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Denver, CO

Demystifying Computing with Magic, continued

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Special Session Overview

- **Motivation**
- **The 7 magic tricks**
  - Real-Time 4x4 Magic Square
  - Left/Right Game
  - The Tricky Dice
  - The Numbers Game
  - Find the Card
  - Guess Your Age
  - Guess Your Birthday Day of Week
- **Reflection**
- **Other References**
- **YOU contribute your tricks**

Magic

is Fun!

but Magic

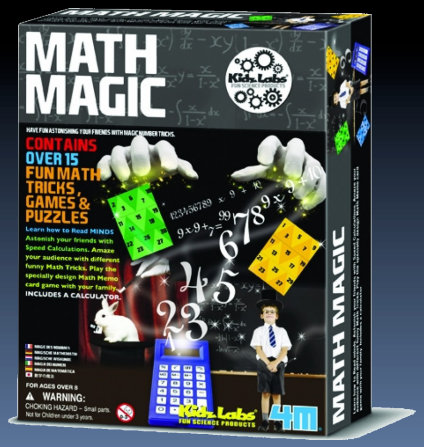
can be much more than fun!

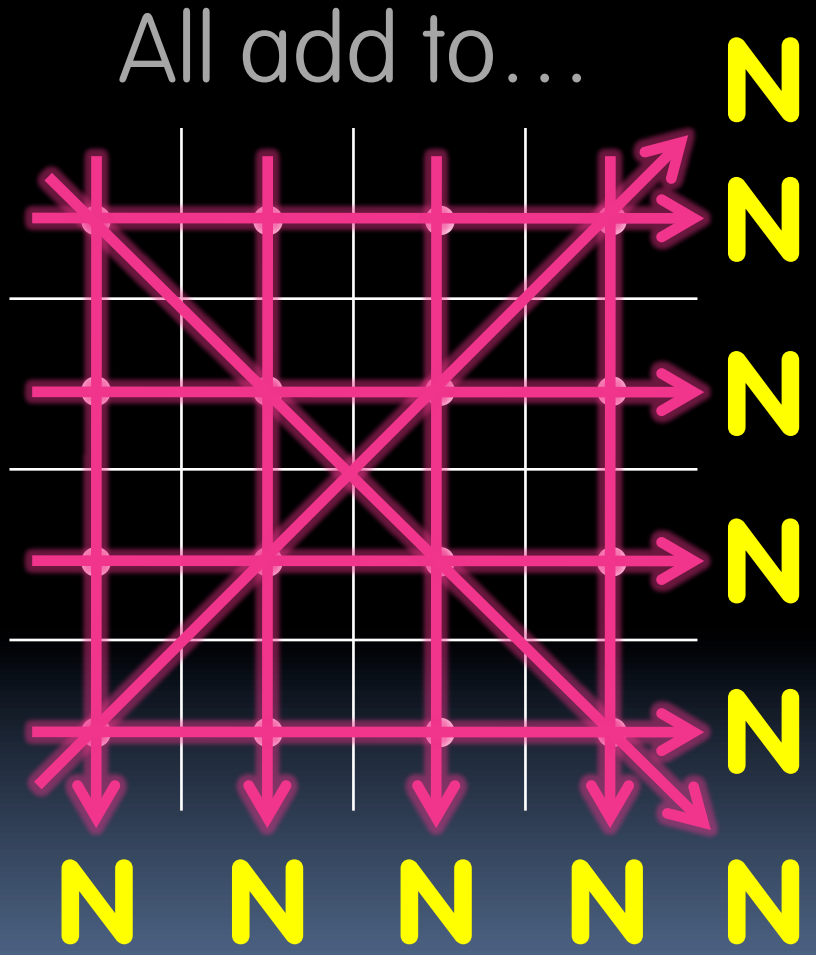
Magic can Motivate, Illustrate, Elaborate…

- **Computing notions**
  - Discrete math terms: e.g., permutations,
  - Problem representation: e.g., binary digits
  - Algorithmic patterns: e.g., sorting
  - General notions: e.g., symmetry
- **Problem solving**
  - Problem decomposition
  - Simplification, Generalization
  - Backward reasoning
  - Analogy (transfer)
  - Problem representation

Real-Time 4x4 Magic Square

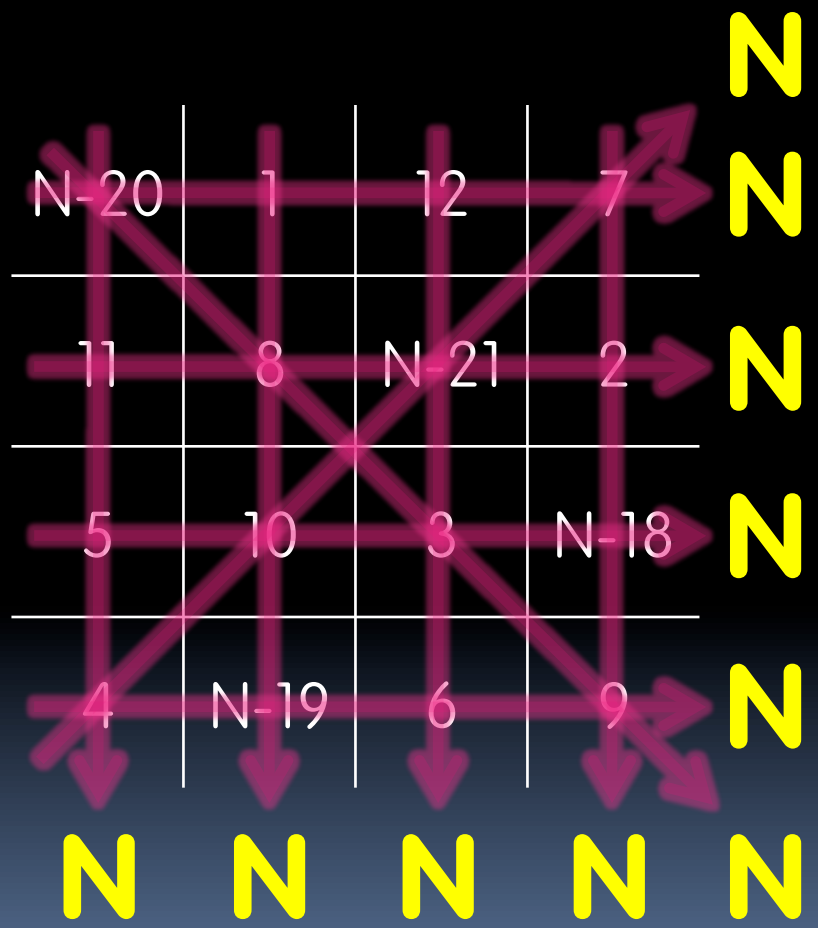
- Volunteer is asked for a number **N** from 25-100
- You create a magic square, where the sum of each row, column and diagonal is **N**
- Source:





