SURVEY and REVIEW

In this issue, we feature two very different articles. The first, "Estimation of Subspace Arrangements with Applications in Modeling and Segmenting Mixed Data" by Yi Ma, Allen Yang, Harm Derksen, and Robert Fossum, is an excellent example of interdisciplinary mathematical research. The work shows how sophisticated mathematical ideas can be used to address a problem which arises in many applications. Moreover, it points to new mathematical research directions.

The basic problem the article addresses is a very important problem in analysis of high dimensional data. Suppose you have a cloud of points in a large dimensional space; the points could represent consumer preferences, for example. The question is to find patterns in the data by identifying a set of hyperplanes which "fit" the data. The challenge is that the number of hyperplanes and their dimensions are to be learned from the data. Thus the task involves "grouping the points," i.e., segmentation, and determining the best approximation for that group of points, i.e., model selection. What makes this hard is that these two steps are tightly coupled.

In their paper, the authors describe a novel method, which they call GPCA (generalized principal component analysis), that solves this problem. The method is a remarkable result based on ideas from statistics, and abstract and computational algebra. Moreover, the approach is justified using statistical arguments. Examples from computer vision and image compression are given.

The subject of the paper is a very "hot" area of research. While there are many other approaches to this problem, this self-contained introduction to GPCA and its mathematical foundation is a must-read.

The second article, "Singularly Perturbed Linear Two-Point Boundary Value Problems" by Robert O'Malley, Jr., is about a classical problem in the study of differential equations involving a small parameter. Problems such as this arise in fluid dynamics, for example, in the study of air flow over an airplane wing. The small parameter in this case is viscosity. The flow field is described by the differential equation, while the boundary conditions correspond to the nature of the flow on the wing and away in the undisturbed parts of the air. This problem, in its canonical form, is the subject of the survey. What is remarkable is the rich and complex behavior exhibited by this innocent-looking linear problem when a coefficient in the differential equation is allowed to change signs.

The asymptotic analysis described in the paper is important because it can provide a closed-form approximate solution. But it is also important because it guides the development of computational methods. Through the asymptotics, the nature of the solution is revealed. In particular, the solution can be exponentially small and exponentially large, as well as having two spatial scales. This behavior is a harbinger of computational nightmares if one were to approach the problem numerically without any consideration to the wild behavior. Thorough understanding of the qualitative nature of the asymptotic solution, revealed by the analysis described, is the foundation of accurate and stable computational schemes for such problems.

The article is a pleasure to read, just like the author's classic text *Introduction to Singular Perturbations*. It gives a nice historical perspective and illustrates the main ideas with concrete examples.

Fadil Santosa
Section Editor
santosa@math.umn.edu