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Demystifying Computing with Magic

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Special Session Overview

- **Motivation**
- **The 5 magic tricks**
  - The 21 Card Trick
  - Magic Hats
  - Guess the Value
  - Josephus Flavius Circle Game
  - Fitch Cheneys Five Card Trick
- **Reflection**
- **Other References**
- **YOU contribute your tricks**

Magic

is Fun!

but Magic

Can be much more than fun

Magic May be Used to

Motivate, Illustrate, and Elaborate on:

- Computing notions
- Problem solving
- Creativity

Computing Notions

- Discrete math terms: e.g., permutations,
- Problem representation: e.g., binary digits
- Algorithmic patterns: e.g., sorting
- General notions: e.g., symmetry

Problem Solving Heuristics

- Problem decomposition
- Simplification, Generalization
- Backward reasoning
- Analogy (transfer)
- Problem representation

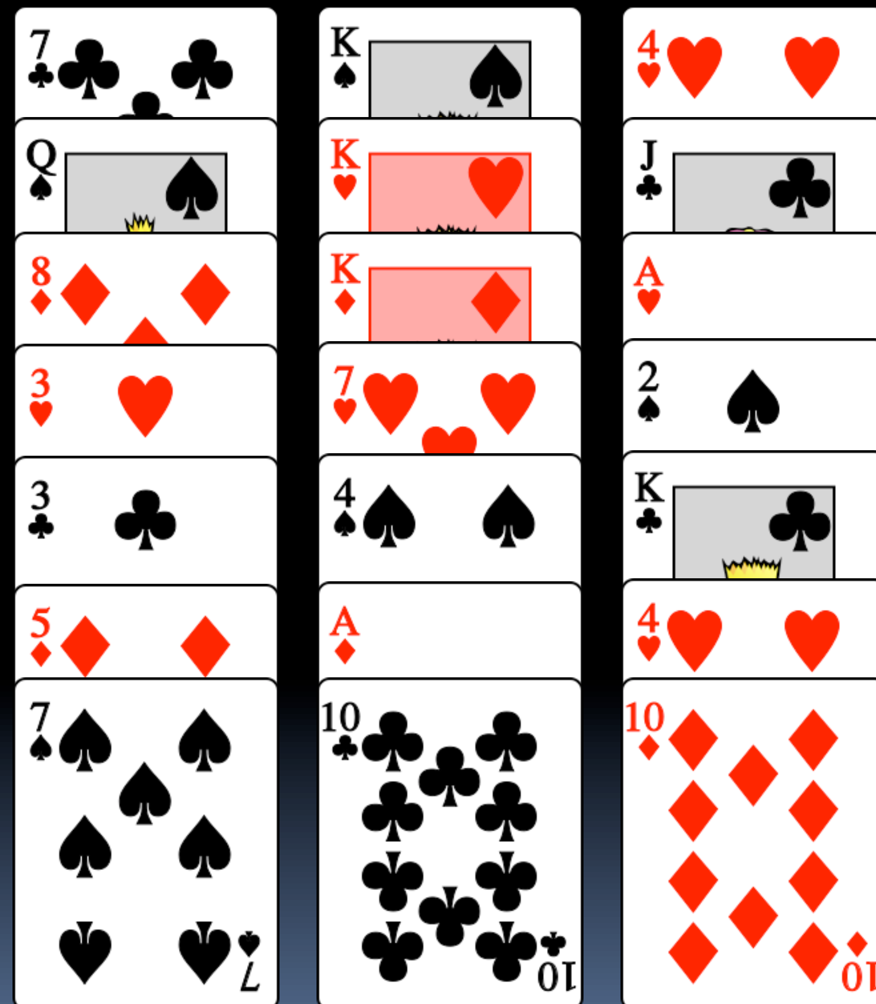
Creativity

In mathematics education, e.g. [Silver 1997]:

- Fluency: diverse directions
- Flexibility: adaptation to the task at hand
- Originality: unfamiliar utilization of familiar notions
- Awareness of possible fixations

