

# Homage to Eva Hild

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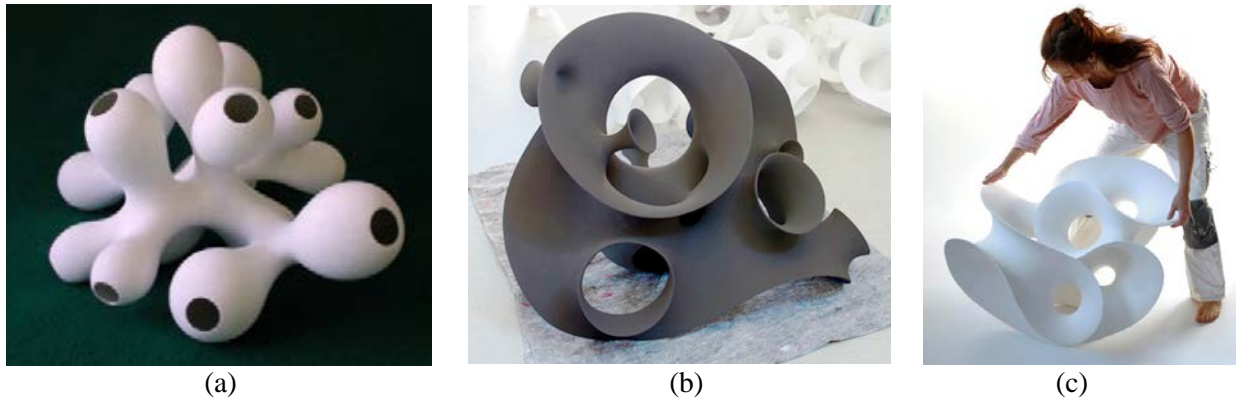
## Abstract

Inspired by the work of Eva Hild, I have developed some auxiliary CAD programs and design processes that make it easier to model her undulating free-form sculptures on a computer, so that similar looking maquettes can be fabricated on a 3D printer. A special focus has been placed on trying to make single-sided 2-manifolds in a similar style.

## 1. Introduction

Eva Hild is a Swedish artist, whom I have never met personally. But for several years, I have been fascinated by pictures of her artwork found on the Internet [5]. Her sculptures (Fig.1) are not only a pleasure to look at, but they invite mental exploration of questions such as: How many tunnels are there? How many separate rims are there? Is this a 1-sided or 2-sided surface? Even more importantly, these sculptures inspire me to create similar shapes. I do not possess the skills to create large ceramic pieces myself, so I aim to create CAD models of such surfaces, and the more promising ones I then realize on a 3D printer.

It turns out that trying to create such shapes with a typical CAD program is difficult and tedious. Those programs have not been designed to create such smooth, free-flowing shapes. If I wanted an exact replica of one of those sculptures, I could travel to Sweden and capture a few hundred thousand surface points with a Kinect [9] or some other 3D scanning device. However, I am more interested in creating my own new shapes, reflecting some of the key features of Hild's creations. Thus, I started an effort to develop auxiliary program modules and processes to define the typical elements found in Hild's sculptures and to compose them in new ways. This paper describes these efforts and shows some initial results.

