibrium Point (NEP) if, for all $i \in I$, the following holds:

$$
\begin{aligned}
J^i(\tilde{f}) &= J^i(\tilde{f}^1, ..., \tilde{f}^{i-1}, \tilde{f}^i, \tilde{f}^{i+1}, ..., \tilde{f}^I) \\
&= min_{f^i \in \mathrm{F}^i} J^i(\tilde{\mathrm{f}}^1, ..., \tilde{\mathrm{f}}^{i-1}, \mathrm{f}^i, \tilde{\mathrm{f}}^{i+1}, ..., \tilde{\mathrm{f}}^I)
\end{aligned}
$$

The importance of an NEP is that it is a point at which no user has an incentive to deviate. However, one problem with an NEP is that it is not necessarily very efficient. In fact, Korillis, Lazar, and Orda [7] give numerical examples with natural cost functions where the difference between the total cost at the system-wide optimal point and that at the NEP could be more than 20 percent.

In our analysis, we use what are called type-C cost functions in [7]. The cost function for user i, J^i, is the sum of link cost functions, J_l^i, i.e., $J^i(\mathrm{f}) = \sum_{l \in L} J_l^i(\mathrm{f}_l)$, and $J_l^i(\mathrm{f}) = J_l^i(f_l^i, f_l) = f_l^i \cdot T_l(f_l)$, where

$$
T_l(f_l) = \begin{cases} \frac{1}{C_l - f_l}, & f_l < C_l \\ \infty, & f_l \geq C_l \end{cases} \qquad (1)
$$

where C_l is the capacity of link l. Such a cost function is very natural in this context as it is directly motivated by the delay formula in an exponential server queue for a given throughput.

Throughout the paper, we will assume that the stability condition is satisfied, i.e., $\sum_{i \in I} r^i < \sum_{l \in L} C_l$. Then, it is easy to see that at any NEP the costs of all users are finite. Otherwise, at least one user with infinite cost can change its own flow configuration to have finite cost.

It turns out that under these assumptions, the routing game is equivalent to a convex game described in [9], and the existence of an NEP is guaranteed. Also, Kuhn-Tucker conditions constitute the necessary conditions for a feasible system configuration to be an NEP. In other words, for every $i \in I$, there must exist a Lagrange multiplier, λ^i, such that, for every link $l \in L$,

$$
\begin{aligned}
f_l^i > 0 &\Rightarrow K_l^i(\mathrm{f}_l) = \lambda^i \\
f_l^i = 0 &\Rightarrow K_l^i(\mathrm{f}_l) \geq \lambda^i
\end{aligned}
$$

where K_l^i is the partial derivative of J_l^i with respect to f_l^i. We will call λ^i user i's marginal cost at the NEP.

Orda, Rom, and Shimkin have proved in [7] that in the parallel link case with type-C cost functions there is a unique NEP. An important consequence of this is that given a set of demands for all the users, there is a unique system flow configuration, $\mathbf{f}'$, that achieves the smallest total system cost, $\sum_{l \in L} J_l(\mathbf{f}')$. For more details on the properties of stage game NEP's, refer to [7]. Unfortunately, in most cases where multiple selfish users compete over the network, the resulting unique NEP does not result in the same link flows as the system-wide optimal point.

2.2 Subgame Perfect NEP

We now assume that the stage game of Section 2.1 is repeated. We will show the existence of a system flow configuration that is a SPNEP of the repeated game and achieves the minimum system total cost C'. Furthermore we will also show the existence of a SPNEP for the repeated game achieving the minimum total system cost, where the cost of *each* user is no more than its cost if play proceeds by repeating the unique NEP of the stage game.

We first need to define user i's reservation cost. This is defined to be

$$\underline{v}_i = max_{\mathrm{f}^{-i}}[min_{\mathrm{f}^i} J^i(\mathrm{f}^i, \mathrm{f}^{-i})] \tag{2}$$

where $\mathrm{f}^{-i} = (\mathrm{f}^1, ..., \mathrm{f}^{i-1}, \mathrm{f}^{i+1}, ..., \mathrm{f}^I)$. In words, if other users collude to punish user i, it can guarantee that it incurs a cost no more than $\underline{v}_i$.