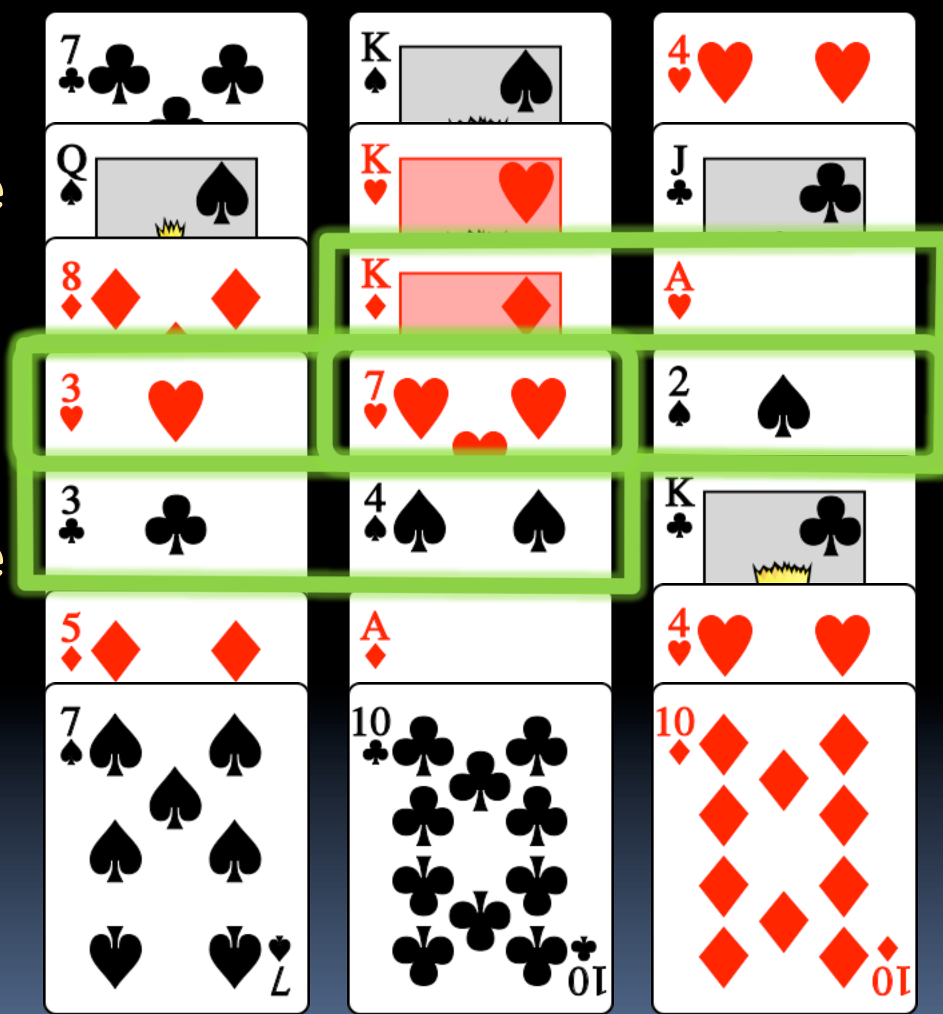
11th Variation, aka the "21 card trick"

- 21 cards in 7x3 grid
- Volunteer picks card, tells column
- Dealer puts that column in middle, redeals by row
- Twice repeated
- Dealer chooses card behind back!