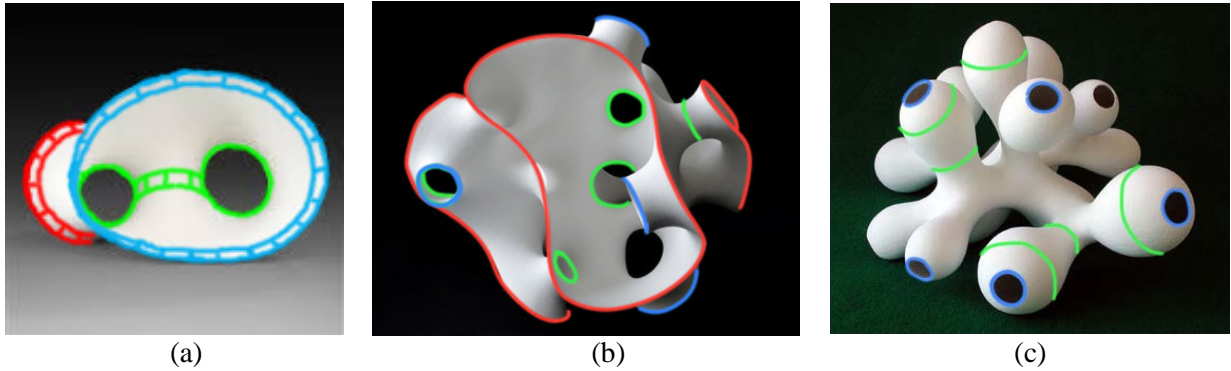


**Figure 1:** *Eva Hild's ceramic creations; (a) bulbous surface with oculus-like openings; (b) various nested funnels; (c) hyperboloid tunnels and undulating rims.*

## 2. Key Features in Hild's Sculptures

From a mathematician's point of view, Hild's sculptures (Fig.1) are smooth 2-manifolds with one or more borders. Some of these borders may be close to circles or ellipses, while others form smooth, undulating, 3D space curves. Some of her sculptures are characterized by bulbous outgrowths with positive Gaussian curvature (Fig.1a); others by giant funnels (Fig.1b) and saddles, or by tunnels in the shape of single-shell hyperboloids with negative Gaussian curvature (Fig.1c); the latter are particularly interesting to me.

After some studies, I found that a good approach for capturing the essence of such a sculpture is to locate some defining key features, in particular, “*rims*,” “*funnels*,” and “*tunnels*.” These features have been marked on a few sculptures in Figure 2. They should be captured in any computer description, and they then form the basis from which the complete sculpture surface can be developed.

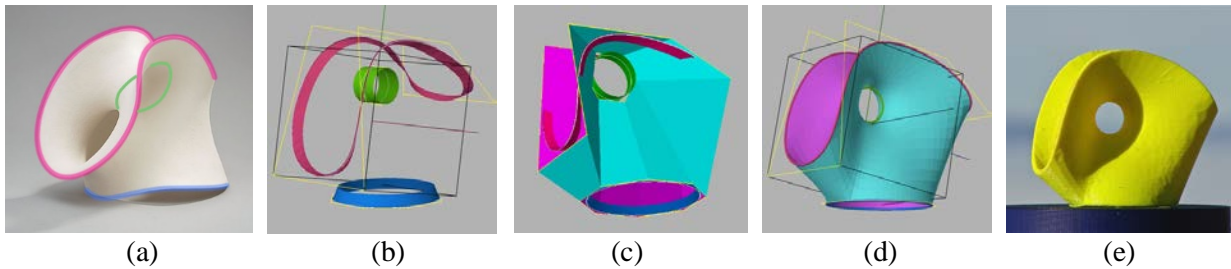


**Figure 2:** Key features marked on some Hild sculptures: red rims; blue funnels; green tunnels.

*Rims* are highlighted by red lines that follow some 3D border curves; in a CAD model, these 3D space curves are represented conveniently as B-splines. *Funnels* are ribbons that follow circular or elliptical openings; they can be generated by circular sweeps or by truncated cones. *Tunnels* are smooth cylindrical segments appearing in a sculpture. They can either represent internal passages (Fig.2b) or the “waists” of some bulbous bellies (Fig.2c). All of these elements can either be defined as 1-manifold curves, or they can be modeled by narrow ribbons, as indicated in Figure 2a. The latter approach is particularly useful for *rims* and for *funnels*, because it makes it possible to specify the tangential direction under which the surface takes off from those borders. For this purpose, the ribbon description must have a way to specify the local azimuth angle of its cross-section with respect to the sweep line. Progressive sweeps with individual control of *azimuth* and *twist*, as implemented in SLIDE [12], are ideally suited for this purpose.

### 3. A Computer-Aided Design Process

I will now describe my current modeling approach with the example of a reconstruction of two relatively simple sculptures by Eva Hild. The first one is *Interruption* [7]; Figure 2a gives a view of its bottom, and Figure 3a is a side view.



