EECS 219C: Computer-Aided Verification

Abstraction & Simulation and other Equivalences

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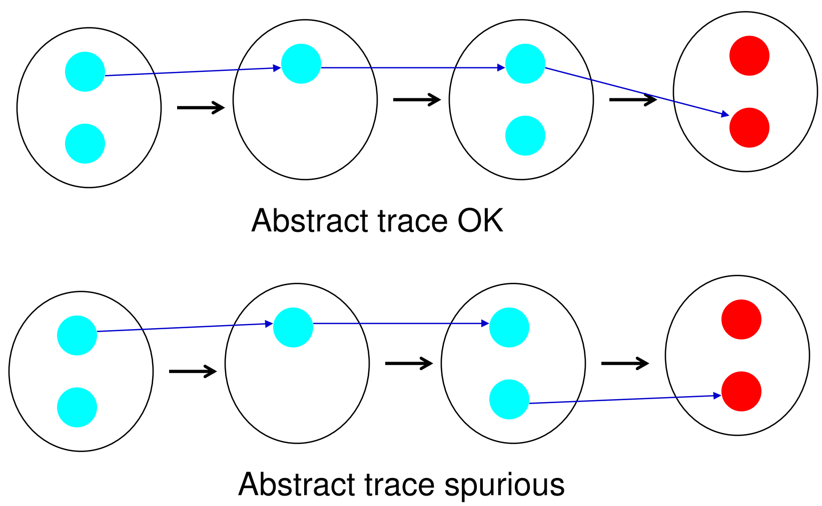
Acknowledgments: Kenneth McMillan

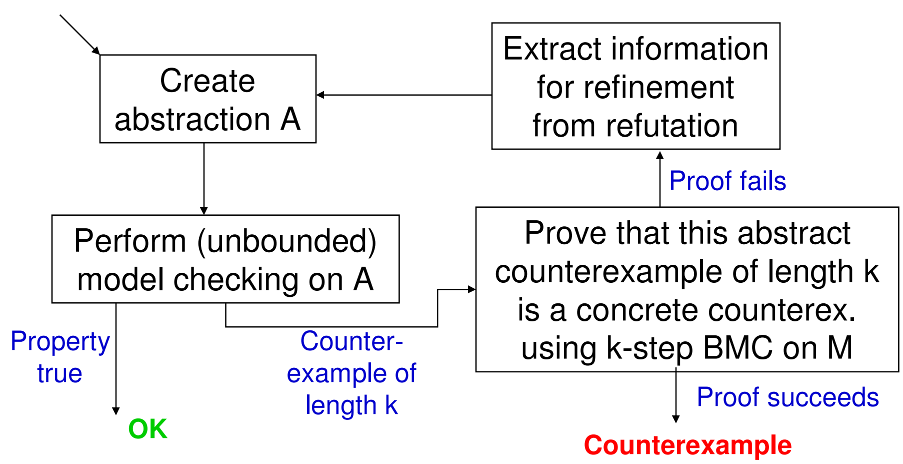
Today's Lecture

- Abstraction in Model Checking
  - Interpolation-based model checking
- Automata-based Property Specification
  - Properties as (Buchi) automata
  - Notions of Trace Containment, Simulation, Bisimulation, Refinement

Abstract/Concrete Error Trace

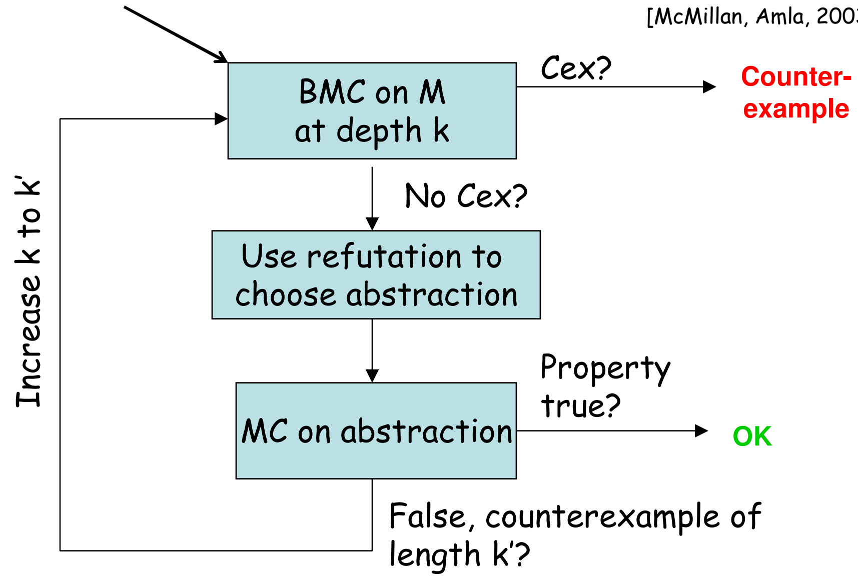


Counterexample Guided Abstraction-Refinement (CEGAR)



Proof-based Abstraction (PBA)

[McMillan, Amla, 2003]