Let $\underline{v} = (\underline{v}_1, \underline{v}_2, ..., \underline{v}_I)$. The *Folk theorem* for repeated games [2] says that any cost vector that strictly Pareto-dominates $\underline{v}$ can be supported by an NEP in the repeated game for discount factors sufficiently close to 1.

Let $\check{f}$ denote the unique stage game NEP and $\check{J}^i$ the cost of user i at the NEP. Then we have :

Lemma 1 $\frac{\check{J}^i}{\check{J}^j} \geq \frac{r^i}{r^j}$ *if* $i \leq j$.

Let C' denote the minimum total system cost that can be achieved given the set of demands for the users, and let R denote the sum of demands of all the users. Also, let $\mathbf{f}' = (\mathbf{f}'_1, ..., \mathbf{f}'_L)$ denote the system configuration that achieves C'. Then we have:

Lemma 2 $\underline{v}_i \geq C' \cdot \frac{r^i}{R}$ *for all* $i \in I$. *If* $\exists\, l \in L$ *such that* $f'_l > f'_{l+1} > 0$, *then* $\underline{v}_i > C' \cdot \frac{r^i}{R}$ *for all* $i \in I$.

Theorem 1 *Playing* $\tilde{\mathbf{f}} = (\tilde{\mathbf{f}}^1, ..., \tilde{\mathbf{f}}^I)$ *every period is a NEP in the repeated game, where*

$$\tilde{\mathbf{f}}^i = (\mathbf{f}'_1 \cdot \tfrac{r^i}{R}, ..., \mathbf{f}'_L \cdot \tfrac{r^i}{R}).$$

Proof: We need to consider two cases. First, suppose $\exists\, l \in L$ such that $f'_l > f'_{l+1} > 0$. Then, by the Folk theorem Theorem 1 follows directly from Lemma 2 since $\tilde{\mathrm{f}}$ yields a cost for each user i that is smaller than the reservation cost of user i. Now, suppose that there exists no such l. Let L^0 be the set of links such that $f'_l > 0$. Then, C_l is same for all $l \in L^0$ [7]. We can show that $\tilde{\mathbf{f}}$ is a stage game NEP. First, K^i_l is the same for all $l \in L^0$ because both C_l and $\tilde{\mathbf{f}}^i_l$ are same on all $l \in L^0$. Hence, in order to show that $\tilde{\mathrm{f}}$ is a stage game NEP, we only need to show that no users are tempted to use any other links not in L^0. Clearly, $K^i_l = \frac{1}{C_l - f'_l} + \frac{f'_l \cdot \frac{r^i}{R}}{(C_l - f'_l)^2} < \frac{1}{C_l - f'_l} + \frac{f'_l}{(C_l - f'_l)^2} = K_l \leq \frac{1}{C_{l'}}$ for all $l \in L^0$ and $l' \notin L^0$. This proves that no users are tempted to use any $l' \notin L_0$. Thus, $\tilde{\mathbf{f}}$ is a stage game NEP, and repeating a stage game NEP is a NEP in the repeated game. In fact, it is a SPNEP. ∎

One thing to notice about $\tilde{\mathbf{f}}$ is that the cost of some users may be greater than their cost in the stage game NEP. Friedman in [1] showed that any cost vector that strictly Pareto-dominates a stage game NEP can be supported by a SPNEP for discount factors sufficiently close to 1. Thus, the following theorem shows that there exists a SPNEP, $\hat{f}$, that achieves C' and yields a cost for each user that is smaller than or equal to its cost at the stage game NEP, i.e., $J^i(\hat{\mathrm{f}}) \leq J^i(\check{\mathrm{f}})$ for all $i \in I$.

