Does It Take Longer to Injection Lock a High-Q Oscillator? [Extended Abstract]

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Injection locking is a phenomenon by which an oscillator locks to a small external oscillatory input signal in both frequency and phase. It has many applications, including the design of high-performance quadrature oscillators [1], injection-locked PLLs [2], frequency dividers [3], optical lasers [4], *etc*. Recently, it has also been used as the central mechanism in oscillator-based general-purpose computing systems [5], [6].

In many of these applications, the speed in which the oscillator's phase locks on to the external signal is of interest to designers. For example, in oscillator-based computing, this speed directly determines how fast bits can flip, a key property in computing. A recent prototype of such computing systems [7] uses CMOS rings oscillators, but the scheme can be generalized to use high-Q LC-type oscillators for better energy efficiency.

Intuitively, high-Q oscillators are slower in response. Which is to say that its amplititude is often more stable and settles more slowly to its steady state. But is it also true for its phase? With a periodic injection, will a high-Q oscillator's phase shift more slowly to its injection-locked state than one with a lower-Q?

To answer this question, we first define the Q factor of oscillators. Then we simulate a simple negative-resistance LC oscillator with an adjustable Q factor and present the results.

I. What's a High-Q Oscillator?

When we refer to an oscillator as having a high Q factor, what we are often trying to say is that it has a stable frequency and amplitude. These properties often translate to better energy efficiency and lower phase noise, so the "quality" Q is higher. However, once we try to write down an exact formula for the Q factor of an oscillator, several confusions arise.

Firstly, Q factor is often defined under the context of (usually second-order) linear resonators, which are systems with damped oscillatory behaviours. There are several definitions. One is the frequency-to-bandwidth ratio of the resonator:

$$Q \stackrel{\text{def}}{=} \frac{f_r}{\Delta f} = \frac{\omega_r}{\Delta\omega}. \tag{1}$$

The formula implies that there is a Bode plot of the system with a resonance frequency, thus is only meaningful for stable linear systems with well-defined inputs and outputs. It is not directly applicable to oscillators which are by-definition autonomous and usually nonlinear. Another definition for Q factor is from the energy perspective:

$$Q \stackrel{\text{def}}{=} 2\pi \times \frac{\text{Energy Stored}}{\text{Energy Dissipated per Cycle}}. \tag{2}$$

This assumes that there is damping in the oscillation, which is not true for self-sustaining oscillators. There are other definitions that directly map Q to a parameter in the transfer function, but they are limited to linear resonators as well.

Another common confusion is that people often simply assume an oscillator to have the same Q factor as the resonator it is using inside. For example, an LC-type oscillator is often said to have the same Q as the RLC circuit in it. However, this is not true either. An obvious conterexample is that the use of a high-Q resonator in a nonlinear oscillator doesn't always result in a high-Q oscillator.

Therefore, in this paper abstract, we first have to define the Q factor of an oscillator. We define it also from the energy perspective. Consider perturbing an amplitude-stable oscillator with a small amount of extra energy. The oscillator will settle back to its amplitude-stable oscillatory state, dissipating (or restoring) some small amount of energy every cycle. Then we define the ratio between the extra energy applied and the energy dissipated (or restored) every cycle as the Q factor of the oscillator:

$$Q \stackrel{\text{def}}{=} 2\pi \times \frac{\text{Extra Energy Applied}}{\text{Extra Energy Dissipated per Cycle}}. \tag{3}$$

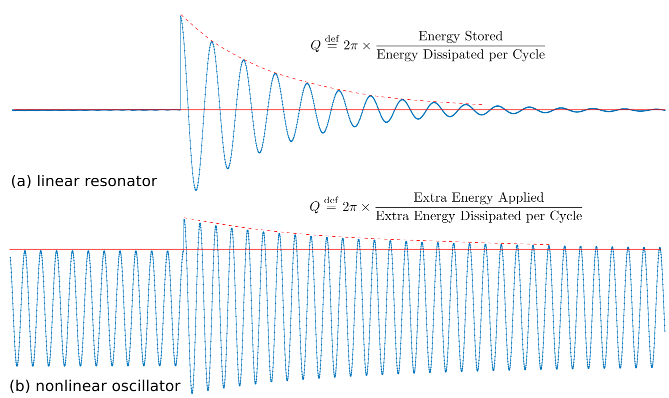

Fig. 1: Illustration of the definition of Q factor for nonlinear oscillators.

The definition is illustrated in Fig. 1. It is analogous to that of the linear resonator, except that instead of zero state, now the oscillator settles to it's amplitude-stable state. The higher the Q factor, the more slowly it's amplitude responds to perturbations, which fits intuition.