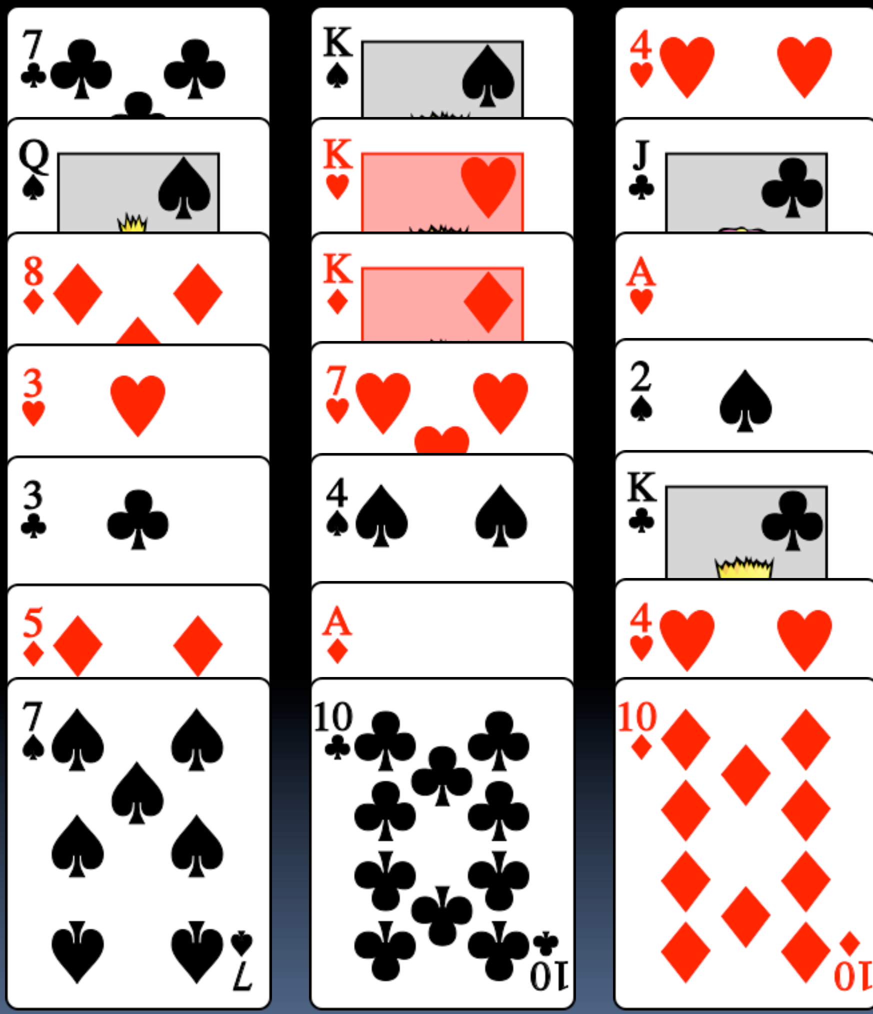
How it works…

- **After the 1st redeal, the cards are #8-14**
  - Redealt by rows, they're in middle 7
- **After the 2nd redeal, they're in #10-12**
  - Redealt by rows, they're in the middle 3
- **After the 3rd gather, it's card #11!**
  - Count 10, and that's it!!



What students learn…

- **Computing Notions**
  - Intro to Algorithms
  - Permutations
  - Markov Chains
  - Convergence
  - Ternary search
  - Correctness proof
- **Problem Solving**
  - Analysis, simulation
- **Creativity**
  - Visualization with arrows
- **Talking points**
  - Max # cards?



Calling the Color of my Hat

Three people, a hat is put on each of them.

Each can see the other two hats but not his own

Each hat may be: Gold, Silver, or Green.

Each person looks at the other two hats.

A the same time each person calls his hat color.

At least one of them is right!

Task Reduction – 2 Hats

4 cases: G G S S G S S G

Asymmetric rules:

- Person-1: Call the color that you see
- Person-2: Call the opposite of what you see

Each person "covers" two cases

But, how to extend to 3 hats?

In the 2-hat case one person "went for" equal colors, and the other – for different colors

So, maybe we'll do the same here …

But, each person sees two hats … maybe they will be of the same color, or – of different colors … so, if "same color" then perhaps one will call this color … and if "different colors" … then will call … ???

Beware of Fixation

The Magic Again, Differently

Version-1: 4 people, 4 hats

Version-2: All of the people are right,

or all of them are wrong

Task Reduction – 2 Hats

4 cases: G G S S G S S G

Asymmetric rules:

- Person-1: Call the color that you see
- Person-2: Call the opposite of what you see

Each person "covers" two cases

Fluency

Seek diverse, relevant observations:

- Asymmetric rules
- Each rule “covers” 2 separate cases
- All the 4 cases are “covered”
- Binary representation of the colors (?)

3 Hats: 3 colors, 27 cases

Each color may be 0, 1, or 2

Therefore, 27 cases: 0 0 0,   0 0 1, … 2 2 2

Each person will "cover" 9 cases (?)

How to split the cases between them?

Originality

Manipulate the three numbers that represent the hats … but, according to what feature?

- Equality of numbers (?)
- Differences between pairs of numbers (?)
- Sums of numbers (?)
- Modulo of numbers (?)

How to manipulate?

Perhaps look at sums?