Theorem 2 *There exists a system flow configuration,* $\hat{f}$*, that yields the optimum total system cost* C' *and a cost for each user that is smaller than or equal to its cost at the stage game NEP. Also, if* $\check{C} = J(\check{f}) > C'$*, then the cost of each user at this configuration is strictly smaller than its NEP cost.*

In fact, if $C' < \check{C}$, then it can be shown that there are uncountably many such strategies because the action space for a user is a continuum.

We need several lemmata to prove Theorem 2. Let $\check{f}$ denote the stage game NEP and f' the flow configuration that achieves the minimum total system cost.

Lemma 3 $\check{f}_l > 0 \Rightarrow f'_l > 0$ *for all* $l \in L$.

Let

$$\check{C}(r) = \sum_{l=1}^{\check{k}} \frac{\check{f}_l}{C_l - \check{f}_l} + \frac{\check{q}}{C_{\check{k}+1} - \check{f}_{\check{k}+1}}, \tag{3}$$

where

$$\sum_{l=1}^{\check{k}} \check{f}_l + \check{q} = r, \; 0 < \; \check{q} \leq \check{f}_{\check{k}+1}. \tag{4}$$

and

$$C'(r) = \sum_{l=1}^{k'} \frac{f'_l}{C_l - f'_l} + \frac{q'}{C_{k'+1} - f'_{k'+1}}, \tag{5}$$

where

$$\sum_{l=1}^{k'} f'_l + q' = r, \; 0 < \; q' \leq f'_{k'+1}. \tag{6}$$

Lemma 4 $\sum_{k=1}^{l} \check{f}_l \geq \sum_{k=1}^{l} f'_l, 1 \leq l \leq$ L.

Lemma 5 $\check{C}(r) \geq C'(r)$ *for all* $0 \leq r \leq R$.

Proof: (Theorem 2) We will give a sketch of proof here. We can think of flow on each link l used at the stage game NEP as a block, K_l, of flow with an associated cost per unit flow, $\beta_l = \frac{1}{C_l - \check{f}_l}$, and amount $\check{f}_l$. From [7], this cost per unit flow is nondecreasing in l. We can create another set of blocks, J_l, with the same amount, $\check{f}_l$, filling up f'_l, $l \in L$, in order of increasing l. J_l may have different cost per unit flow, α_l, from β_l. But, Lemma 5 and the monotonicity of $C_l - f'_l$ and $C_l - \check{f}_l$ tell us that either $J_l \leq K_l \ \forall\ l \in L$ or for some initial number of l's, $J_l \leq K_l$ and for the other l's, $J_l > K_l$. In the first case assigning a fraction, $\frac{\check{f}^i_l}{\check{f}_l}$, of J_l, $\forall\ l \in L$, to user i yields to user i a cost smaller than or equal to its NEP cost because the cost per unit flow of each block, J_l, is smaller than or equal to that of K_l. In the latter case, let $A = \{l : \alpha_l < \beta_l\}$ and $B = \{l : \alpha_l > \beta_l\}$. By Lemma 5, if $B \neq \emptyset$, then $A \neq \emptyset$. Starting with the smallest $l \in B$, for each $l \in B$, by exchanging some flow of J_l on links with higher cost per unit flow with same amount of flow of another $J_{l'}$, $l' \in A$, $l' < l$, on links with lower cost per unit flow, we can reduce the cost per unit flow of J_l until $J_l = K_l$. This procedure is always possible by Lemma 5. After the procedure terminates, since $\alpha_l \leq \beta_l$, $\forall\ l \in L$, for the same reason as in the first case, assigning a fraction, $\frac{\check{f}^i_l}{\check{f}_l}$, of J_l, $\forall\ l \in L$, to user i again yields to user i a cost smaller than or equal to its NEP cost. ∎

3 General Networks

3.1 Model and Problem Formulation

In this section we consider a network $G(V, L)$, where V is a finite set of nodes and $L \subseteq V \times V$ is a set of directed links. Without loss of generality we can assume that there is at most one link between each pair of nodes in each direction. As before we have a finite number of selfish users $I = \{1,2,...,I\}$ that share the etwork, and the demand for user i is denoted by r^i. Again, assume that users are ordered in order of decreasing demand.