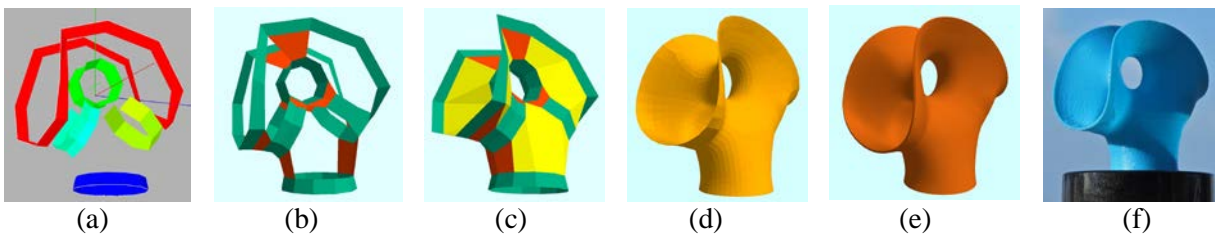
**Figure 3:** Modeling “*Interruption*” in SLIDE: (a) Hild’s sculpture with key features marked; (b) key features modeled; (c) connecting control mesh; (d) subdivision surface; (e) a first 3D print.

In Figure 3a, I placed a (blue) funnel at the bottom, fitted a (red) free-form B-spline curve along the top rim, and inserted a (green) tunnel to define the central opening. Figure 3b shows these defining features by themselves, modeled in the Berkeley SLIDE environment [12]. The next step is to specify polyhedral mesh components that properly connect the three feature geometries to one another. For a simple structure, one can specify the necessary faces with a text editor, referring to the proper control vertices that define the key features (Fig.3c). However, this approach is rather tedious and error-prone in any CAD environment that

does not support it with some interactive graphics technique. When the topology of the whole surface has been defined in this manner, the composite mesh is subjected to two or three steps of Catmull-Clark subdivision [2] to produce a smooth, tangent-continuous surface geometry (Fig.3d). Finally, two offset surfaces are generated, lying at distance  $t/2$  above on either side of this subdivision surface, so as to create the geometrical description of a physical object of thickness,  $t$ , that can be fabricated on a 3D printer (Fig.3e).

To make this construction process more amenable, my research students have helped me to create a new CAD environment, called NOME (Non-Orientable Manifold Editor) [14], which provides these crucial editing capabilities. As in SLIDE, the defining features can be parameterized, allowing the user to fine-tune the shapes and placement of these features. There are pre-programmed generators for *funnels* and *tunnels* with all their defining variables. Once these features have been placed reasonably well, the user can construct the connecting mesh in a point and click manner. The user may select some number,  $n$ , of vertices and then create a (typically irregular, non-planar)  $n$ -sided facet spanned by those vertices. The key characteristic is that this facet behaves like a rubber sheet, staying attached to the different *tunnels*, *funnels*, or *rims*, as these elements are repositioned and re-shaped. The parametrization of the key features remains active through the whole design process. Even after subdivision smoothing has been applied and offset surfaces have been generated, those parameters can still be changed, and the user will see immediately the effect that this has on the final geometry to be sent to the 3D printer. This is a crucial feature that distinguishes NOME from most other CAD tools.