In the 2-hat case the sum could be 0, 1, or 2:

- Person-1 “covered” the integers: 0 and 2

- Person-2 “covered” the integer: 1

Here there are 7 options for the sum:
0, 1, 2, … 6.   How to split them?

How to divide {0,1,2,3,4,5,6} ?

One person will "cover" 3 integers, and the other two people – will "cover" 2 integers each

What should be the integers of the 1st person?

{1, 2, 3}?

Or: {1, 3, 5}?

Or: {0, 3, 6}?

Learn from the 2-Hat case

In the 2-hat case one "went for odd" and the other "went for even"

So, maybe the 1st person here will "go for" {1, 3, 5}?

How will the other two split {0, 2, 4, 6}?

Maybe: {0 2} and {4 6}?
Or: {0 6} and {2 4}?

Flexibility

We may transfer the "odd/even" in the 2-hat case into here not just as "odd/even", but as the remainders of 2. So, in the 3-hat case we may look at remainders of 3.

The 1^{st} person may "go for" {0, 3, 6}

The 2^{nd} – to {1, 4}

And the 3^{rd} – to {2,5}

How will each play his part?

Person-1 will “go for” 0 modulo 3

Person-2 will “go for” 1 modulo 3

Person-3 will “go for” 2 modulo 3

Example 1

We define: Gold = 0, Silver = 1, Green = 2

Example:

Three Hats: Gold, Gold, Green

Person-1 sees: 0, 2 calls: 1 (Silver)

Person-2 sees: 0, 2 calls: 2 (Green)

Person-3 sees: 0, 0 calls: 2 (Green)

Example 2

We define: Gold = 0, Silver = 1, Green = 2

Example:

Three Hats: Gold, Green, Silver

Person-1 sees: 2, 1 calls: 0 (Gold)

Person-2 sees: 0, 1 calls: 0 (Gold)

Person-3 sees: 0, 2 calls: 0 (Gold)

Reflection

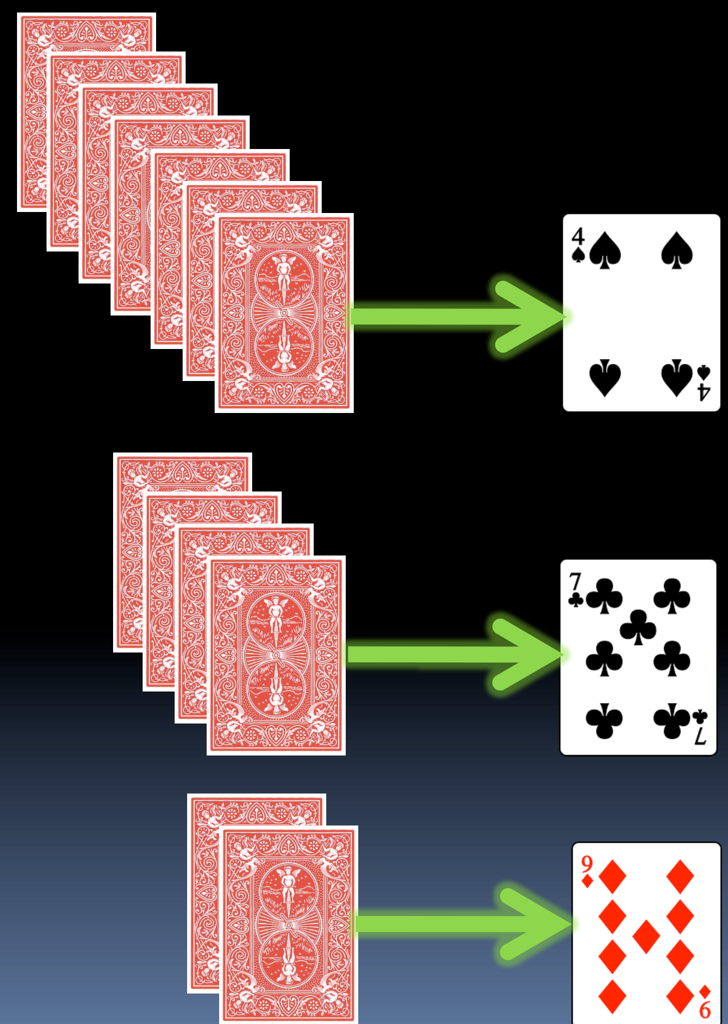
Computing notions: Numeric representation, disjoint sets, complement, modulo arithmetic