A user sends its demand from its source node to its destination by splitting its flow on the different paths that connect its source and destination. A user is able to decide how to split its flow as in parallel links case. An important difference between the parallel link network and more general network is that in the general networks, many paths can come together at some node, share certain links, and split again at another node. This makes the analysis of the network much harder. Some of the properties that hold in the parallel link networks and were crucial in proving some of the lemmata and theorems in that case, no longer hold for more general networks.

We will first discuss single source-destination pair case. We discuss the multiple source-destination pairs case in the next subsection.

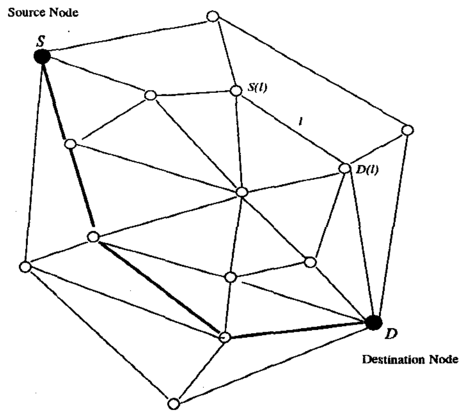

Figure 2: Single source-destination pair case.

3.1.1 Single Source-Destination Pair Case:

In the single source-destination pair case every user has the same source and destination nodes. Let P denote the set of paths available from the source node to the destination node, and L_p the set of links on path $p \in P$. Again since we are faced with selfish users, we have a noncooperative game played by users that attempt to minimize their own costs. The first thing we investigate is the existence of an NEP in the stage game. The existence is guaranteed by the same result in [9] as in the parallel link networks. Kuhn-Tucker conditions can be written as follows: for every $i \in I$, there exists a set of Lagrange multipliers $\{\lambda^i_u\}_{u \in V}$ such that, for every link $(u, v) \in L$:

$$f^i_{uv} > 0 \Rightarrow \lambda^i_u = K^i_{uv}(\mathrm{f_{uv}}) + \lambda^i_v$$
$$f^i_{uv} = 0 \Rightarrow \lambda^i_u \leq K^i_{uv}(\mathrm{f_{uv}}) + \lambda^i_v$$

Uniqueness of the NEP has been proven for only a few special cases. However, the uniqueness of the system flow configuration that achieves the minimum total system cost is guaranteed by Theorem 5 in [7]. Therefore, the minimum total system cost, C', and the corresponding system flow configuration are well defined.

Throughout this section, we will assume that f' is such that there are two paths used under f' with different cost per unit flow, $\sum_{l \in L_p} \frac{1}{C_l - f_l}$. We will first show the existence of a system flow configuration that is a NEP and achieves the total system cost, C'. Then, we will show that any such system flow configuration is a SPNEP.

Theorem 3 *There is a NEP in the repeated game that achieves the minimum total system cost,* C'.

In order to give the proof of the theorem, we need the following.

Lemma 6 *Given a fixed total demand $R < C$, where C is the minimal cut capacity from the source node to the destination node, there exists a uniform bound on the total system cost at the unique NEP regardless of distribution of demands among any finite number of users.*

