1. Prove that the Bell state $|\psi^-\rangle$ is rotationally invariant: i.e. $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|vv^\perp\rangle - |v^\perp v\rangle)$.

2. You are given one of two quantum states of a single qubit: either $|\phi\rangle = |0\rangle$ or $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\phi\rangle$ or $|\psi\rangle$ with 50% probability each, what is the probability that your measurement correctly identifies the state? Can you generalize your result to distinguish between two arbitrary quantum states $|\phi\rangle$ and $|\psi\rangle$ on two qubits?

3. Suppose you have two entangled qubits in the Bell state $|\psi^-\rangle$. You apply the teleportation protocol to the first qubit. What is the result?

4. Consider a CNOT gate whose second (target) input is $|0\rangle - |1\rangle$. Describe the action of the CNOT gate on the first (control) qubit.

   Now show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

5. Show that given a circuit with n inputs and m gates, there is an equivalent reversible circuit with $O(n \log m)$ inputs and $O(m^{\log_2 3})$ gates.

   Can you generalize your construction to reduce the number of gates to $O(m^{1+\epsilon})$.