How it Works; What Students Learn

- **Mechanics**
  - Make-Magic-Square(N)
  - Have a grid with the value already written faintly (or memorize it)
- **What they learn**
  - Value of lookup table
  - Correctness proof
  - Decomposition
  - Algebraic representation
  - Value of randomness
    - 8 rotations-and-flips



LEFT/RIGHT CARD GAME [IOI'96]

A line of 2N integer cards

Two players, alternating turns

Each player, on her turn, takes a card from one of the line ends (left or right)

The winner is the player whose sum of N cards is larger

THE MAGIC

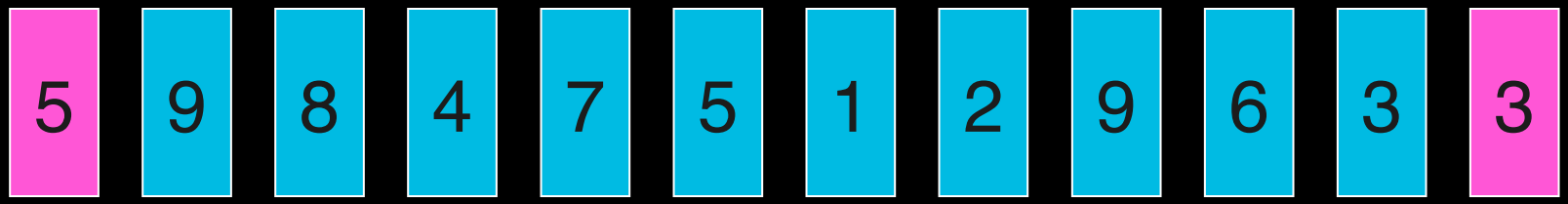
A volunteer from the audience arranges the line of 2n cards as she wishes

The magician looks at the line for a few seconds; then turns his back to the cards

The player plays against the magician, while the magician does not see the line of cards

The magician is the 1st player, never loses

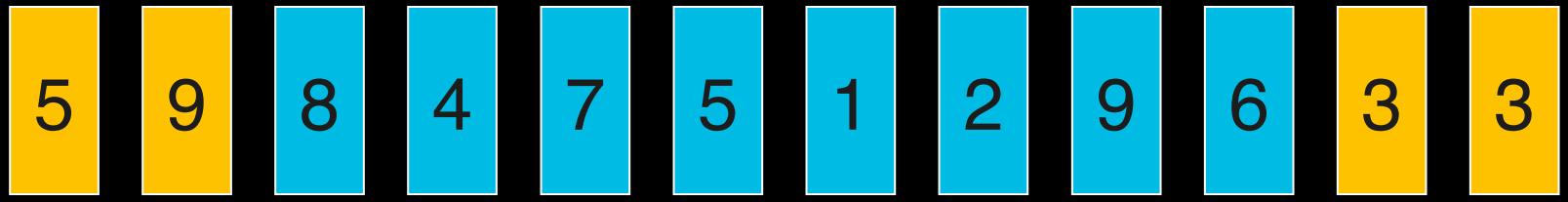
A GREEDY HEURISTIC



Take the larger end

But, maybe the larger end yields a good move for the opponent?

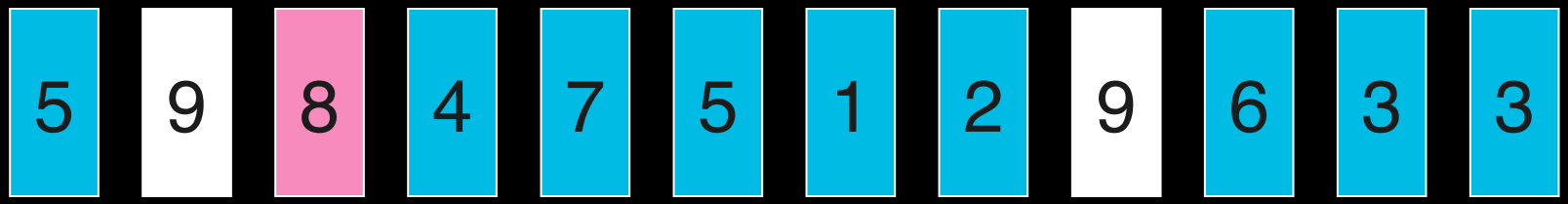
ANOTHER GREEDY HEURISTIC



Take the card from the end with the better "delta"

But, is it sufficient to look locally?