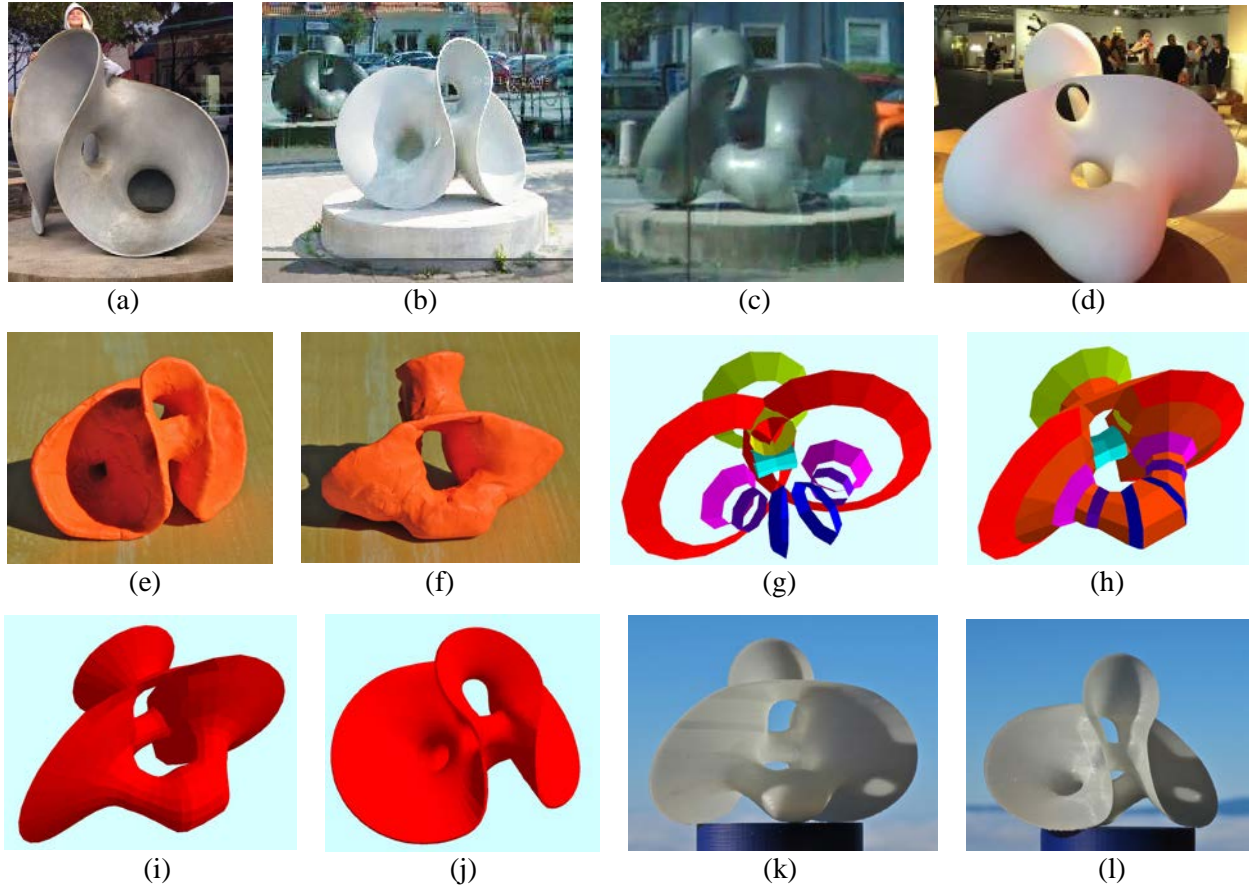
Proper placement of the three defining elements shown in Figure 3 and interactive fine-tuning of their locations, sizes, and shapes, allowed me to capture quickly the topology and basic geometry of *Interruption*. However, the resemblance between the original and my model was not very close; the leftward pointing tunnel entrance in the lower half of Figure 3a has a rather elliptical shape in my model, while in Hild's ceramic it is much more circular. To obtain a better match in geometry, I introduced two additional *tunnels* (Fig.4a). This makes for a more complicated connecting mesh; but in the NOME environment, it is easy to construct it. First, a few connecting quads are introduced, shown in red in Figure 4b; then the various mesh patches are filled in, shown in yellow in Figure 4c. The rest of the process is almost automatic: a subdivision surface (Fig.4d) and two offset surfaces (Fig.4e) are produced. Even at this stage, the various parameters are still fully in effect. For instance, by lowering the position of the blue funnel, the tubular base of the sculpture can be elongated. When a satisfactory shape has been achieved, the geometry is saved in .STL format, which can then be sent to a low-end 3D-printer [13] to obtain a physical maquette (Fig.4f).



