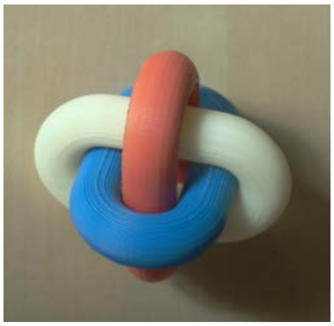
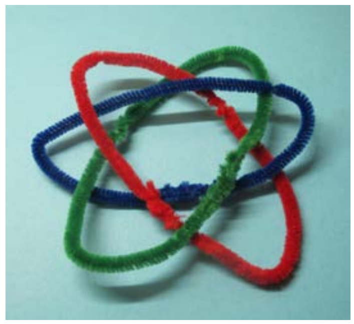
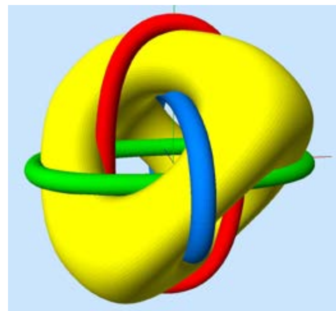
“A Dozen Ways of Showing the Same Topology”

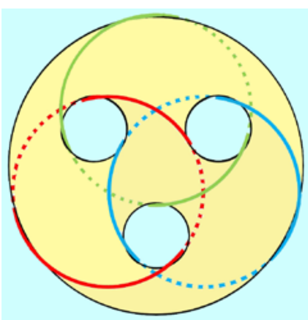
All images depict the three Borromean rings. Figures (C) through (L) show them embedded in a genus-3 handle-body. These images are a subset of a larger series of models created to study how the embedding of the three rings will change, when the surface of the handle-body is turned inside-out (everted) by pulling the whole skin through a small puncture made in a suitable location in the skin of the handle-body. The results may look different, depending on the initial shape of the handle-body and on where the puncture has been placed. The process of actually discovering what happens to the Borromean rings when the surface in which they are embedded is everted was far less “linear” than might be implied by the image sequence below.


(A) Borromean rings.

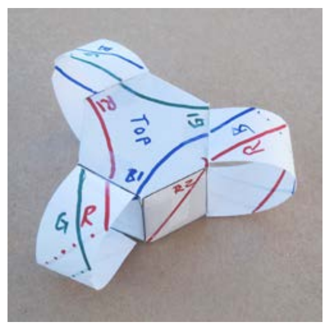

(B) Flexibly interlinked rings.


(C) Embedding in Tetrus.


(D) Deformable Tetrus.


(E) Rings on 3-hole Donut.


(F) Prism with 3 handles.

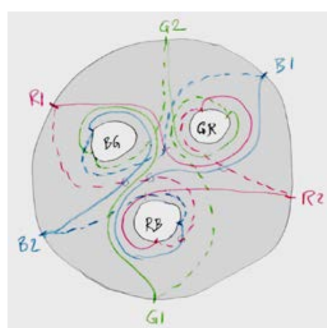


(G) Cardboard model.

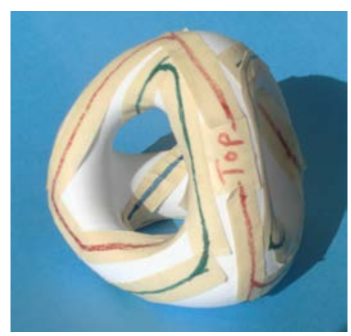

(H) 4-pillar Pagoda.


(I) Plastic bag model of (F).


(J) Everted bag model.



(K) Everted embedding.



(L) Eversion based on (H).

(A) The Borromean rings in a solid, unmovable arrangement. Even though no pair of ovals is interlinked, the assembly as a whole cannot be taken apart.
(B) The Borromean rings implemented in a flexible, deformable manner using red, blue, and green pipe-cleaners. This model allows one to study how the rings can be re-arranged without opening any one of them.
(C) The most symmetrical way of embedding the three Borromean rings in a genus-3 handle-body. A computer-graphics rendering shows the three entangled ovals placed onto the surface of a *Tetrus* – a fattened and nicely rounded tetrahedral wire-frame.
(D) A flexible arrangement of the same topology that keeps the rings fixed in reference to the yellow-white *Tetrus* frame. This model allows one to push a tetrahedron vertex through the opposite tunnel, thereby changing the order in which the rings circle around one another.
(E) A schematic drawing showing the same embedding on a *3-hole Donut*. This shape can be obtained by squashing the 3-sided pyramid structure of the *Tetrus* into a flatter configuration. The three ovals have now become undulating loops. They keep the original topological linking, but emphasize the 3-fold rotational symmetry.
(F) The goal is to make a model that can actually be everted physically. For this structure, which comprises a prismatic stem with three attached handles, it is known how to do this. Marker pen traces on a 3D-print model show the embedding equivalent to that of models (C) through (E).
(G) It was actually quite difficult and error prone to translate the embedding in Figure (E) onto the prism model (F). In this paper model, the three handles are made of simple ribbons that permit experimental twisting through ±180° to give better insight as to how many times each Borromean loop has to wind around each handle.
(H) An alternative way to re-shape the original *Tetrus* handle-body. The resulting *4-Pillar Pagoda*, is a "squarish" form of the *genus-3 Lawson surface*, which is a shape that can also be everted through a puncture placed at the "North pole." Colored adhesive plastic strips placed on a 3D-print depict the embedding of the Borromean rings.
(I) A physical model of the *Prism with 3 handles* made from plastic-bags. It uses four of the cylindrical wrappers that our newspaper is delivered in. They are pasted together with their top and bottom ends, respectively, to form two 4-way tubular junctions. In the top junction, at the "North pole," a star-shaped cut serves as an expandable puncture. The embedding of the three Borromean rings is represented by traces drawn with colored marker pens.
(J) The same plastic-bag model after the whole skin has been pulled through the puncture. After smoothing out the everted plastic sleeves as much as possible, the prism and each handle turn into half-length "reversed socks," ending in a "mouth" as can be found in the standard depiction of a Klein bottle. The four partly reversed cylinders now form a new 4-way tubular junction at the model center.
(K) A sketch of the everted embedding of the four Borromean rings as observed on the surface of model (J). During the whole eversion process, the 3-fold rotational symmetry has been maintained; but the embedding has changed. In the original model, all three rings pass once through each of the three holes. In the everted model, each tunnel is visited by only two of the three rings.
(L) The alternative embedding resulting from the eversion of model (H), which breaks the 3-fold symmetry. The green ring now behaves differently from the other two. The everted handle-body has been re-shaped into a *Tetrus*, and the new embedding can be seen by the colored marks on the 3D-print model. Two of the six handles carry all three rings; the remaining pair of opposite edges ("Top" and "Bottom") accommodate just two colors, but carry two passages for each of them.

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