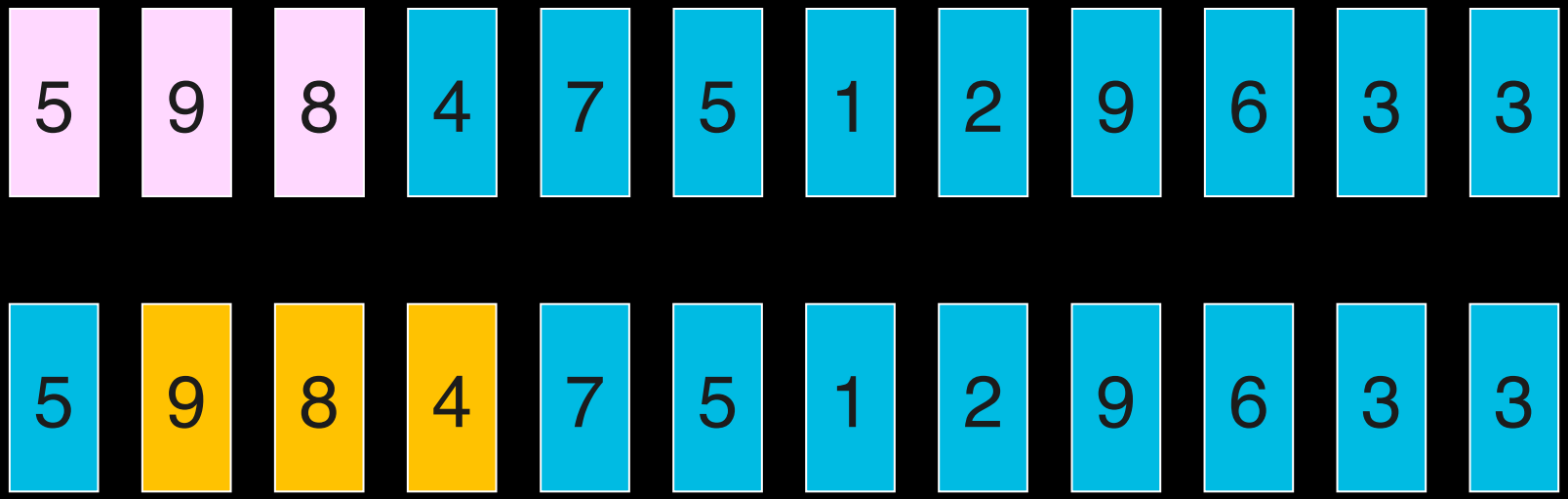
EXTREME-VALUES HEURISTIC



Identify the locations of the larger cards, and try to get these cards

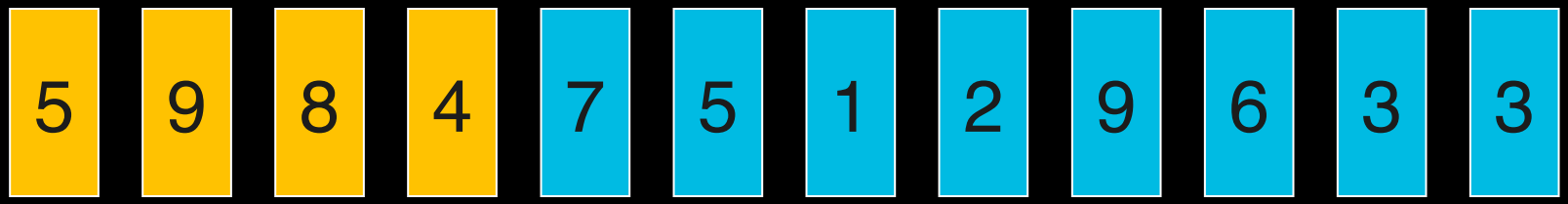
But, how to guarantee getting them?

A DETERMINISTIC APPROACH



Apply dynamic programming - solve the game for: all pairs, all triples, all quadruples …

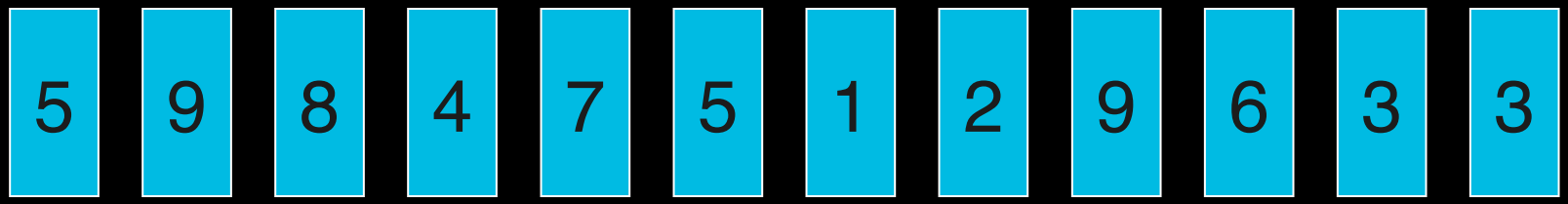
DYNAMIC PROGRAMMING



Not simple to program

Requires $O(N^2)$ time, $O(N)$ space (or even $O(N^2)$ space, if not experienced)

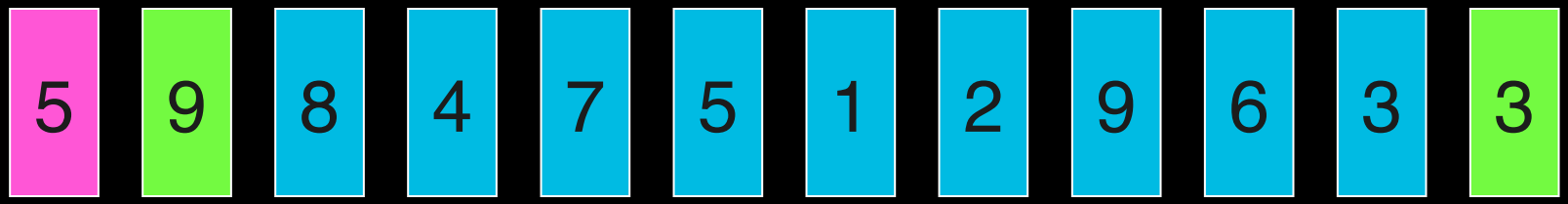
SEEK A PATTERN



Employ divergent thinking / creativity

The game involves selection from alternative locations; so maybe look, for a moment, only on locations and not on values

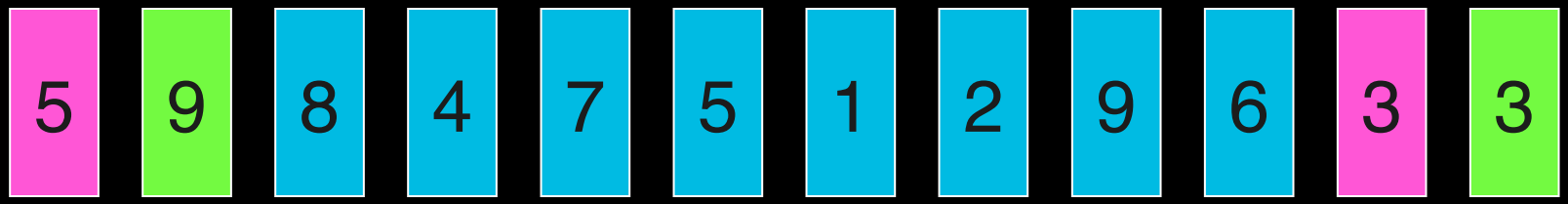
IDENTIFY A PATTERN



The leftmost card is initially in an odd location and the rightmost card – in an even location

If the leftmost card is removed, then the two ends will be initially-even locations (2^{nd} , 12^{th})

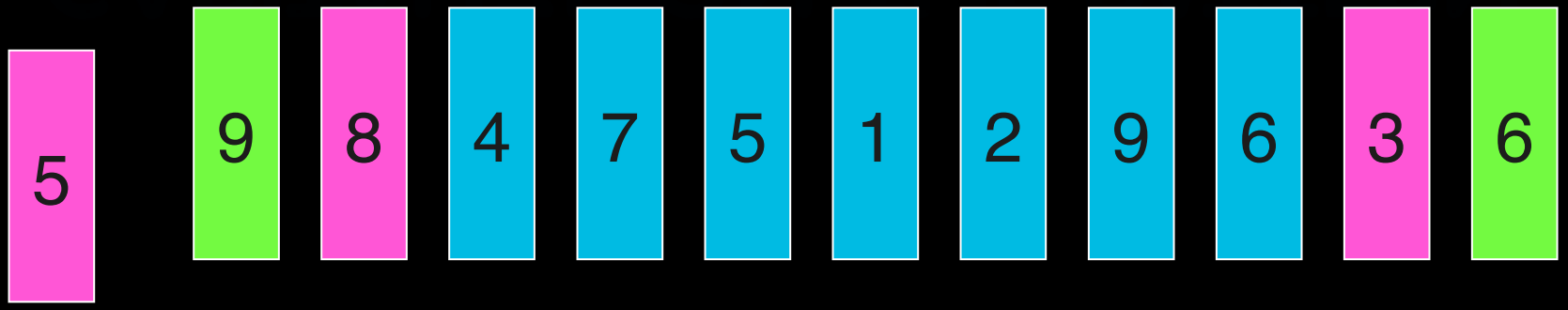
EXTEND THE PATTERN



If the leftmost card is removed, then the two ends will be initially-even location (2^{nd} , 12^{th})