Problem solving: Simplification and generalization, problem representation

Creativity: Diverse attempts, sum utilization, observation of “odd/even” as remainders

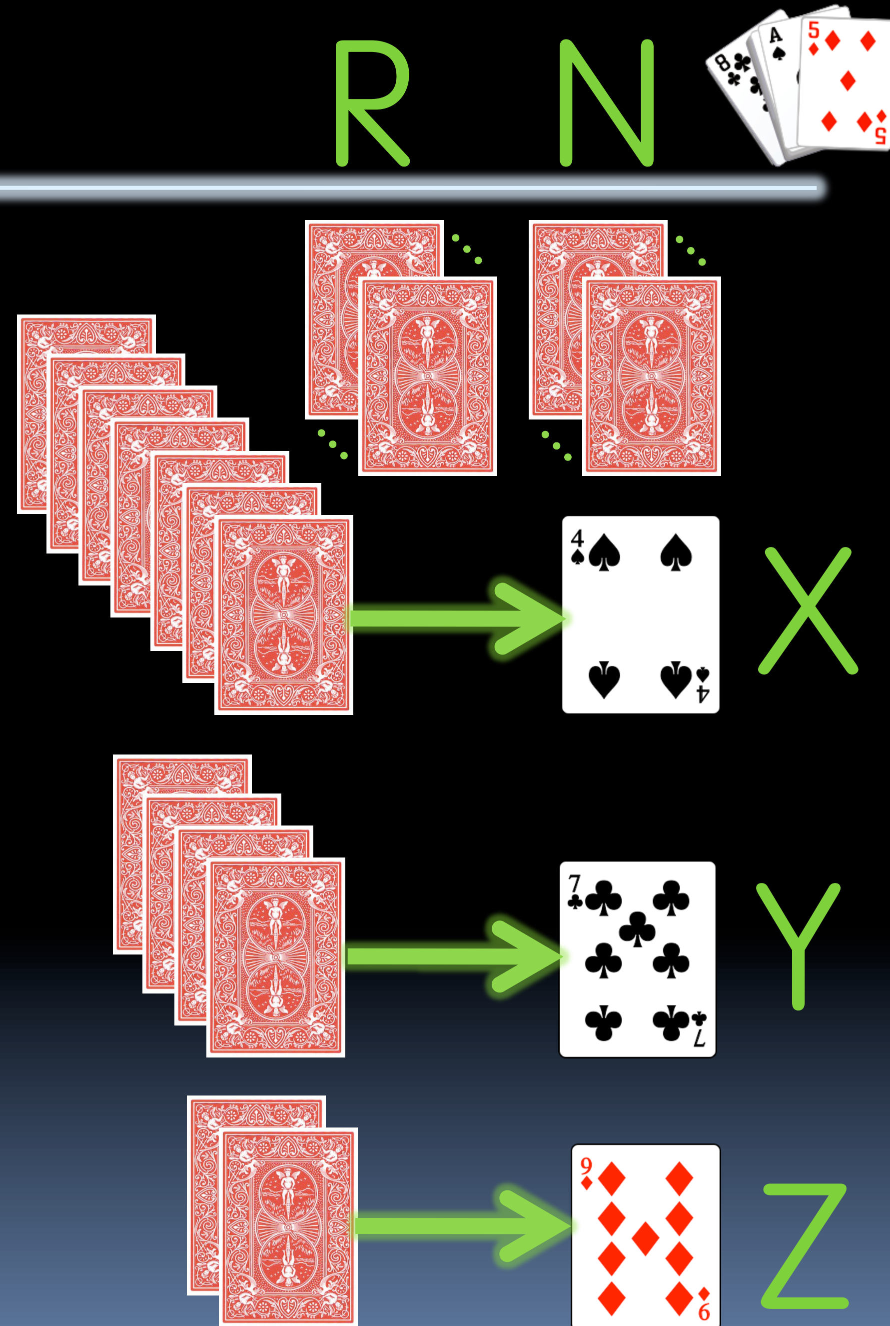
Guess the Value

- Remove N cards
- Remaining turned face down in piles from top card to 10
  - E.g., "7, 8, 9, 10"
    - (4 cards)
  - E.g., "9, 10"
    - (2 cards)
- Audience keeps 3 piles, # cards picked = sum 3 top cards