**Figure 4:** Modeling “Interruption” in NOME: (a) augmented key features; (b) first quad faces added; (c) connecting mesh; (d) level-2 subdivision surface; (e) offset surface; (f) resulting 3D print;

Figure 5 shows this same design process applied to “Hollow” [6], a more complicated sculpture by Eva Hild (Figs.5a-5d). To start with, I had difficulties understanding the complete connectivity of this sculpture. Two years ago, when I first started this quest, I could find dozens of front-view pictures on the Web, but not a single view from the back. The best I could do, was to go into Google Street View [4] and virtually position the viewpoint, so that a reflection of the backside (Fig.5c) could be seen in the plate-glass windows of the building in front of which the sculpture is located. Only recently could I find a direct view of the backside (Fig.5d). To get a clear understanding of the topology of this sculpture, I formed a small, crude clay model (Figs.5e, 5f). With this in hand, I could then specify a whole set of properly sized and spaced

*funnels* and *tunnels* that outline the geometry of this sculpture (Fig.5g). NOME then made it easy to fill in the connecting mesh fragments. It offers a utility that allows the user to select two whole border curves and then invoke an automatic zipping routine. This routine fills the space between the two border curves with a ribbon made of quadrilateral and triangular faces, trying to keep the number of these facets small, and the lengths of the connecting edges as short as possible (Fig.5h). After subdivision smoothing (Fig.5i) and surface thickening (Fig.5j), a 3D print can be obtained (Figs.5k, 5l) that has a fairly close resemblance to Hild’s original.

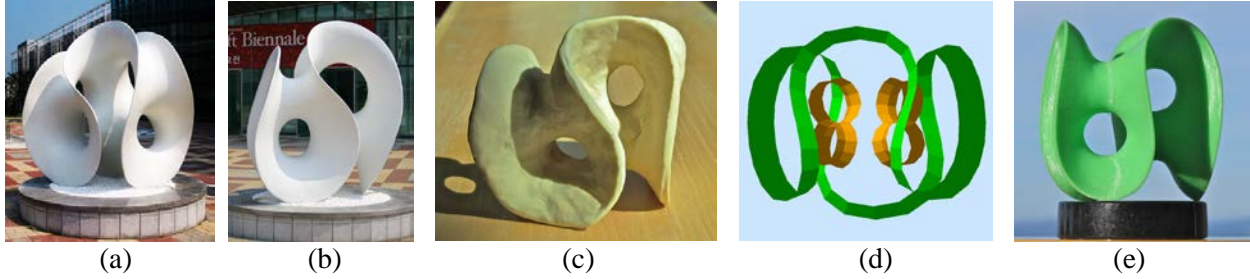


**Figure 5:** “Hollow”: (a),(b) front views; (c) reflected back view; (d) newer back view of sculpture; (e) clay model front and (f) back view; (g) key elements positioned; (h) connecting mesh; (i) a first subdivision step; (j) 3 subdivisions plus offset surfaces; (k) 3D maquette, back and (l) front view.

#### 4. Deriving New Topologies

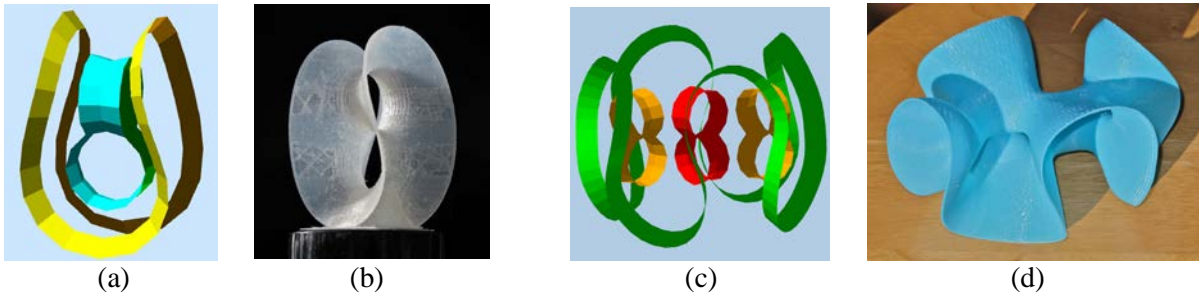
With the NOME editing utilities [14] at my fingertips, I could now focus on creating new geometries inspired by Hild’s creations. In a first experiment, I started from Hild’s “Whole” (Figs.6a, 6b) [8]. A clay model helped me again to understand the connectivity of this sculpture (Fig.6c) and to make a corresponding skeletal CAD model (Fig.6d). To model this structure, I created a new auxiliary geometrical shape, a “cross-tunnel” (Fig.7a). Two instances of this new feature were then surrounded with an undulating 3-period Gabo curve [10] to form the complete feature model (Fig.6d) that defines my version of this sculpture. In contrast to Hild’s sculpture, which has an organically flowing, slightly irregular shape, my model has strict “ $C_{2h}$  symmetry” (Schönflies notation), also known as “ $2^*$ ” symmetry (Conway notation), featuring a vertical mirror plane (most easily visible in Fig.6d) and a horizontal  $C_2$  rotation axes. Figure 6e shows the resulting 3D print.