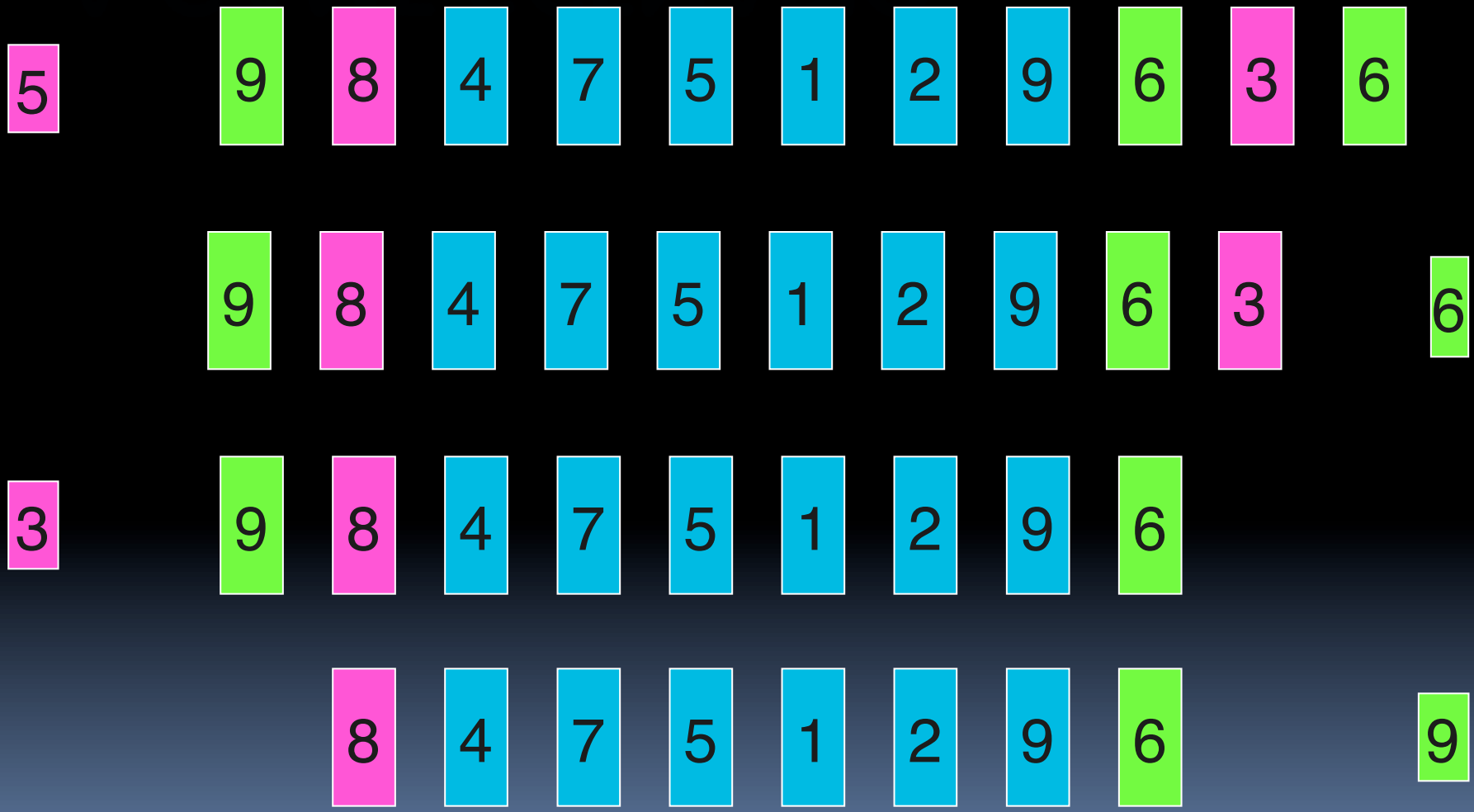
If the rightmost card is removed, then the two ends will be initially-odd location (1^{st} , 11^{th})

CAPITALIZE ON THE PATTERN

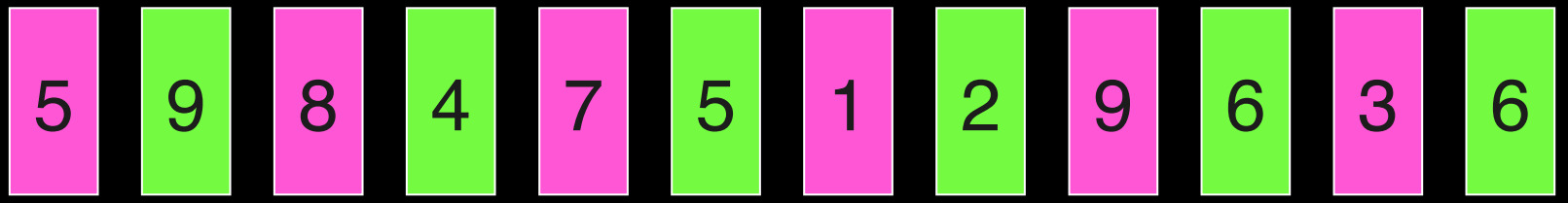


After I remove the card from the 1st location, both ends will be the 2nd and 12th locations; after my opponent will remove a card, there will again be an end of an initially-odd location; I may remove it, and again leave ends of initially-even locations

A GAME SCENARIO



GAME INVARIANT

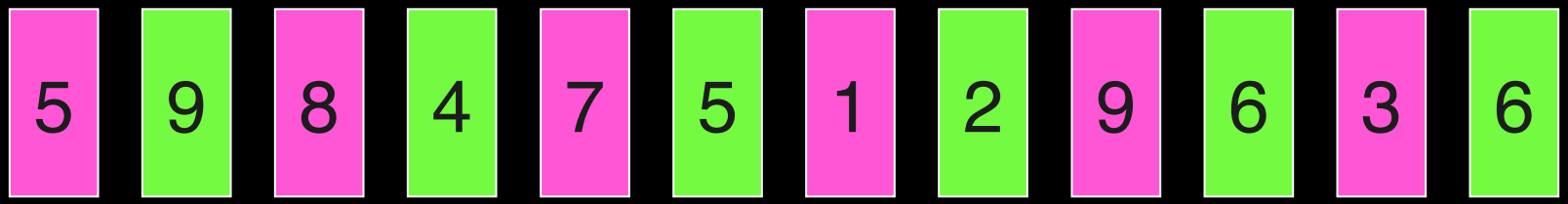


Following the above, I may play as follows:

"after each of my turns, both ends will be initially-even locations", or:

"after each of my turns both ends will be initially-odd locations"

GAME STRATEGY



Sum separately the values in the even locations (here – 32) and the values in the odd locations (here – 33).

