EFFECT OF NOISE ON LONG TERM MEMORY IN CELLULAR AUTOMATA WITH ASYNCHRONOUS DELAYS BETWEEN THE PROCESSORS *

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SUMMARY

Cellular automata are simple computational models which are capable of exhibiting a wide range of complex dynamical behavior, see [4]. The computation is considered as proceeding synchronously via identical processors at each site on a regular lattice, usually $\mathbf{Z}^p$, and the computational rule is assumed to be spatially homogeneous. The interest in studying such automata comes from several points of view. For example, there is a belief that the complex dynamical behavior exhibited by these automata is a good model for the natural statistical behavior of physical systems such as gases, consisting of large numbers of interacting elements. Another powerful source of reawakened interest in cellular automata has been the development of parallel computational systems employing different types of regular architectures, e.g. [1].

A remarkable property of certain cellular automaton updating rule is that they can admit more than one invariant configuration, representing the ability to maintain long term memory. Further, it is known that there are automata whose long term memory persists under noise. This ability is particularly important from the point of view of the automaton as a computational device, where the initial configuration is the input on which the processors perform their calculations. For the operation to be reliable in a noisy or unreliable environment, we would require the system to remember enough relevant information about its initial configuration over arbitrarily long periods of time, [2].

In this paper we deal with a class of cellular automata called monotonic binary tessellations (MBT's). Let $V \stackrel{\text{def}}{=} \{(s,t) \in \mathbf{Z}^{p+1} : t \geq -t_W\}$, where $t_W > 0$. Given $\mathrm{v} \in V$, let $U(\mathrm{v}) = \{\mathrm{v} + (u_1, t_1), \ldots, \mathrm{v} + (u_r, t_r)\}$, where $u_i \in \mathbf{Z}^p$ and $t_i < 0$, $1 \leq i \leq r$. An MBT evolves according to the rule

$$\mathrm{x_v} = \phi(\mathrm{x}_{\mathrm{v}+(u_1,t_1)}, \cdots, \mathrm{x}_{\mathrm{v}+(u_r,t_r)}),$$

where $\mathrm{x_v} \in \{0,1\}$, with initial conditions prescribed on $W \stackrel{\text{def}}{=} \{(s,t) \in V: -t_W \leq t < 0\}$. Here ϕ is a monotonic function, i.e., $\phi(\mathrm{x}_1, \cdots, \mathrm{x}_r) \geq \phi(\mathrm{x}'_1, \cdots, \mathrm{x}'_r)$ if $\mathrm{x}_i \geq \mathrm{x}'_i$ for $i = 1, \ldots, r$. Note that the all-zero configuration and the all-one configuration are both invariant for an MBT.

We study MBT's with asynchronous communication delays in computation, [1]. Let $V' \stackrel{\text{def}}{=} \{(s,t) \in \mathbf{Z}^{p+1} : t \geq -d - t_W\}$, with initial conditions on $W' \stackrel{\text{def}}{=} \{(s,t) \in V': -d - t_W \leq t < 0\}$. An asynchronous scenario is given by delays $\tau_1(\mathrm{v}), \ldots, \tau_r(\mathrm{v})$ for each $\mathrm{v} \in V' - W'$, where $0 \leq \tau_i(\mathrm{v}) \leq d$, $1 \leq i \leq r$. Given a scenario, the automaton evolves according to the rule

$$\mathrm{x_v} = \phi(\mathrm{x}_{\mathrm{v}+(u_1,t_1-\tau_1(\mathrm{v}))}, \cdots, \mathrm{x}_{\mathrm{v}+(u_r,t_r-\tau_r(\mathrm{v}))}),$$

written $\mathrm{x_v} = \phi(\mathrm{x}_{U^\tau(\mathrm{v})})$. Note that for all τ, the all-zero and all-one configurations are both invariant.

For a fixed τ and $\epsilon \in (0,1)$, $M_\epsilon^\tau(\mathrm{z})$ denotes a set of probability measures on the σ-algebra generated by the cylinder subsets of $\{0,1\}^{V'}$. A measure $\mu \in M_\epsilon^\tau(\mathrm{z})$ if and only if for any finite A, $\mu(\mathrm{x_v} \neq \phi(\mathrm{x}_{U^\tau(\mathrm{v})}) \forall \mathrm{v} \in A) \leq \epsilon^{|A|}$ and $\mu(\mathrm{x_v} = 0 \forall \mathrm{v} \in W') = 1$. We say the all-zero trajectory is a stable τ-trajectory if

$$\lim_{\epsilon \to 0} \sup_{\substack{\mu \in M_\epsilon^\tau(\mathrm{z}) \\ \mathrm{v} \in V'}} \mu(\mathrm{x_v} = 1) = 0.$$

Roughly speaking, the memory of the all-zero trajectory persists even when there is noise, if it is small enough. A parallel definition can be made for the all-one trajectory.

Our main theorem states that, for a fixed d, the all-zero state is a stable τ-trajectory for all τ if and only if

$$-\bigcap_{q=1}^{Q} \bigcup_{\{\alpha \in \mathbf{R}: \alpha \geq 0\}} \{\alpha \mathrm{v} : \mathrm{v} \in \mathrm{conv}(C(Z_q))\} = \{0\}.$$

Here $Z_1, \cdots, Z_Q$ are the minimal zero sets in $U(0)$, namely : (a) $(\mathrm{x_v} = 0 \forall \mathrm{v} \in Z_q) \Longrightarrow \phi(\mathrm{x}_{U(0)}) = 0$, and (b) Z_q does not contain a proper subset with property (a). Further $C(Z_q)$ denotes $\{(s, t-i) \in V' : i = 0, \cdots, d; (s,t) \in Z_q\}$ and $\mathrm{conv}(Z_q)$ is the convex hull of Z_q. This result is a simple generalization of a deep result of Toom to the asynchronous context, [3].

Through the use of examples and the above theorem, we examine the effect of asynchronism on computation by MBT's, in particular the mechanisms by which asynchronism can force an originally nonergodic MBT to become ergodic (loss of memory).

REFERENCES

[1] D. P. Bertsekas and J. N. Tsitsiklis, Parallel and Distributed Computation, Prentice Hall, 1989.

[2] N. Pippenger, "Developments in the synthesis of reliable organisms from unreliable components," in The Legacy of John von Neumann, J. Glimm, J. Impagliazzo, and I. Singer (editors), American Mathematical Society, 1990.

[3] A. L. Toom, "Stable and attractive trajectories in multicomponent systems," in Multicomponent Random Systems, R. L. Dobrushin and Y. G. Sinai (editors), Marcel Dekker, 1980.

[4] S. Wolfram, "Universality and complexity in cellular automata," Physica, vol. 10D, pp. 1–35, 1984.

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