Antenna Design Notes

A Note on the Shaping of Dual Reflector Antennas

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***Abstract*—A technique based on a recent paper [3] for designing shaped dual reflector antennas that appears to be significantly better than the traditional method [1], [2] is presented. The main advantages derived are extreme computational simplicity and the ability to utilize very accurate feed pattern expressions. A typical practical problem is solved using [1] and our method to bring out these features.**

The analytical procedure for determining the reflector profiles in a symmetrical-shaped dual reflector antenna system based on ray optics is very well known [1], [2]. Here we present a computationally efficient approach based on a paper by Lee *et al.* [3]. The approach is efficient because the new scheme involves a single first-order differential equation rather than the coupled differential equations of the earlier scheme. The method outlined also provides certain significant advantages.

The geometry of the subreflector and main reflector is shown in Fig. 1. For a point source of illumination application of the law of conservation of energy, Snell's law at the subreflector, and the equal path length law lead to the following shaping equations:

$$\frac{\int_0^{\theta} F(\theta') \sin\theta' \, d\theta'}{\int_0^{\theta_c} F(\theta') \sin\theta' \, d\theta'} = \frac{\int_0^{R} I(R')R' \, dR'}{\int_0^{R_m} I(R')R' \, dR'} \tag{1}$$

$$x = R \tag{1a}$$

$$\frac{\partial\rho}{\partial\theta} = V/Q \tag{2}$$

where

$$Q = (a\cos\theta - c\sin\theta)/\rho \tag{2a}$$

$$V = l + a\sin\theta + c\cos\theta \tag{2b}$$

$$l = \sqrt{a^2 + c^2} \tag{2c}$$

$$a = x - \rho\sin\theta \tag{2d}$$

$$c = z - \rho\cos\theta \tag{2e}$$

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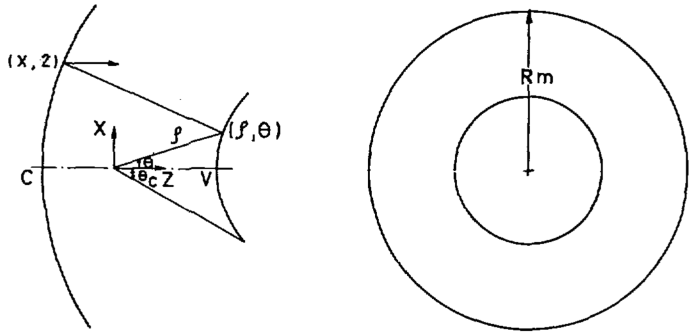

Fig. 1. Dual-reflector antenna geometry.

and

$$\rho + l - z = \text{path length of the central ray (at } z = 0). \tag{3}$$

Here instead of using Snell's law at the main reflector, (1a) assumes the rays leave the main reflector parallel to the z axis. It may be emphasized that Snell's law is authomatically satisfied when the incident and reflected wavefronts are defined and the equal path length condition is invoked [4].

Besides the feed pattern $F(\theta)$ and the desired aperture distribution $I(R)$, the input data required for the formalism is 1) θ_c, 2) R_m = 0.5 (diameter of the main dish), and 3) the distances of the feed phase center from each of the two reflectors (which define the path length of the central ray). Thus the distance between the feed and the subreflector is part of the program's input data, which is what allows the use of very accurate feed pattern expressions.

Shaping was carried out for 1) $\theta_c = 15.2^\circ$, 2) $R_m = 400$ cms, and 3) CF_1 = 152.4 cms; F_1V = 173.06 cms. Without loss of accuracy (1) was integrated in closed form using $\sin\theta = \theta - \theta^3/6$. After verifying that the radiation pattern of a high performance feed like a corrugated conical horn [5] over the main beam can be closely approximated by a Gaussian function, $F(\theta)$ was taken as $e^{-\beta\theta^2}$ with 10 dB taper at 15.2°. Uniform aperture illumination was assumed.

The numerical computations were carried out as follows.

i) For each θ in steps of 0.1° the value of R was calculated.

ii) ρ_{n+1} at any stage was determined using

$$\rho_{n+1} = \rho_n + \left[\frac{\partial\rho}{\partial\theta}\right]_n (\theta_{n+1} - \theta_n)$$

where ρ_n, Q_n, and V_n are known.

iii) a) $x_{n+1} = R_{n+1}$ which is known.
b) z_{n+1} is determined from (3), knowing x_{n+1}, ρ_{n+1} and θ_{n+1}. In determining c to find l, z_n is used.

iv) Q_{n+1} and V_{n+1} are determined subsequently and we go on to calculate ρ_{n+2}.

Comparison of the proposed scheme with the traditional Galindo-Williams technique [1], [2] is made in Tables I and II. (No plot was made since the two match very closely.) The

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TABLE I
COMPARISON OF THE SUBREFLECTOR PROFILES

	$\rho \cos \theta$		$\rho \sin \theta$	
$\theta°$	Proposed Scheme	Galindo–Williams	Proposed Scheme	Galindo–Williams
15.2	189.87	189.97	51.59	51.61
14.4	188.67	188.68	48.44	48.32
13.8	187.77	187.85	46.12	46.24
13.1	186.70	186.69	43.45	43.37
12.5	185.79	185.85	41.19	41.33
11.6	184.43	184.37	37.86	37.78
11.0	183.52	183.51	35.67	35.73
10.0	182.03	181.94	32.10	32.02
9.0	180.59	180.52	28.60	28.66
8.0	179.20	179.04	25.18	25.09
7.0	177.89	177.74	21.84	21.84
6.0	176.70	176.51	18.70	18.55
5.0	175.64	175.42	15.37	15.34
4.0	174.73	174.49	12.22	12.21
3.0	174.00	173.73	9.12	9.13
2.0	173.48	173.16	6.06	6.06
1.0	173.17	172.78	3.02	3.02
0.0	173.06 (defined)	≅172.50 (max. error)	0.00 (defined)	≅0.00 (max. error)

TABLE II
COMPARISON OF THE MAIN REFLECTOR PROFILES[1]

	$-Z$	
X	Proposed Scheme	Galindo–Williams
0.0	152.40 (defined)	≅153.03 (max. error)
17.0	152.26	152.77
34.0	151.70	152.06
51.0	150.72	150.96
79.0	148.19	148.27
117.0	142.92	142.80
138.0	139.11	138.89
178.0	130.04	129.67
215.0	119.60	119.09
228.0	115.46	114.91
260.0	104.18	103.59
271.0	99.96	99.37
285.0	94.41	93.74
318.0	80.04	79.40
333.0	73.00	72.40
369.0	54.77	54.37
375.0	51.49	51.21
382.0	47.68	47.47
383.0	47.20	46.94
390.0	43.29	43.15
394.0	41.10	40.97
397.0	39.38	39.33
398.0	38.83	38.79
399.0	38.27	38.24
400.0	37.70	37.70

[1] Because the two schemes were implemented with different free variables, the tables are not regularly spaced in θ and X, respectively.

input parameters for the analytical solution were determined from the results of our shaping since the end points of the reflectors have to be specified here. Shaping was carried out using the Runge–Kutta method. Another advantage of the new scheme recognized during shaping is that since computation starts from the center, numerical error is minimal in the central regions which handle most of the energy.

The attractive features of the technique based on the Lee–Parad–Chu paper [3] are thus very clear. The computational efficiency assumes special significance in connection with calculating the scattered patterns of the subreflector and the main reflector [6].

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