Then, start with the end of the higher sum, and just "imitate" the opponent's moves

LESSON LEARNED

Greedy algorithms (be careful …)

Dynamic programming

Invariance

Values vs locations (addresses)

Divergent thinking

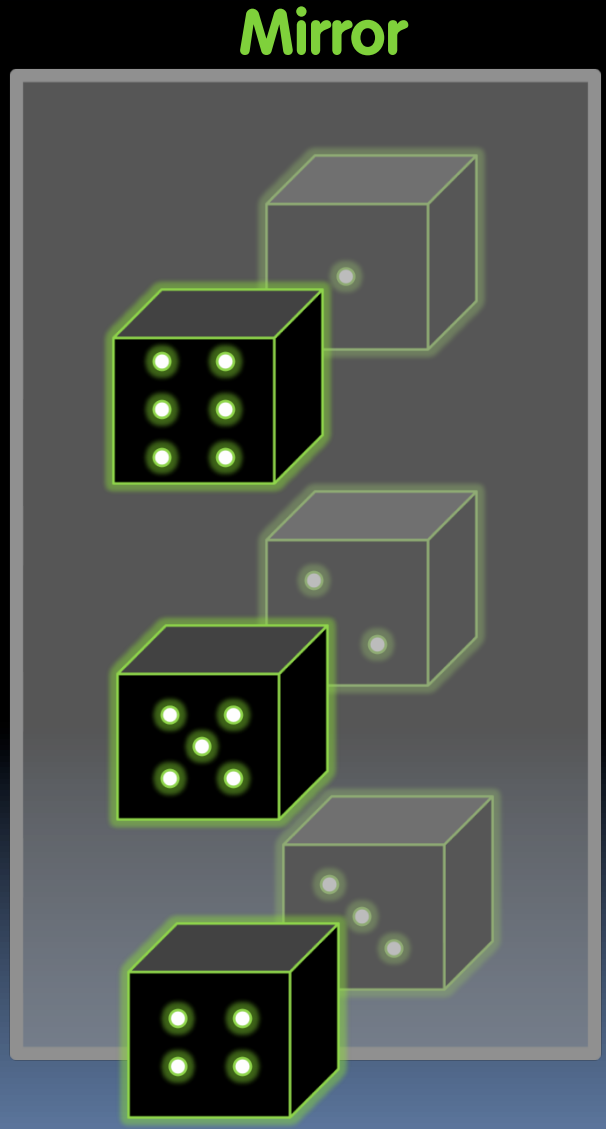
The Tricky Dice

- **Participant puts dice in a glass**
  - Magician guesses the numbers on the bottom
- **Source: Math Magic**

How it Works; What Students Learn

- **Mechanics**
  - Opposite die sides sum to 7
  - Answer is (7-die$_1$, 7-die$_2$)

- **Students Learn**
  - Good design (in how the pips on a die are chosen)
  - Simple algorithm
  - Complements

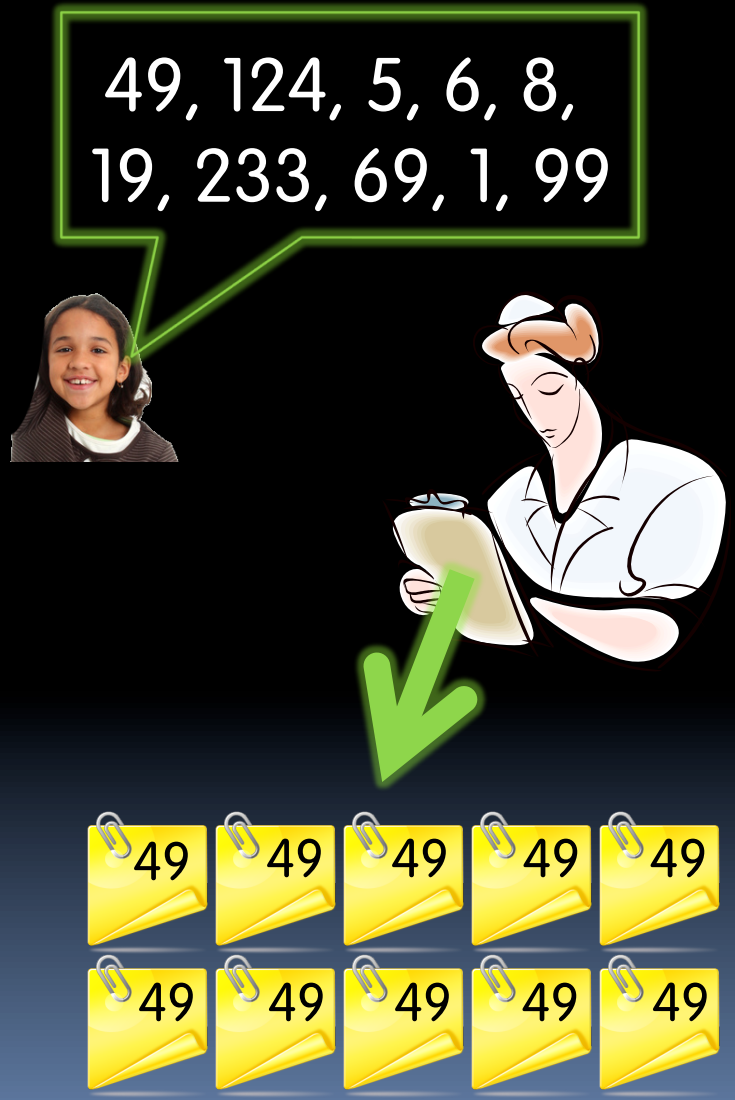


The Numbers Game

- **Participant calls out 10 numbers**
  - Magician writes numbers on paper, turns them over
  - Participant picks random one, magician guesses number
- **Source: CS4FN** www.cs4fn.org/magic/numbersgame.php

How it Works; What Students Learn

- **Mechanics**
  - Write 1st number on all cards.
  - You never promised anything more than that!
- **Students Learn**
  - HCI design principle
    - You developed a mental model
    - That wasn't how things worked
    - It's important to make the critical parts of the internal system visible to the user so they see what state it's in.