**Figure 6:** (a),(b) Hild's "Whole"; (c) clay model; (d) skeleton CAD model; (e) 3D print.

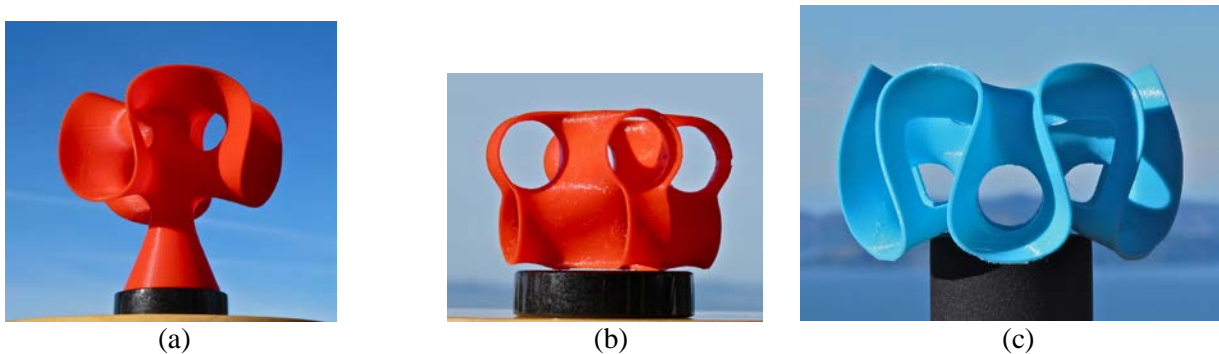
Figures 7a & 7b show what happens when I use just a single *cross-tunnel* and augment it with a simple 2-period Gabo curve (as found in the seam of a baseball). In yet another variation, I have placed three such *cross-tunnels* side by side and wrapped a 4-period Gabo curve around them, which is stretched left-to-right to fit more gracefully around the central structure (Fig.7c). NOME was used to fill the voids between the Gabo-ribbon and the inner core formed by the three *cross-tunnels*, and to obtain a 3D-print (Fig.7d).



**Figure 7:** "Wrapped Cross-Tunnels": (a),(b) one cross-tunnel; (c),(d) three cross-tunnels.

Figure 8 shows sculptures that result, when three or more *cross-tunnels* are placed in a circular arrangement. The sculpture shown in Figure 8a starts with 3-fold symmetrical arrangement of three cross-tunnels, where the lower tunnels point in the radial direction. This core is surrounded by a 3-period Gabo curve that smoothly connects to the various tunnel openings. An additional large funnel is placed at the bottom of this arrangement to produce a stand for this sculpture.

Figure 8b is an arrangement of four cross-tunnels; but here all upper tunnels point in the same direction (towards the viewer). A 6-period Gabo curve has been wrapped around this core. It naturally connects to half of all the tunnel openings, but it also makes an extra up/down transition in between (Fig.8c). This leads to a somewhat convoluted inner border curve, for which I have not found a good way to let it terminate in one or more additional *funnels*. Fortunately, this odd border is not readily visible from the outside. This model has overall 8-fold  $D_{2d}$  symmetry ("2\*2" Conway notation).