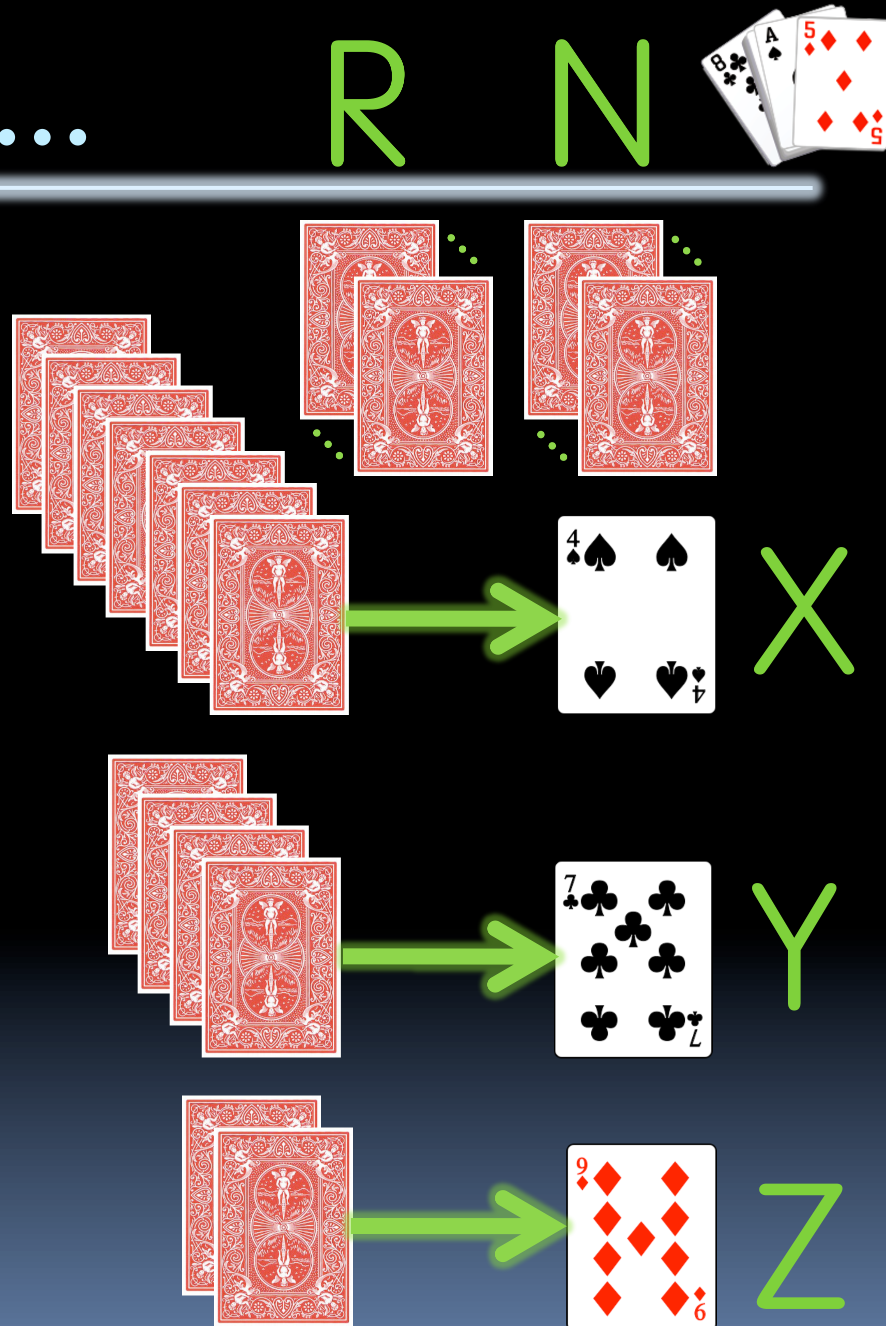
How it Works

- **3 Piles, top cards**
  - X, Y, Z
- **# cards in Piles?**
  - (11-X), (11-Y), (11-Z)
  - 33 – X – Y – Z
- **Remaining Cards R?**
  - R = 52 – Removed – Piles
  - R = 52 – N – (33-X-Y-Z)
  - R = 19 – N + X + Y + Z
- **What N s.t. R = X+Y+Z?**
  - N = 19



What students learn…

- **Computing Notions**
  - Intro to Algorithms
  - Correctness proof
  - Error-handling
    - What happens when you see a face card?
- **Problem Solving**
  - Importance of a good problem representation
  - Notion of complement
    - Number of cards = 11 – top
- **Creativity**
  - Algebraic representation



The Josephus Problem

N people in a circle, numbered 1..N.