FIND THE CARD

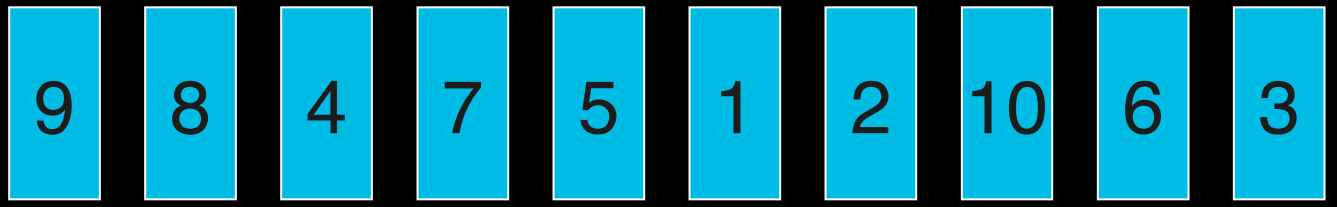
A volunteer from the audience arranges the cards 1..N in some permutation

Magician-1 looks at the line for a few seconds; leaves it as is, or swaps two cards

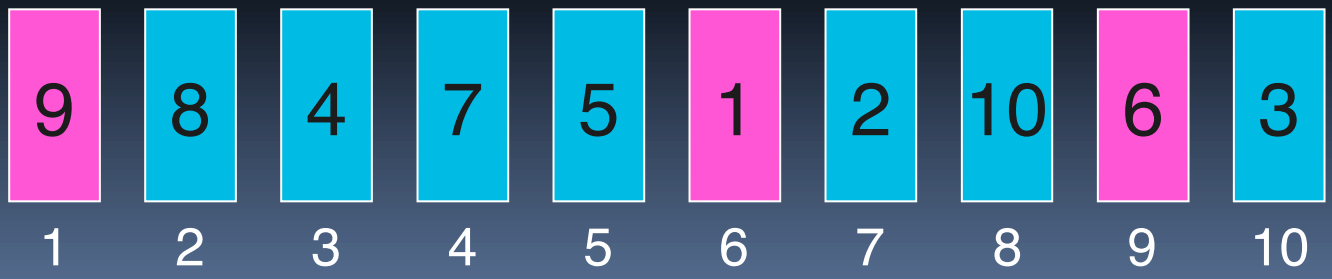
Magician-2 enters the room, and the audience calls out any of the integer 1..N

Magician-2 finds the called integer in at most N/2 attempts

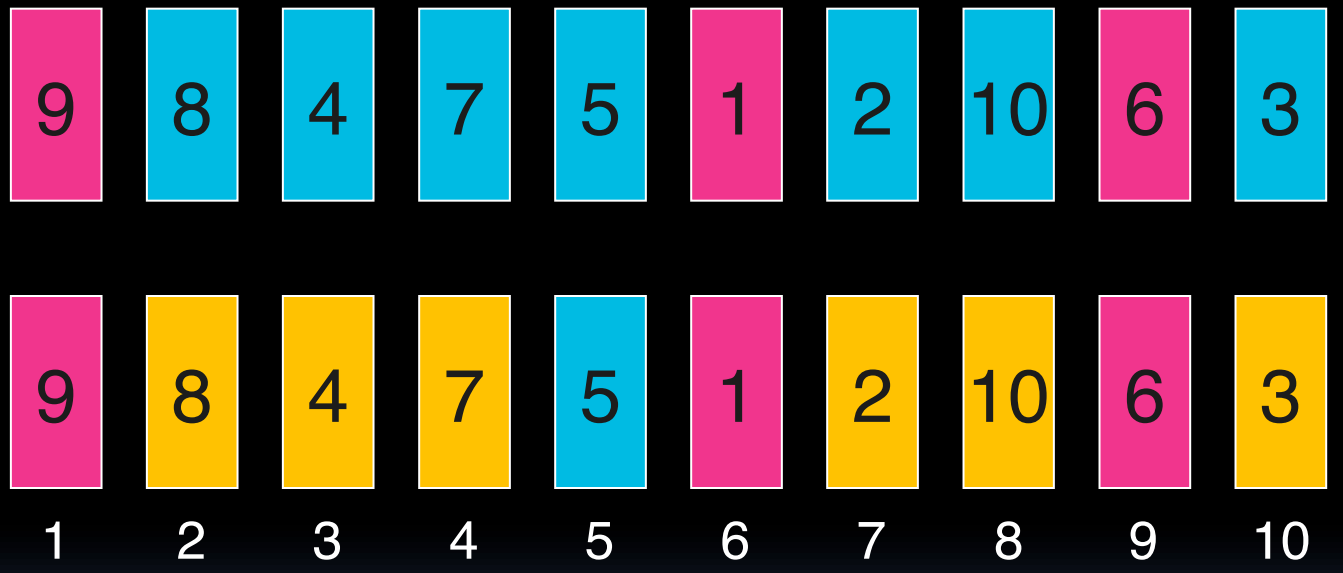
DIVERGENT THINKING



Each value is in the range of the card locations; look at values as pointers (?)

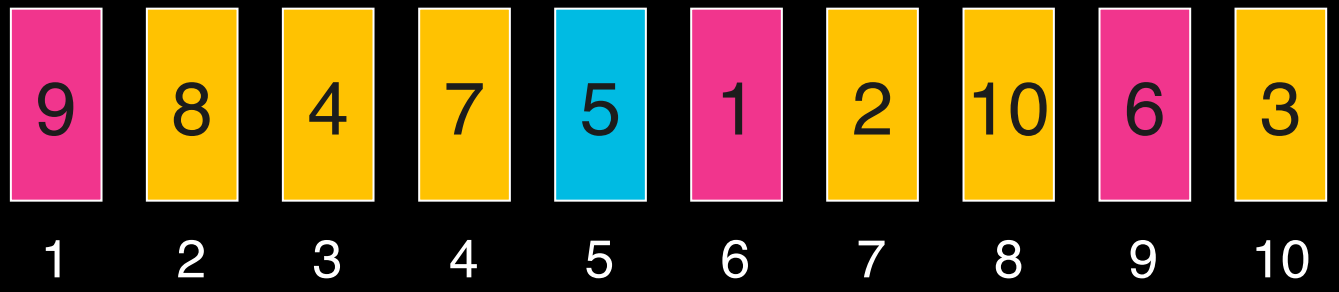


CYCLES OF POINTERS



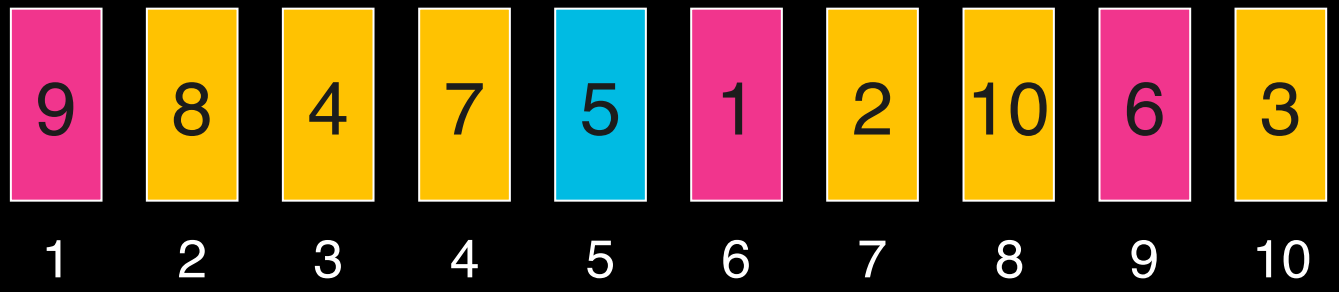
Each card points-at exactly one other card, and is pointed-by one other card

