Optimal Sequences, Power Control and Capacity of Synchronous CDMA systems with Linear Multiuser Receivers

Pramod Viswanath, Venkat Anantharam and David Tse
Dept. of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA94720 email: {pvi,ananth,dtse}@eecs.berkeley.edu

Abstract — The Capacity of networks with multiuser receivers together with efficient usage of the resources, is analyzed in the uplink scenario of a single cell synchronous Code Division Multiple Access (CDMA) system. The capacity of the system is the maximum number of users per unit processing gain admissible in the system such that each user has its quality-of-service (QoS) requirement (expressed in terms of its desired signal-to-interference ratio) met. We characterize the capacity of a single cell for both the optimal linear receiver (MMSE receiver) and the matched filter receiver structures. This characterization of the capacity has a simple "effective bandwidth" interpretation. We also identify the "optimal" signature sequences and transmit powers to be allocated so that the users meet their QoS requirement.

I. Introduction

We consider a single cell synchronous CDMA system and focus on the uplink. The users are distinguished from each other by their signature sequences or codes. Let the processing gain of the system be L. We say that M users are admissible in the system if one can allot transmit powers and signature sequences to the users and linear receiver structure for each user at the base-station so that the signal-to-interference ratio (SIR) of each user is greater than its SIR requirement. We are interested in characterizing the maximum admissible number of users per unit processing gain, called the *capacity* of the system. In practice, two linear receivers are of particular interest: the MMSE receiver which is the best linear receiver (in the sense of maximizing the SIR of each user, see [5]) and the matched filter receiver. Our main results are as follows:

1. With the MMSE linear receiver for each user, M users with SIR requirements $\beta_1, \dots, \beta_M$ are admissible in the system with processing gain L if and only if $\sum_{i=1}^{M} \frac{\beta_i}{1+\beta_i} < L$.
2. With the matched filter receiver for each user, M users with SIR requirements $\beta_1, \dots, \beta_M$ are admissible in the system with processing gain L if and only if $\sum_{i=1}^{M} \frac{\beta_i}{1+\beta_i} < L$. Thus we have the somewhat unexpected result that by using perfect power control and choosing "good" signature sequences there is no loss in capacity by using the, a priori inferior, matched filter receiver.

   This allows us to characterize the admissibility of the users via a notion of *effective bandwidth*. If we consider $\frac{\beta_i}{1+\beta_i}$ as the effective bandwidth of a user with SIR requirement β_i then M users are admissible in the system with processing gain L if and only if the sum of the effective bandwidths of all users is less than or equal to L.
3. We identify the "optimal" signature sequences and transmit powers to allot to the users so that the users' SIR requirements are met.

For related literature, see [10, 4, 5, 7] on multiuser receiver structure design and analysis and [3, 1, 9] on convergent power control algorithms for channelized systems and CDMA systems with fixed signature sequences. The problem of identifying good signature sequences has been studied in [6] in the context of a spread-spectrum system with conventional receiver and equal received power for all users. Different from these works, we study here the joint optimization problem of designing multiuser receiver structure, power control and spreading sequences, and obtain simple characterizations of the resulting system capacity.

II. Model and Definitions

In the multiaccess symbol synchronous CDMA system, user i spreads its information on the common channel through modulation using its signature sequence, denoted by s_i, a unit vector in $\mathcal{R}^L$. As is traditional in multiaccess systems, we model the information transmitted by the users as independent zero mean random variables, say X_i. We denote the received power of user i as $E\left[X_i^2\right] = p_i$, the product of the transmit power of user i and the path gain from user i to the receiver (base-station) The ambient noise is assumed to be white gaussian independent of the users, denoted by say $W \sim N\left(0, \sigma^2 I\right)$. Then the received signal at the base-station can be written as

$$Y = \sum_{i=1}^{M} s_i X_i + W$$

Let $S = [s_1 \dots s_M]$ and $D = \text{diag}\{p_1, \dots, p_M\}$. When the symbol of user i is decoded using a linear receiver represented by c_i, the achieved SIR of user i is given by

$$SIR_i = \frac{(c_i, s_i)^2 p_i}{\sigma^2 (c_i, c_i) + \sum_{j \neq i} (c_i, s_j)^2 p_j} \tag{1}$$

We say that M users are admissible in the system with processing gain L if we can find powers p_i, signature sequences s_i and linear receivers c_i for each user i so that $SIR_i \geq \beta_i$ where β_i is the QoS requirement of user i. Under the choice of MMSE linear receiver, $c_i = \left(SDS^t + \sigma^2 I - p_i s_i s_i^t\right)^{-1} s_i$ (see [11]) and under the matched filter receiver $c_i = s_i$. The corresponding SIR_i for the MMSE linear receiver can be verified to be $s_i^t \left(SDS^t + \sigma^2 I - p_i s_i s_i^t\right)^{-1} s_i p_i$ (see [11]).

III. Capacity Characterization

Our main result is the following. It is true for both the scenarios of choice of MMSE linear receiver and matched filter receiver.

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Theorem III.1 *M users with SIR requirements $\beta_1, \ldots, \beta_M$ are admissible in the system with processing gain L if and only if $\sum_{i=1}^{M} \frac{\beta_i}{1+\beta_i} < L$.*

This allows us to consider the quantity $\frac{\beta_i}{1+\beta_i}$ as the *effective bandwidth* of the user i. Thus M users are admissible in the system if and only if the sum of their effective bandwidths is less than the processing gain of the system, allowing a very compact description of the capacity region.