Abstraction and Reachability

- An abstraction expands the set of states reachable from the initial state
  - OVER-APPROXIMATION
- Instead of starting by abstracting states, one can directly abstract the transition relation
  - Each time you compute the set of next states, you get an over-approximation of the actual set of next states
  - Gives a way of computing an over-approximation of the set of reachable states

Abstraction using Interpolation

- Abstraction is extracting sufficient/relevant information from a system *to prove a given property*.
- This notion is in some sense closely related to a notion of "interpolant" and a lemma called "Craig's interpolation lemma"

Interpolation Lemma (Craig, 57)

- If $A \wedge B$ = false, there exists an *interpolant* A' for (A,B) such that:

$$A \Rightarrow A'$$

$$A' \wedge B = \text{false}$$

A' refers only to common variables of A,B

- Example:
  - $A = p \wedge q$, $B = \neg q \wedge r$, $A' = q$

Interpolants from Proofs

(Pudlak,Krajicek,97)

- Interpolant A’ for $A \wedge B$:

$$A \Rightarrow A'$$

$$A' \wedge B = \text{false}$$

A' refers only to common variables of A,B

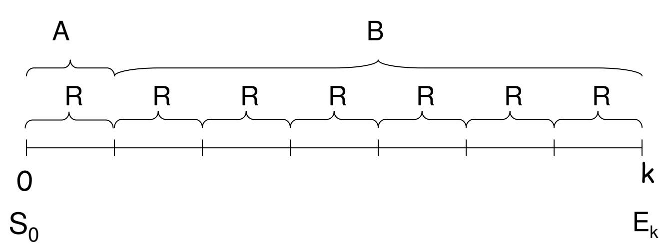
- Interpolants can be obtained from proofs
  - given a resolution-based refutation (proof of unsatisfiability) of $A \wedge B$,

A' can be derived in time linear in the proof

Interpolation based Model Checking

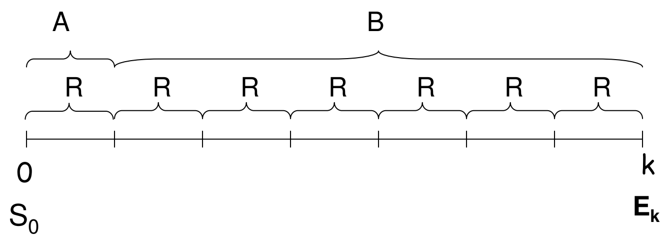
(McMillan, 2003)

- Main Idea: Pose the problem of over-approximating the set of next states as finding an interpolant



$S_0(v_0) \wedge R(v_0, v_1) \wedge R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$

Interpolation based Model Checking



$S_0(v_0) \wedge R(v_0, v_1) \wedge R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$

$A = S_0(v_0) \wedge R(v_0, v_1)$

$B = R(v_1, v_2) \wedge \ldots \wedge R(v_{k-1}, v_k) \wedge E_k(v_k)$

A' is a function of v_1 s.t.

1. $A \rightarrow A'$
2. $A' \wedge B$ is unsat

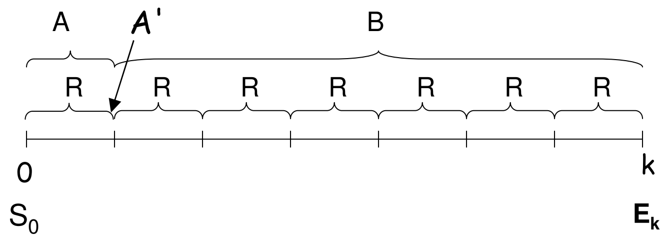
What set of states does A' represent?

Interpolation based MC

For a fixed k:

1. Set Z initially to S_0
2. Do BMC starting from Z for k steps
   - If SAT: have we found a counterexample?
   - If UNSAT, continue
3. Use interpolation to compute over-approximation of next states of Z and add them back into Z
   – Can newly added states lead to error states in k-1 steps? In k steps?
4. If Z does not increase
   – We've reached a fixed point. Is the property true?
5. Otherwise, back to step 2