Starting with person-2, every second person leaves the circle.

What is the number of the (last) survivor?

Fluency

Seek diverse, relevant observations:

- All the even numbers leave first
- If the circle is of an even number of people, then person-1 survives the next cycle
- The answers for N=2, 3, 4, 5, 6, … may help
- Binary representation (?)

Decompose the Problem

Separate between: N is a power of 2, or not

If N is a power of 2: each cycle will leave an even amount of people; person-1 will always be skipped, and remain last

If N is not a power of 2: person-1 will not remain last … but how can we tell the last?

Flexibility

Solve the general case by capitalizing on the special case, of "N is a power of 2"

Reason backwards: if we knew the number of the person whose deletion yields a power-of-2 remaining people … then we can tell the last survivor

Telling the Survivor's Number

D = the difference between N and the closest power-of-2 smaller-than or equal-to N

After D people will leave, the circle will include a power-of-2 people

Thus, the number of the last survivor is 2D+1

Examples

Example-1: N=41 → D=9

The 9^{th} person to leave is number 18. At this point, 32 people will remain, so the 1^{st} person in this remaining cycle will survive.

The survivor: number 19.

Example-2: N=60 → D=28

The survivor: $2\times D+1 = 2\times 28+1 = 57$

Binary Representation

Example-2: N=60 → D=28

The survivor: $2 \times D+1 = 2 \times 28+1 = 57$

N=60 in binary: 1 1 1 1 0 0

D=28 in binary: 0 1 1 1 0 0

2D in binary: 1 1 1 0 0 0 (shift left)

2D+1 in binary: 1 1 1 0 0 1 (cyclic shift left)

Reflection

Computing notions: Binary representation, powers-of-2, complementing cases

Problem solving: Problem decomposition, backward reasoning

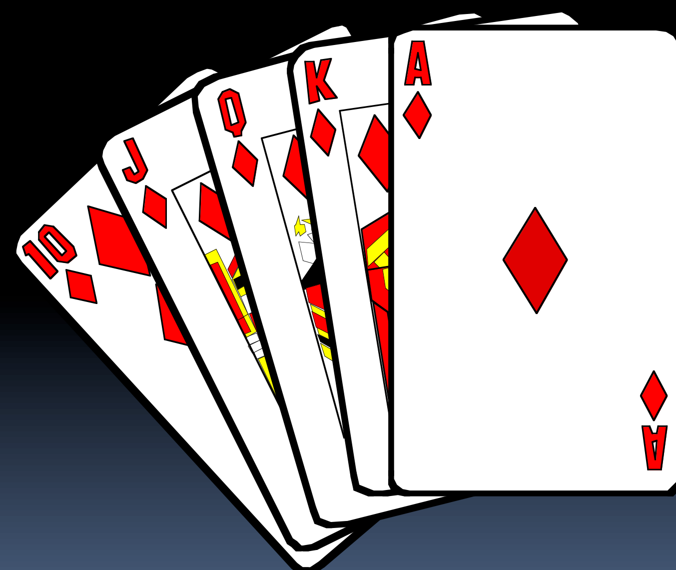
Creativity: Capitalizing on case-1's solution for solving case-2,   2D = binary shift left

Fitch Cheneys Five Card Trick

1. (assistant off-stage)
2. Audience chooses 5 cards from deck, gives to Dan
3. Dan picks 1, gives back to audience
4. Dan puts his 4 in some order, leaves
5. Assistant enters, says audience card

Aha! #1 : This is encoding / decoding!

- Just like Pig Latin to confuse parents
- This is really information passing between mathemagician and assistant
- But how <u>did</u> they pass on that info?



