

HW 2: Equivalence of DFAs & NFAs; Regular Expressions

Assigned: February 1, 2010

Due: February 8, 2010

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (8 points)
 - (a) Give an NFA recognizing all binary strings which both do not contain 11 as a substring and do not end with 1.
 - (b) Following the equivalence proof, convert this to a DFA.
 - (c) Simplify the DFA given in part (b) so that every state is reachable by some possible computation of the machine.
 - (d) Give a regular expression for this language. You are not required to use the equivalence proof.
2. (6 points) If $\underline{w} = w_1w_2 \dots w_n$ is a string, define $\underline{w}^{1/2}$ to be the first half of \underline{w} . I.e. $\underline{w}^{1/2} = w_1w_2w_3 \dots w_{\lfloor \frac{n}{2} \rfloor}$. For any language L , define $L^{1/2} = \{\underline{w}^{1/2} : \underline{w} \in L\}$.

Prove that if L is regular, then $L^{1/2}$ is regular.

Hint: If a DFA recognizing L has state space Q , construct an NFA with state space $Q \times Q$ where the first coordinate goes forward and the second coordinate goes backward.

3. (4 points)
 - (a) Assume A is a language recognized by a DFA M , and let L be an arbitrary language. Define B to be the set of all strings that can be padded by a string in L and end up in A . Formally $B = \{x : \exists y \in L, xy \in A\}$. Show that B is also recognizable by a DFA M_L that has the same states and transition function as M .
 - (b) Is the same true for NFAs?

4. (4 points) Explain in words what languages the following regular expressions represent.

Let $\Sigma = \{0, 1\}$.

- (a) $0\Sigma\Sigma^*1\Sigma \cup 1\Sigma\Sigma^*0\Sigma$
 - (b) $(0^*10^*10^*)^*$
5. (8 points)

- (a) Two regular expressions are equivalent if their corresponding languages are the same.

Let A be a regular expression. Prove that $A \circ \varepsilon = A$.

- (b) Let L be a language comprising all strings w such that w contains an even number of 1s, an odd number of 0s, and no occurrences of the sub-string 10. Write down a regular expression that generates L . Justify your answer.

- (c) In the C programming language, comments appear between delimiters such as $/*$ and $*/$. Let L denote the language of all valid delimited comment strings. A member of L must begin with $/*$ and end with $*/$ but have no intervening $*/$. For simplicity, assume that the characters within the comments that are not $/$ or $*$ are only the symbols 0 and 1, so the alphabet Σ is $\{0, 1, *, /\}$.

Give a regular expression that generates L .