Proof: (Theorem 3) Let $\underline{v}_i$ denote the reservation cost of user i. We first need to show that $\underline{v}_i \geq \frac{r^i}{R} \cdot C'$. By splitting $r^{-i} = R - r^i$ into n identical users and letting $n \to \infty$, from the K-T conditions and Lemma 6, we can show that as $n \to \infty$, at a NEP with user i and n identical users, $\bar{f}(n)$, the identical users use only the paths with the smallest cost per unit flow, $\sum_{l \in L_k} \frac{1}{C_l - \bar{f}_l(n)}$. Thus, as $n \to \infty$, user i's cost per unit flow cannot be smaller than that of the identical users. Since the total system cost at the NEP is no smaller than C', $\underline{v}_i \geq \frac{r^i}{R} \cdot C'$.
If $\underline{v}_i > \frac{r^i}{R} \cdot C'$, $\forall\, i \in I$, the existence of such NEP follows directly from *Folk theorem* because proportional sharing is strictly Pareto-dominated by the reservation cost vector [2]. If $\underline{v}_i = \frac{r^i}{R} \cdot C'$, $\forall\, i \in I$, then since any NEP of the stage game is Pareto-dominated by the reservation cost vector and the total system cost at the stage game NEP is no smaller than C', there exists a stage game NEP that achieves C' [7] and the stage game NEP cost of each user equals its proportional sharing cost. Repeating the stage game NEP is a NEP in the repeated game. If $\underline{v}_i > \frac{r^i}{R} \cdot C'$ for some users and $\underline{v}_i = \frac{r^i}{R} \cdot C'$ for the other users, an additional argument involving exchanging flow between the users is required to complete the proof. ∎

The rational feasible cost region is the subset of the feasible cost region that is Pareto-dominated by the reservation cost vector. Suppose V^* is the rational feasible cost region. Then, we say that V^* has full dimensionality if there exists $v \in V^*$ such that $\exists \varepsilon > 0$ such that all v' for which $| v'_i - v_i | < \varepsilon$ for all $i \in I$ are in V^*. This means that the interior of V^* is nonempty in R^I.

We may show that any NEP of the repeated game is also a SPNEP of the game, by showing that the full dimensionality holds [2].

Theorem 4 *There exists a SPNEP in the repeated game that achieves the minimum total system cost, C'.*

3.1.2 Multiple Source-Destination Pairs Case: In this section we will discuss the case where not all users have same source-destination pair. Again the existence of a NEP is guaranteed by the result due to Rosen [9]. However, the uniqueness of the system flow configuration that achieves minimum total system cost hasn't been proved yet. Rather than attempting to prove or disprove such uniqueness, we will show that even when there exists a unique system flow configuration that achieves the minimum total system cost, C', and the full dimensionality condition is satisfied, there may not exist a NEP of the repeated game that achieves C', however close the discount factor is to 1.

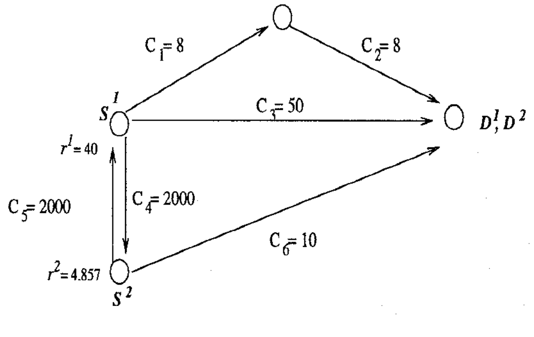

Figure 3: Multiple source-destination pairs case.

Consider the network with two users as shown in Fig 4. The reservation costs of user 1 and user 2 are 4.326 and 0.6416, respectively. Also, if we consider a global user who attempts to minimize the overall system cost, the unique system flow configuration that minimizes the total cost is $f = (f_1, f_2, f_3, f_4, f_5, f_5) =$ (1.5,1.5,38.5,0,0,4.857), which yields 3.81 and 0.944 for user 1 and 2, respectively. It is easy to see that this is the unique system flow configuration that achieves C' because in order for any system flow configuration to be optimal, it cannot send any flow on link 4 or link 5. Notice that even though the total system cost is smaller, the unique system flow configuration, f, requires user 2 to incur a cost that is greater than its reservation cost. Therefore, there exists no NEP of the repeated game that achieves C' and yet is Pareto-dominated by the reservation cost vector. This proves that it's is not always possible to find a NEP that achieves the minimum total system cost in multiple source-destination pairs case.