Intuition

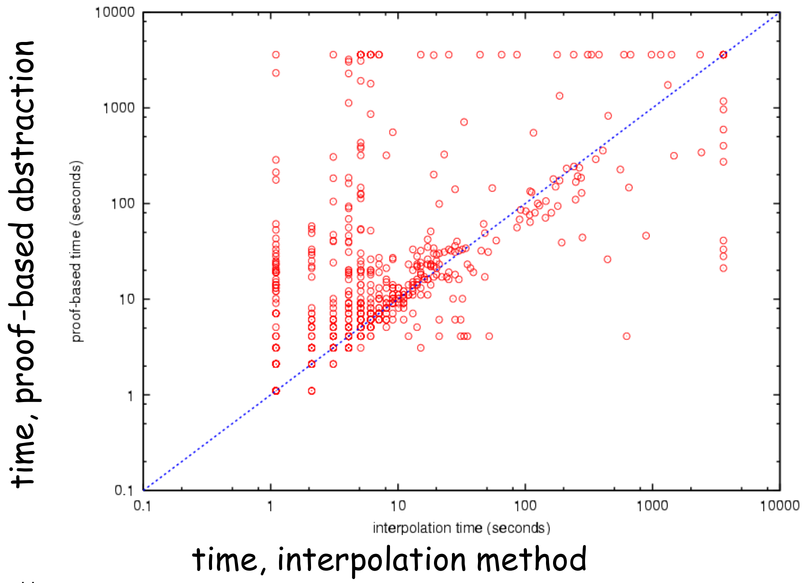


- A' tells us everything the prover deduced about the image of S_0 in proving it can't reach an error in k steps.
- Hence, A' is in some sense an abstraction of the image relative to the property *and* the bound k

Refinement

- Model checking may fail for a fixed k
  - May add a state that reaches error in k steps (getting SAT in step 2 with Z != S_0)
- Refinement is just increasing k
  - How big can k get?

Proof-based Abstract. vs Interpolation



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Source: Nina Amla

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Properties as Automata

- Often properties themselves are finite-state machines
  - E.g. two versions of the same system, an optimized "implementation", and a simple-and-correct "specification"
- How do we formalize the notion of "implementation satisfies specification"?

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Properties as Automata

- Often your properties themselves are finite-state machines
  - E.g. two versions of the same system, an optimized "implementation", and a simple-and-correct "specification"
- How do we formalize the notion of "implementation satisfies specification"?
  - All behaviors (traces) of the implementation are also traces of the specification

TRACE CONTAINMENT

(traces are projected over a common set of atomic propositions)

Abstraction A and Original System M

- All traces of M are also traces of A
- If A satisfies an LTL property, does M also satisfy that property?
- How about for CTL*?

Abstraction A and Original System M

- All traces of M are also traces of A
- So any LTL property that A satisfies will also be satisfied by M
- Holds good for any CTL* property that
  - Has all negations appearing only over atomic propositions
  - Has only the “A” quantifier, not the “E” quantifier
  - ACTL*

Simulation --- Intuition

- Two finite state machines M and M’
- M’ simulates M if
  - M’ can start in a similarly labeled state as M
  - For every step that M takes from s to t, M’ can mimic it by stepping to a state with similar label as t

Simulation

- $M = (S, S_0, R, L)$ and $M' = (S', S_0', R', L')$
- A relation $H \subseteq S \times S'$ is a simulation relation between M and M' means that:
  For all (s, s'), if H(s, s') then:
  – $L'(s') = L(s) \cap AP'$
  – For every state t s.t. R(s, t) there is a state t' such that R'(s', t') and H(t, t')
- M' simulates M if
  – there exists a simulation relation H between them, and
  – For each $s_0 \in S_0$, there exists $s_0' \in S_0'$ s.t. $H(s_0, s_0')$

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Simulation and Trace Containment

Are they the same? If not, which implies which?

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Bisimulation

- M and M' are bisimulation equivalent (bisimilar) if
  - M simulates M' and vice-versa
  - Note: atomic proposition sets must be identical
- Are bisimulation and trace equivalence the same thing?

(Bi)Simulation and (A)CTL*

- If M' simulates M, then any ACTL* property satisfied by M' is satisfied by M
- If M' and M are bisimilar, any CTL* property satisfied by one is also satisfied by the other

Verification

- How do we check for:
  - Trace containment?
  - Simulation?
  - Bisimulation?
- Assume that your machines are given as Kripke structures/Buchi automata
  - For the latter, all accepting paths correspond to runs

Verification

- How do we check for:
  - Trace containment?
    - Can be done using LTL model checking (see MC Sec. 9.6)
  - Simulation?
    - Iterative computation → next slide
  - Bisimulation?
    - Effectively same as simulation check (just done in two directions) [see Ch. 11 of MC]

Simulation Checking

- We attempt to compute the largest relation H such that

  For all (s, s'), if H(s, s') then:

  – L'(s') = L(s) ∩ AP'

  – For every state t s.t. R(s, t) there is a state t' such that R'(s', t') and H(t, t')
- Then, check whether every initial state of M is related by H to an initial state of M'

Simulation Checking

- We attempt to compute the largest relation H such that

  For all (s, s'), if H(s, s') then:

  – L'(s') = L(s) ∩ AP'

  – For every state t s.t. R(s, t) there is a state t' such that R'(s', t') and H(t, t')
- Compute sequence $H_0, H_1, \ldots, H_k$ where:
  – $H_0(s, s')$ iff $L'(s') = L(s) \cap AP$

  – $H_{n+1}(s, s')$ iff
  - $H_n(s, s')$, and
  - $\forall t \{ R(s, t) \Rightarrow \exists t' (R(s', t') \wedge H_n(t, t'))$

  (How to implement this? Why will it terminate?)

Simulation vs. Trace Containment

- Why would we want to use one over the other?

Next class

- Other optimizations in model checking:
  - Compositional reasoning
  - Symmetry reduction
- Mu-calculus