Note that because the oscillator is nonlinear, when we are measuring the Q in the way shown in Fig. 1, the size of the extra amplitude introduced will affect the measurement. But

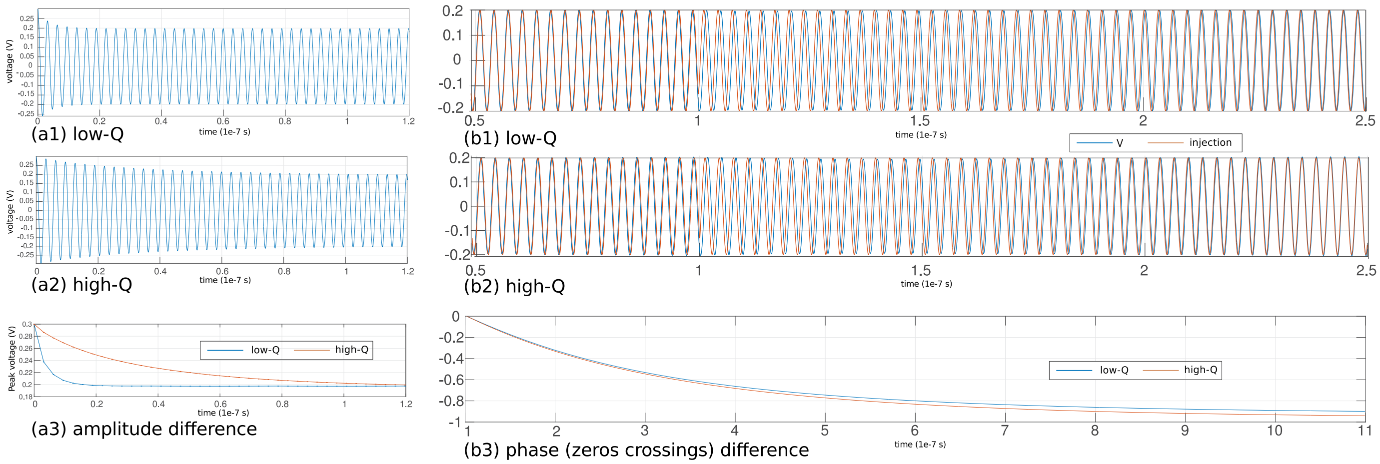

Fig. 2: Simulation results of amplitude (a1-3) and phase shifting (b1-3) of high-Q and low-Q oscillators. In (b3), we plot the zero-crossing differences between the v signals and injection signals in (b1) and (b2).

as the extra amplitude gets smaller and smaller, the Q factor defined in (3) should converge. We can then define the Q factor using this limit. The limit can be analyzed using techniques developed for Linear Period Time Varying (LPTV) systems. It can be estimated both analytically and numerically given the oscillator's DAEs. Therefore, the Q factor in (3) not only fits intuition, it is also a quantity that can be conveniently characterized and analyzed.

II. Does It Take Longer to Injection Lock a High-Q Oscillator?

By definition, a high-Q oscillator settles more slowly in amplitude. But is it also true for its phase? To rephrase the question: as Q factor becomes higher, does it also take longer to injection lock the oscillator's phase?

To study this question, we consider a simple negative-resistance LC oscillator shown in Fig. 3. Different nonlinearities in $f(v)$ result in different Q factors. Intuitively, as $f(v)+\frac{1}{R}$ gets "flatter", the oscillator will appear more like an LC tank with no resistance, thus the Q gets higher.

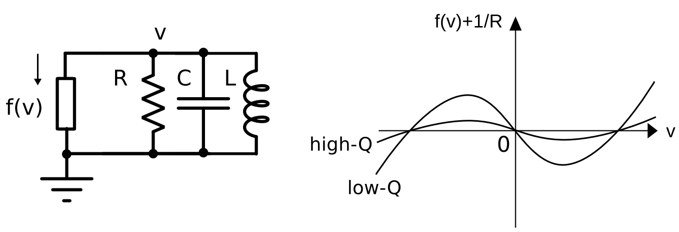

Fig. 3: A simple negative-resistance LC oscillator. Different choice of f(v) will result in different Q factor of the oscillator.

We simulate the LC oscillator with $L = 0.5nH$, $C = 0.5nF$, $f(v) = K \cdot (v - \tanh(1.01 \cdot v))$. We choose K values as 1 and 20, the former results in a high-Q oscillator, the latter low-Q. The simulation results in Fig. 2 show that the high-Q one settles much more slowly in amplitude. Then we apply a small injection current $I(t) = 1mA \cdot \cos(\omega_0 t + \pi/2 * u(t - 100ns))$ at the only non-ground node of the circuit, where $\omega_0 = \frac{1}{\sqrt{LC}}$, $u(t)$ is the step function. In this way the injection signal shifts its phase to 90° after $100ns$. The oscillator's phase will follow this change by shifting gradually through the mechanism of injection locking. The results in Fig. 2 indicate that the difference in the phase shifting behaviour between the high-Q and low-Q oscillators is marginal.

Therefore, at least in this simple LC oscillator experiment, the shifting speeds of amplitude and phase seem to be decoupled. This result is intriguing as the speed in the phase shift doesn't seem to be sacrificed as we use more energy efficient oscillators. This property is very appealing in many injection-locking-related applications.

In our further research, we will investigate the question in more detail, including simulations on other types of oscillators, other potential trade-offs as we use energy efficient high-Q oscillators, possible numerical issues with the injection locking simulations, more rigorous analytical expression of oscillator's Q factor using LPTV theories, *etc.*

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