

HW 9: Space Complexity; PSPACE; L*Assigned: April 21, 2010**Due: April 28, 2010*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

For all parts of this assignment, write down all the steps of your algorithm and carefully count its space complexity. There is no need to give a Turing machine description.

1. (6 points) [Sipser 8.11] Show that, if every NP-hard language is also PSPACE-hard, then $PSPACE = NP$.

Also show that any PSPACE-hard language is also NP-hard.

2. (6 points) [Sipser 8.12] Show that $TQBF$ restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

3. (8 points)

Consider the following game played by two players on a directed graph G , with two vertices of G designated as s and t : the two players take turns and color an edge of the graph in each turn. The first player colors green and the second player colors red. An edge is colored only once. The first player wins if there is a directed path consisting of green edges from s to t ; otherwise the second player wins.

Show that

$$GreenPathGame = \{\langle G, s, t \rangle \mid \text{the first player has a winning strategy}\}$$

is in PSPACE.

4. (10 points) [Sipser 8.17, 8.18]

- (a) (4 points)

Let A be the language of properly nested parentheses. For example $(())$ and $((())())$ are in A , but $)()$ is not.

Show that $A \in L$.

- (b) (6 points)

Let B be the language of properly nested parentheses and brackets. For example, $((())())$ is in B but $([])$ is not. Show that B is in L .

[Hint: Use your solution to part (a); match open/close symbols of the same type using counters.]