Let us now consider the system flow configuration ((1.33,1.33,35.368, 3.302,0, 3.302), (0,0, 3.428, 0,3.428, 1.429)) which yields $\check{J}^1 = 4.1839$ and $\check{J}^2 = 0.5772$ for user 1 and 2, respectively. It is easy to see that $\exists \epsilon > 0$ such that all $\underline{J}$ such that $| \underline{J}^i - \check{J}^i | < \epsilon$ for $i \in \{1,2\}$ are in the rational feasible cost region. This proves that the above example satisfies the full dimensionality condition.

Suppose that we are given a network with a finite number of users. Let $S = \{s_1, \cdots, s_m\}$ be the set of source-destination pairs, and I_k the set of users that have source-destination pair s_k. Class user k is a user whose source-destination pair is s_k and has demand $r^k_{class} = \sum_{i \in I_k} r^i$. Each class user can be considered to represent the users in the original network with same source-destination pair. We will now consider the NEP's of the network with only class users. Let C' be the smallest among the total system costs achieved by such NEP's.

Theorem 5 *There is a NEP in the repeated game with the original users, that achieves C'.*

An interesting question now would be whether C' is the smallest total system cost that can be achieved by any NEP or an even smaller cost is achievable by some NEP. It turns out in most cases there exists a NEP that achieves a smaller cost than the minimum among the NEP costs with class users. However, it proves to be very difficult to characterize the set of NEP's that achieves the minimum total system cost among NEP's.

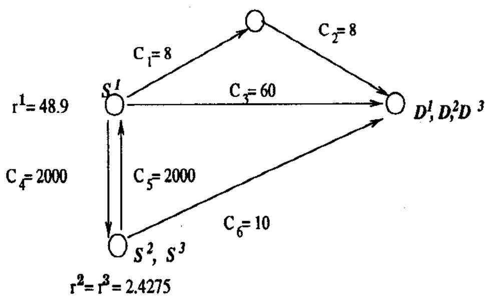

Figure 4: Multiple source-destination pairs case.

Let us look at the network in Fig 5. There are three users, and user 2 and 3 have same source-destination pair and same demand while user 1 has a different source-destination pair. If we consider a global user who attempts to minimize the overall system cost, the unique system flow configuration that yields the minimum total system cost C' is $f' =$ (1.5,1.5,47.4,0,0,4.857). Note that f' requires user 2 and 3 to use link 6 only, which has much higher cost per unit flow than the path along link 3 and 5. Now, let us investigate the network with class users. Suppose class user 2 represent user 2 and 3. The reservation costs of user 1 and class user 2 are 4.778 and 0.5963, respectively. The minimum cost that is achievable among the NEP's in the repeated game with class users can be shown to be 5.1807 and the corresponding system flow configuration is $\hat{f} = (\hat{f}^1, \hat{f}^2) =$ ((1.2765,1.2765,44.15,3.4735,0,3.4735), (0,0, 3.18,0,3.18,1.677)). Note that both links 4 and 5 are used in $\hat{f}$. Let us go back to the original game now. Suppose user 1 and 2 use flow configuration (0,0,48.9,0,0,0) and (0,0,1.76425,0,1.76425,0.66425). Then, user 3's best reply is (0.329, 0.329,1.0495,0,1.495,1.495), which yields a cost 0.3392 for user 3. Since user 2 and 3 are identical, the reservation costs of user 2 and 3 is greater than or equal to 0.3392. Repeating $\check{f}$ $= (\check{f}^1, \check{f}^2, \check{f}^3) =$ ((1.2765,1.2765,44.65,2.9735,0,2.9735), (0,0,1.34,0,1.34,1.0885), (0,0,1.34,0,1.34,1.0885)) yields a cost 4.517, 0.330, and 0.330 for user 1, 2, and 3, respectively, and, hence, is a NEP that achieves a total system cost smaller than 5.1807. This phenomenon is because user 2 and 3 can punish each other, and the presence of the threat drives users to operate at a NEP closer to the system-wide optimal point. This proves that in some cases there is a NEP that achieves a smaller total system cost than the smallest NEP cost with class users.

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