EACH CARD IN ONE CYCLE



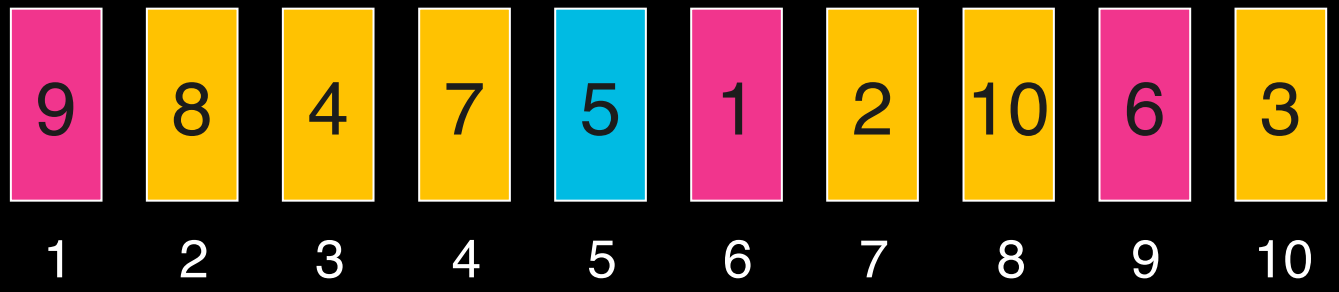
Each card is in exactly one cycle; the card in the i^{th} location is in a cycle with the card whose value is i, which is it's predecessor in the cycle

FOLLOW THE CYCLE OF CARD i



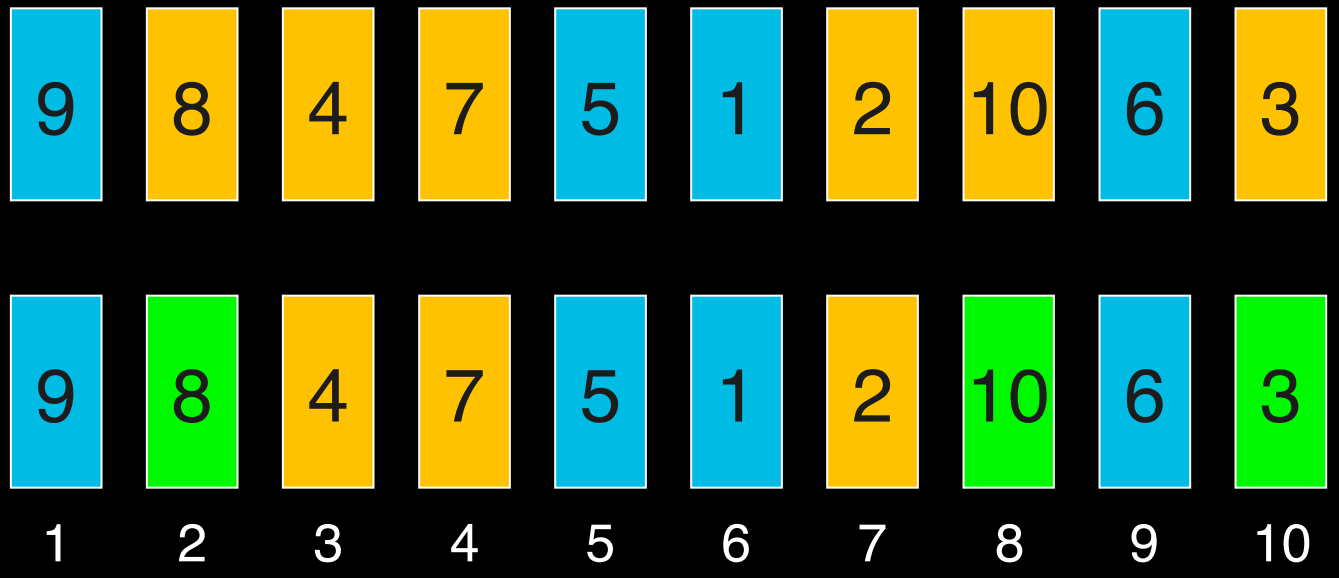
Thus, we may reach the card with value i, by following the pointers starting from the i^{th} card; the amount of work will be the length of the cycle that we follow

AT MOST ONE LONG CYCLE

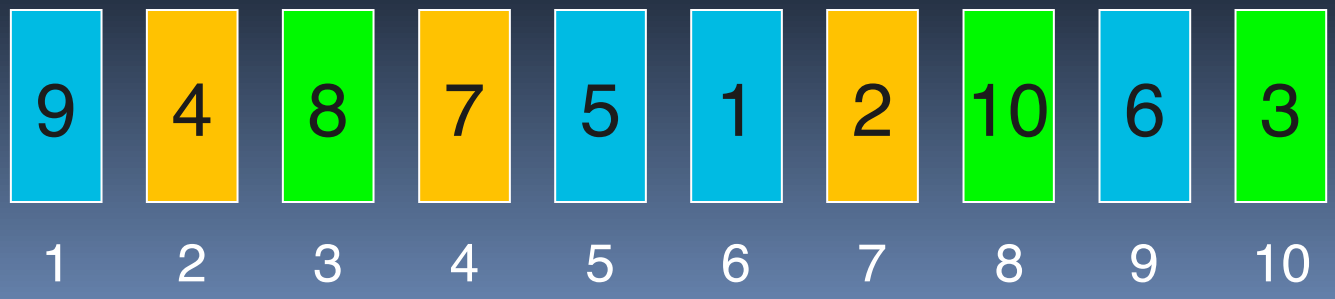


There may be at most one cycle of length larger than N/2; if such a cycle exists, then we may break it into two cycles, by swapping two of its pointers

