1. Let u, v be distinct elements of Z_2^n. Let $f : Z_2^n \to Z_2^n$ be a four to one function such that for every $x \in Z_2^n$, $f(x) = f(x+u) = f(x+v) = f(x+u+v)$. Give an efficient quantum algorithm that takes as input a program for computing f, and outputs $\{u, v, u+v\}$.

2. Show how to implement the conditional rotation gate given by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$

   $(0 \leq \phi < 2\pi)$ using elementary quantum gates.

3. Let $v \in \mathcal{C}^{2^n}$ be an arbitrary unit vector; i.e. $v = \sum_x \alpha_x |x\rangle$ with $\sum_x |\alpha_x|^2 = 1$. Give a quantum circuit, with polynomial in n gates, that maps v to the vector $|0^n\rangle$.

4. Let $f : \{0,1\}^n \to \{0,1\}$ be a function such that either $f(x) = 0$ for all n-bit strings x, or $f(x) = 1$ for at least ϵ fraction of the n-bit strings x. Show that any quantum algorithm that takes as input a black box program for computing f must take $\omega(1/\sqrt{\epsilon})$ steps to decide which of the two conditions f satisfies.

5. **Extra credit** Let $f : \{0,1\}^n \to \{0,1\}^n$ be a bijection. Show that any quantum algorithm that takes as input a black box program for computing f must take $\omega(2^{n/3})$ steps to compute the least significant bit of $f^{-1}(0^n)$.