J♦ 5♠ A♣ 4♥

Aha! #2 : All cards are fully ordered

- **This isn't as easy with 52 random objects**
- **How should <u>we</u> order cards?**
  - Ranks are ordered 2-10, J, Q, K (A?)
  - Suits are already ordered: ♦ ♣ ♥ ♠
  - But, which first?
  - A♦ A♣ A♥ A♠ 2♦ 2♣ or A♦ 2♦ 3♦ 4♦

Aha! #3 : Independently find rank, suit

- **With the four cards, we can either…**
  - Use all 4 cards to choose that card (this is what often comes to mind first!)
  - Use some to choose rank, some for suit.
- **Big idea: decouple hidden card into 2 dimensions, rank and suit**

Aha! #4 : Pigeon-hole the suit (5 > 4)

- With 5 cards, but only 4 suits,
  at least 2 have to have the same suit!
- Thus, the suit of the hidden card same as suit of, say, leftmost card
- But that only leaves us
  with 3 cards for
  encoding the rank (1 out of 13!)

Aha! #5 : Hidden card is 1 of 12, <u>not</u> 13

- Since the suit-revealing card is up, then there **are only <u>12</u> cards left**
- Hmm, how do I specify from among 12 cards by reordering the other <u>3</u>?

Aha! #6 : Permutation is n!, 3! = 6

- With 3 cards left, can choose 3! = 6 things by reordering them…Aha! #2
- But there are 12 cards there!
- All we need is 1 bit, Rodney…
- Do we backtrack or continue?

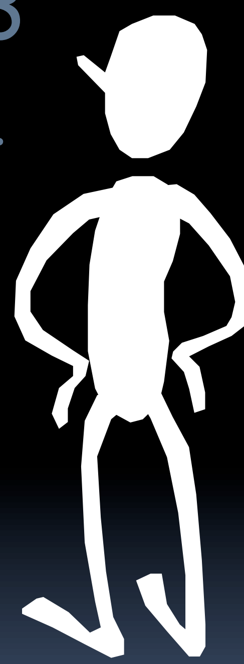
1=123
2=132
3=213
4=231
5=312
6=321

Aha! #7 : 1 bit = which card we hide!

- We had a choice of that bit in step 3
- Which card did Dan give to audience?
- **Which card should we hide?**
  - If we know that we'll be able to specify an additional number from 1 through 6, say as an offset.

Aha! #8 : See a (mod) 13-hr clock

- Same-suit cards are hands on a clock
- Find acute angle
- Show “earlier” card
- Hide “later” card
- (Earlier + [1-6]) mod 13 ≡ Later



Let's do one together, shall we?

- Audience hands us

J♦ A♣ 4♥ 5♠ 3♦

1. Which two same suit? J♦ & 3♦
2. Which do we hide among J & 3? 3♦
3. Place J♦ on the left, reorder others
4. Want 5 (312), so J♦ 5♠ A♣ 4♥

What do students learn from this?

- **Computing Notions**
  1. Information theory, compression
  2. Full ordering of a set (52 cards)
  3. Decomposition (rank, suit)
  4. Pidgeon-hole principle (5 > 4)
  5. Off-by-one matters (12 not 13)
  6. Permutation and combinatorics (3! = 6)
  7. Constraints (1 bit left)
  8. Modulo arithmetic (modulo 13)
- **Problem Solving**
  - Solution decomposition into 8 aha stages
- **Creativity**
  - Recognition that two same-suit cards are no more than 6 away

Fitch Cheneys Five Card Trick Variant

1. (assistant off-stage)
2. Audience chooses 5 cards from deck, gives to Dan
3. Dan picks 1, gives back to audience
4. Dan throws 1 away
5. Dan puts his 3 in some order, makes it “harder” by flipping some, leaves
6. Assistant enters, says audience card

Tremendous Resource : CS4FN

- **Paul Curzon, Peter McOwan, Jonathan Black @ Queen Mary, University of London**
  - CS4FN magazine
  - Two free books on Magic and CS!
  - Some online apps
- **If you'd like to contribute tricks, contact them…**

And in conclusion… Magic May be Used to

Motivate, Illustrate, and Elaborate on:

- Computing notions
- Problem solving
- Creativity

Audience Participation
Do YOU have any magic to share?