

Some Thorax Design Parameters *

Ronald S. Fearing
 Department of EE&CS
 University of California
 Berkeley, CA 94720-7440

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Abstract

Some worthwhile parameters from bending beams, etc.

1 Cantilever Model

Consider a single end clamped cantilever beam as in Fig. 1. The rectangular beam has cross-sectional moment of inertia

$$I = \frac{wh^3}{12} . \quad (1)$$

The deflection at the end of the beam with force F is

$$y = \frac{Fl^3}{3EI} = \frac{4Fl^3}{Ewh^3} , \quad (2)$$

where E is Young's modulus. The equivalent stiffness is

$$F = ky = \frac{wh^3E}{4l^3}y \quad (3)$$

The maximum stress (at the support) is

$$\sigma = \frac{6Fl}{wh^2} = \frac{3Ehy}{2l^2} . \quad (4)$$

2 Scaling from Mockup

Consider dimensional scaling by a factor s from large scale mockup to MFI sized device, under various scaling assumptions, which preserve the ratio $\frac{y}{l}$. The width w and length l must scale with s to make the thorax the correct size. The thickness h can be chosen to some extent, as the mass budget allows it. The thickness can be solved from eq. 2:

$$h = \left(\frac{4Fl^3}{Ewy} \right)^{\frac{1}{3}} . \quad (5)$$

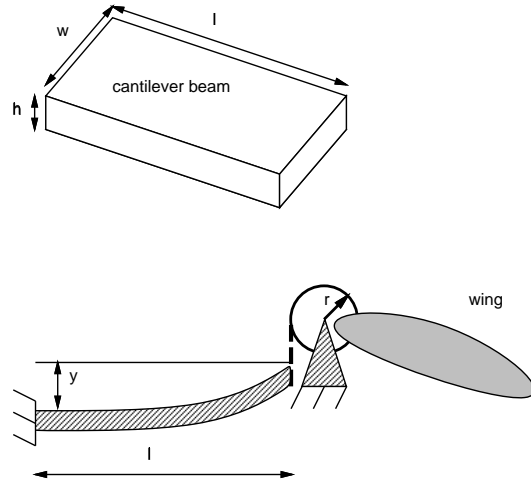


Figure 1: Cantilever beam driving wing pulley.

2.1 Force scales as s^3

Assumption: the required wing force scales as the mass and hence the volume of the insect. Denoting the scaled variables by subscript 1, thus $F_1 \sim s^3 F$, $l_1 \sim s l$, $w_1 \sim s w$ and $y_1 \sim s y$. Thus the required thickness is

$$h_{s^3F} = s^{\frac{4}{3}} h . \quad (6)$$

If the mockup is reduced in size by a factor of 64, the thickness of the band would be reduced by a factor of 256. The peak stress decreases:

$$\sigma_{s^3F} = s^{\frac{1}{3}} \sigma . \quad (7)$$

2.2 All dimensions scale as s

The required force to obtain deflection y is

$$F = \frac{wh^3E}{4l^3}y . \quad (8)$$

If all dimensions scale as s , the required force is just:

$$F_1 = s^2 F . \quad (9)$$

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property	music wire	poly silicon	cuticle	resilin
Elastic Modulus $E Nm^{-2}$	200×10^9	140×10^9	10×10^9	2×10^6
yield stress Nm^{-2}	4×10^9	0.7×10^9	0.4×10^9	
ultimate extension	2%	.5%	2-3%	300%
endurance limit Nm^{-2} ($> 10^6$ cycles)	10^9 0.5%	0.5%		
$Q = \frac{\pi}{\text{lossfactor}}$	200	10^4	30	> 100
density mg/mm	7.9	2.3	1.3	
reference	Oberg 88	Keller 98	Jensen 62	Jensen 62

Table 1: Material properties for thorax

The peak stress is unchanged:

$$\sigma_1 = \frac{6s^2 Fsl}{s w s^2 h^2} = \sigma . \quad (10)$$

3 Material Limits

Material limits for likely thorax materials are given in Table 1. Note that peak stress must be limited to maintain life. This can be done both with thin materials, and be keeping the Q low since high Q increases dynamic stresses.

4 Structural Damping Measures

(From “M3D III: Mechanics and Mechanisms of Material Damping”, edited by A. Wolfended and V.K. Kinra, ASTM 1997.)

Valid for small values of damping: $\tan \phi < 0.1$:

$$Q^{-1} = \frac{\Psi}{2\pi} = \eta = \frac{\delta}{\pi} = \tan \phi = \phi \quad (11)$$

and

$$Q^{-1} = \frac{E''}{E'} = 2\zeta = \frac{\Delta W}{2\pi W} = \frac{\lambda\alpha}{\pi}. \quad (12)$$

Where

- Q = quality factor
- Ψ = specific damping capacity
- η = loss factor
- δ = logarithmic decrement
- ϕ = phase angle by which stress leads strain
- E'' = loss modulus
- E' = storage modulus
- ζ = damping ratio
- ΔW = energy loss per cycle

- W = maximum elastic stored energy
- λ = wavelength of elastic wave
- α = attenuation .

References

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