

# Capacity for a Power Controlled CDMA System with Linear Receivers

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**Abstract** — Capacity of networks with multiuser linear receivers together with efficient usage of the resources is analyzed in the uplink scenario of a single cell synchronous Code Division Multiple Access (CDMA) system. The capacity of the system is the maximum number of users per unit processing gain admissible in the system such that each user has its quality-of-service (QoS) requirement (expressed in terms of its desired signal-to-interference ratio) met. We characterize the capacity of a single cell for both the optimal linear receiver (MMSE receiver) and the matched filter receiver structures. This characterization of capacity has a simple “effective bandwidth” interpretation.

## I. INTRODUCTION

We consider a single cell synchronous CDMA system and focus on the uplink. The users are distinguished from each other by their signature sequences or codes. Let the processing gain of the system be  $L$ . We say that  $M$  users are admissible in the system if one can allot transmit powers and signature sequences to the users and linear receiver structure for each user at the base-station so that the signal-to-interference ratio (SIR) of each user is greater than its SIR requirement. We are interested in characterizing the maximum admissible number of users per unit processing gain, called the *capacity* of the system. Here two linear receivers are of particular interest: the MMSE receiver which is the best linear receiver (in the sense of maximizing the SIR of each user) and the matched filter receiver (the decorrelator is not of relevance in this context).

## II. MAIN RESULTS

Our main results are as follows:

1. With the MMSE linear receiver for each user,  $M$  users with SIR requirements  $\beta_1, \dots, \beta_M$  are admissible in the system with processing gain  $L$  if and only if  $\sum_{i=1}^M \frac{\beta_i}{1+\beta_i} < L$ .
2. With the matched filter receiver for each user,  $M$  users with SIR requirements  $\beta_1, \dots, \beta_M$  are admissible in the system with processing gain  $L$  if and only if  $\sum_{i=1}^M \frac{\beta_i}{1+\beta_i} < L$ . Thus we have the somewhat unexpected result that by using perfect power control and choosing “good” signature sequences there is no loss in capacity by using the, a priori inferior, matched filter receiver.

This allows us to characterize the admissibility of the users via a notion of *effective bandwidth*. If we consider

$\frac{\beta_i}{1+\beta_i}$  as the effective bandwidth of a user with SIR requirement  $\beta_i$ ; then  $M$  users are admissible in the system with processing gain  $L$  if and only if the sum of the effective bandwidths of all users is less than or equal to  $L$ .

3. We identify the “optimal” signature sequences and transmit powers to allot to the users so that the users’ SIR requirements are met.

For related literature, see [4] on multiuser receiver structure design and analysis and [1] on convergent power control algorithms. The problem of identifying good signature sequences has been studied in [2] in the context of a spread-spectrum system with conventional receiver and equal received power for all users. Different from these works, we study here the joint optimization problem of designing multiuser receiver structure, power control and spreading sequences

## III. DISCUSSION

Recently, [3] analyzed the capacity of the system (with the same modeling assumptions) when the signature sequences are chosen independently from a common distribution. It is interesting to compare our results with the corresponding ones in [3] which are asymptotic and valid for a large system. There, it is shown that for a system employing MMSE receivers and containing  $M$  users (each user having SIR requirement  $\beta$ ) and processing gain  $L$  as  $M \rightarrow \infty$  and  $L \rightarrow \infty$  and  $\frac{M}{L} \rightarrow \alpha$ , the users have their SIR requirements met (in the sense of probability) if and only if  $\alpha < 1 + \frac{1}{\beta}$ . Thus for a large system this suggests that using random sequences is as good as choosing signature sequences optimally for the users. However, when the receiver structure used is the matched filter, [3] shows that in a large system,  $\alpha$  users per unit processing gain have their SIR requirements met if and only if  $\alpha < \frac{1}{\beta}$ . Thus, asymptotically, 1 user per unit processing gain is lost when sequences are random and the matched filter receiver is used.

## REFERENCES

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