The formal proof can be found in [11]; here we shall outline the proof for the scenario when all the SIR requirements are identical, equal to say β. The proof of the necessity of the above condition can be obtained in [11], here we shall focus on the sufficiency. Let $\frac{M\beta}{1+\beta} < L$. If the number of users is less than or equal to the processing gain, then the choice of orthogonal signature sequences allows us to decompose the system such that each user does not interfere with any other user and thus with no transmit power constraints arbitrary SIR requirements can be met. Hence, we assume that $M > L$. We shall now allot signature sequences and powers for the users so that each user has achieved SIR equal to the requirement β. We let the received power $p_i = p = \frac{\sigma^2 \beta L}{L(1+\beta) - M\beta}$ for each user i. We choose signature sequences $s_1, \ldots, s_M$ such that $SS^t = \frac{M}{L} I$. Such sequences were first introduced in [6] and were denoted WBE sequences since they meet the so-called Welch-Bound-Equality (see [12]); such sequences are known as *tight frames* in the wavelet literature (see [2]) and [11] develops a formal methodology to construct these sequences for arbitrary $M \geq L$. With this choice of signature sequences and powers, it can be verified that the linear MMSE receiver c_i (as in (1)) is just the same as the matched filter receiver s_i. It is now easily verified that the achieved SIR of each user is exactly β. Thus we have the somewhat surprising result that there is no loss in capacity by using the a priori inferior matched filter receiver when the signature sequences and powers are appropriately chosen. It is also shown in [11] that this choice of signature sequences and powers gives the lowest possible allocation of sum of received powers of all users. This observation is then used to characterize the effect of a transmit power constraint on the capacity region.

IV. Conclusions and Discussion

We have characterized the capacity of a synchronous CDMA system using linear receiver structures for the uplink. The downlink scenario is very similar and the exercise of capacity characterization is done in [11]. However, complete synchronization is idealistic and in practice one has asynchronous reception of the users' symbols by the base-station and multipaths are also present. Furthermore, the assumption of perfect power control made in this study is also not valid in practical systems. Nevertheless, these results give some insights to the best one can achieve. We are currently studying the capacity of systems with asynchronous reception. It also must be emphasized that these results are for the case of a single base-station. In a cellular system with many base-stations, the characterization of the capacity region (now a mobile will have to be distinguished by its path gains to the different base-stations) is an important problem. In particular questions such as how does the multiple base-station receiver capacity compare with the single receiver capacity will be answered by such a characterization. Our preliminary results in this direction show that the answer depends on the path gains of the mobiles to the base-stations and we discover an important property of these systems - the need to do a kind of "channel-sharing".

Recently, in [8], the authors analyzed the capacity of the system (with the same modeling assumptions - symbol synchronous, single cell, additive white gaussian noise) when the signature sequences are chosen independently from a common distribution. Such a system models current CDMA systems such as IS-95 where pseudo-noise sequences span many symbol periods. It is interesting to compare our results with the corresponding one in [8]. The results in [8] are asymptotic and are valid for a large system (i.e., a system with a large processing gain and large number of users). There, it is shown that for a system employing MMSE receivers and containing M users (each user having SIR requirement β) and processing gain L as $M \to \infty$ and $L \to \infty$ and $\frac{M}{L} \to \alpha$, the users have their SIR requirements met (in the sense of probability) if and only if $\alpha < 1 + \frac{1}{\beta}$. Thus for a large system this suggests that using random sequences is as good as using the optimal WBE sequences for the signature sequences of the users. However, when the receiver structure used is the matched filter, in [8] the authors show that in a large system, α users per unit processing gain have their SIR requirements met if and only if $\alpha < \frac{1}{\beta}$. Thus 1 user per unit processing gain is lost asymptotically when random sequences are used and the matched filter receiver is used.

References

[1] Foschini G. J., Miljanic Z. "A simple distributed autonomous power control algorithm and its convergence" IEEE Trans. Vehic. Techn, Vol. 40., No. 4.:641–646, 1993.

[2] V K Goyal, M Vetterli and N T Thao, "Quantized overcomplete expansion in $\mathcal{R}^N$: Analysis, synthesis and algorithms", *IEEE Trans. on Info. Theory*, Vol. 44, Jan 1998, pp 16-31.

[3] Hanly S. V. "An algorithm for combined cell-site selection and power control to maximize cellular spread spectrum capacity" *IEEE JSAC, special issue on the fundamentals of networking*, Vol. 13, No. 7 September, 1995.

[4] Lupas, R. and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple access", *IEEE Trans. on Information Theory*,IT-35, Jan., 1989, pp.123-136.

[5] Madhow, U. and M. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA", *IEEE Trans. on Communications*, Dec., 1994,pp.3178-3188.

[6] Rupf, M and J. L. Massey, "Optimum sequence multisets for Synchronous code-division multiple-access channels", *IEEE Transactions on Information Theory*, Vol 40, No. 4, pp. 1261-1266, July 1994.

[7] Rupf, M., F. Tarkoy and J. Massey, "User-separating demodulation for code-division multiple access systems",*IEEE JSAC*, June,1994, pp.786-795.

[8] D. Tse and S. Hanly, "Multiuser Demodulation: Effective Interference, Effective Bandwidth and Capacity", UCB/ERL Memo. 98/5.

[9] Ulukus, S. and Yates R, "Adaptive power control and MMSE interference suppression", to appear in *Wireless Networks*, special issue on Multiuser Detection in Wireless Communications.

[10] Verdú, S., "Optimum multiuser asymptotic efficiency ", *IEEE Trans. on Comm.*, COM-34, Sept.1996, pp. 890-897.

[11] P Viswanath, V Anantharam and D Tse, "Optimal sequences, power control and capacity of spread-spectrum systems with linear receivers", UCB/ERL Memo. 98/6.

[12] Welch, L.R. "Lower bounds on the maximum cross correlation of signals", *IEEE Transactions on Information Theory*, vol.IT-20, pp.397-399, May, 1974.