1. Derive the following property of the two-dimensional Fourier transform: if an object is rotated through an angle θ, its Fourier transform is rotated through the same angle θ.
2. A two-dimensional circularly symmetric function is completely specified by either its cross section or by its projection. Show how each of these one-dimensional functions can be reconstructed from the other.
3. Suppose we have a biased coin, with the probability of a '0' being $\frac{1}{2} + \epsilon$ and that of a '1' being $\frac{1}{2} - \epsilon$. Using the usual $0, 1$ notation and then the $+1, -1$ notation, prove that if we toss the coin k times, the probability of the parity of the results being *even* is $\frac{1}{2}(1 + (2\epsilon)^k)$.
4. Let $f, g, h : \mathbb{F}_2^n \to \mathbb{F}_2$. Relate $Prob_{x,y} f(x)g(y) = h(x \cdot y)$ to $\max_\alpha \left|\hat{f}_\alpha\right|$, $\max_\beta |\hat{g}_\beta|$ and $\max_\gamma \left|\hat{h}_\gamma\right|$.
5. Let H be a subgroup of Z_m, the group of integers mod m. Let K be a coset of H. If $f : Z_m \to \mathbb{R}$ is a function such that $f(x) = 1$ for all $x \in K$ and $f(x) = 0$ for all $x \notin K$, determine the Fourier transform of f (over the group Z_m).
6. Given a boolean function $f : Z_2^n \to Z_2$ (i.e. given an oracle for computing f), give an efficient algorithm to estimate all Fourier coefficients of f that have magnitude at least ϵ. Your algorithm should run in time polynomial in $1/\epsilon$ and $1/\delta$, where each significant (greater than ϵ magnitude) fourier coefficient is estimated to with in δ.