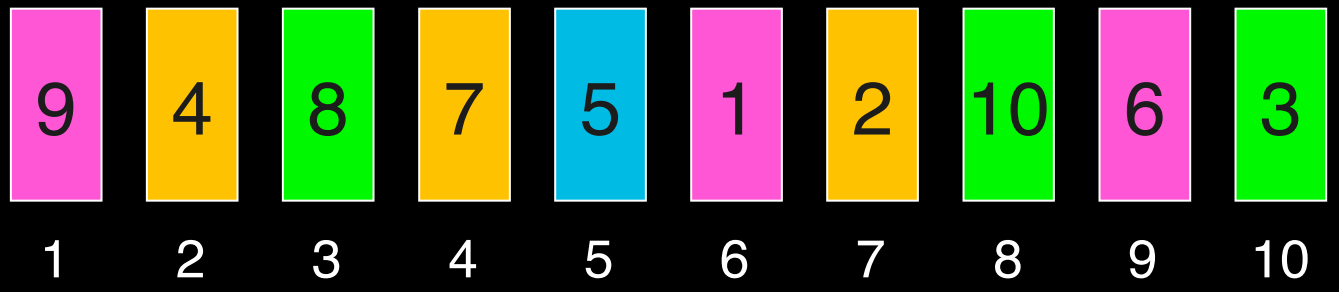
BREAK ONE CYCLE INTO TWO



Swap the cycle's middle with its start



BOUND MAX CYCLE LENGTH



So, if there is a cycle of length larger than N/2, break it after at most N/2 hops into two smaller cycles; each of the cycles will not exceed the length of N/2

LESSON LEARNED

Values as locations (pointers)

Cycles of values in a permutation

A cycle may be broken into two cycles by swapping the locations of two of its pointers

Divergent thinking

Guess Your Age

- Volunteer is handed several cards with "random" numbers written on them
  - Returns the cards with age listed on them
- Magician glances and says the person's age
- Source: Math Magic

A	8 9 10 11 12 13 14 15
B	4 5 6 7 12 13 14 15
C	2 3 6 7 10 11 14 15
D	1 3 5 7 9 11 13 15

A, B, D

How it Works; What Students Learn

- **Mechanics**
  - Every card is a column in the binary table
    - E.g., "2^1 = 2 card has { 2, 3, 6, 7, 10, 11, 14, 15 }
  - The first number in the list is the number of the card.
  - Just add those together
- **Students Learn**
  - Intro to Algorithms
  - Binary numbers
  - Encoding
  - Lookup table for quick calc!

	A	B	C	D
N	2^3	2^2	2^1	2^0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

A, B, D

8+4+1
13

Guess the birthday day of the week

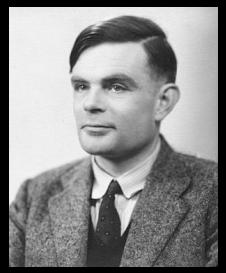
- Volunteer is asked for their birthday
- Magician says what day of the week it was
- Source: `en.wikipedia.org/wiki/Zeller%27s_congruence`

How it Works

- **What they learn:** Don't nec generalize
  - Alg, value of lookup table, modulo

- **Mechanics**
  - Complicated general alg:

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor - 2J\right) \mod 7,$$



Easier: M, D, Y in 1900s

- Turing: June 23, 1912 (6/23/12)
- Sunday!

- **Algorithm**
  - If M =1 or 2, Y = Y - 1

(F(M) + D + Y + Y/4) % 7

- 1-index: 1=Mon, 2=Tue, etc.
- Y/4 fast by 20s: 0,5,3,1,-1

Example

- (4+23+12+3) % 7
- (4 + 2 + -2 + 3) = 7 (Sunday!)

Y	Y/4%7
0	0
20	5
40	3
60	1
80	-1

M	F(M)	My memory trick
1	1	"1 and 3 pass through"
2	4	24 hours in a day
3	3	"1 and 3 pass through"
4	6	4-6 are flips
5	7	Heinz 57 sauce
6	4	6-4 are flips
7	6	Spirit of 76
8	2	8 is made up of 2s
9	5	Working 9-to-5
10	0	Only # with 0 … is 0!
11	3	11 in Binary is 3
12	5	Working 12-5 (afternoons)

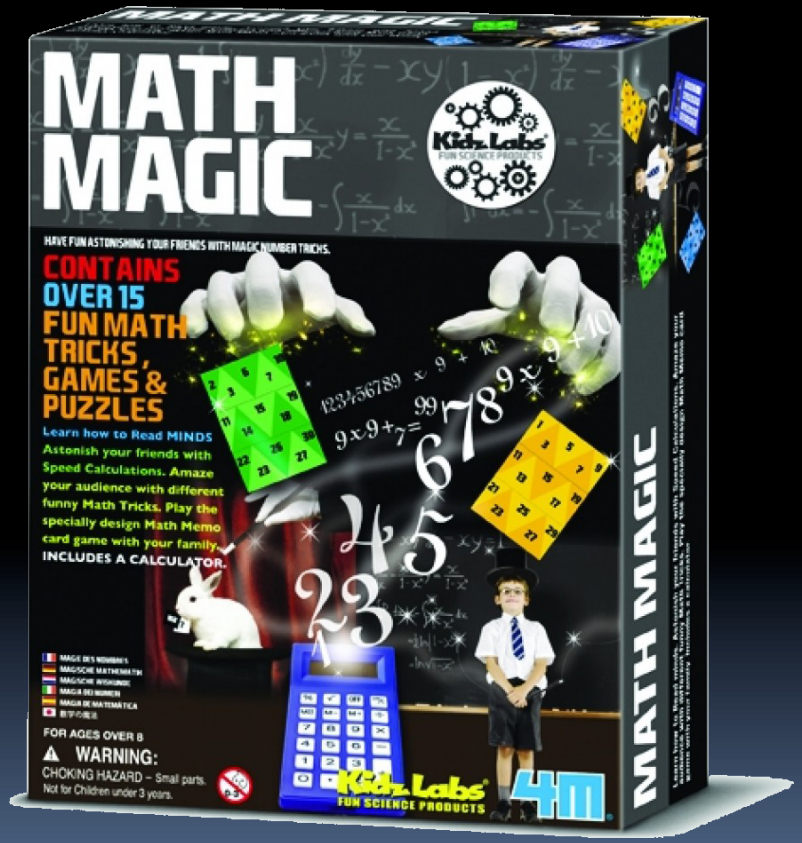
Great Resource : CS4FN

- Paul Curzon, Peter McOwan, Jonathan Black @ Queen Mary, University of London
  - CS4FN magazine
  - Two free books on Magic and CS!
  - Some online apps
- If you'd like to contribute tricks, contact them…

Great Resource : Math Magic

www.parents-choice.org/product.cfm?product_id=29300

- **Kids Labs, $12**
- **They include dice, cards, calculator, templates, …**
  - Some of our tricks we demonstrated today were from this great resource!



AND IN CONCLUSION…
MAGIC MAY BE USED TO

Motivate, Illustrate, and Elaborate on:

- Computing notions
- Problem solving
- Divergent Thinking

Audience Participation

Do YOU have any magic to share?