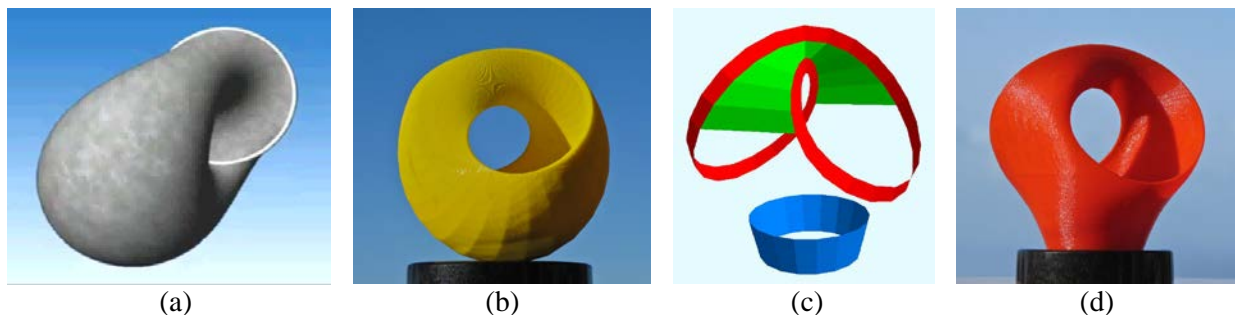
**Figure 8:** (a) 3 cross-tunnels in a 3-fold symmetrical cycle, enhanced with a 3-period Gabo curve; (b) 4 cross-tunnels with alternate orientations, (c) enhanced with a 6-period Gabo curve.

## 5. Single-Sided 2-Manifolds

Most Hild sculptures seem to be two-sided. I have not yet found a picture of one that is clearly single-sided; but it is difficult to analyze the more complex ceramic pieces from just one or two images. This prompted me to try to construct some Hild-like 2-manifolds that are single-sided and non-orientable – like a Möbius band. I pursued three different approaches. And for each conceptual approach, I also asked myself, what might be the simplest possible geometry that could result from it, and which would still exhibit some Hild-like appearance.

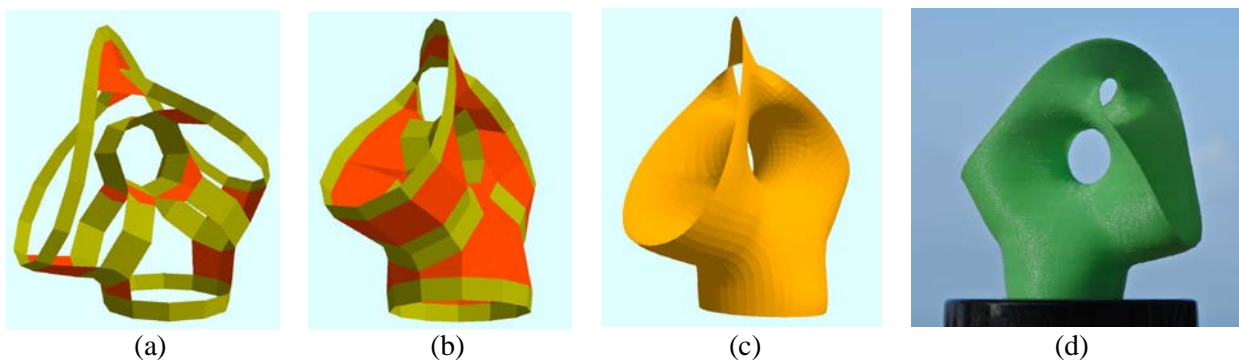
### *Möbius Rim*

A first approach is derived from the “Sue-Dan-ese” Möbius band (Fig.9a) [1]. It has a single circular border connected to a roughly spherical bag. But it does not look very Hild-like; its rim does not resemble a funnel, and part of it is obscured by the geometry of its bag-shaped body. Thus, I let the circular rim be transformed into a loopy figure-8-shape, letting the border follow the tangential pull of the attached surface (Fig.9b). This leads to an undulating 3D border curve that is more akin to what is found in Hild’s sculptures. In Figure 9c, this new Möbius rim (red) is combined with a bottom funnel (blue) on which the sculpture may rest stably; part of the connecting mesh is shown in green. Figure 9d shows the resulting smooth surface.



**Figure 9:** (a) *The Sudanese Möbius band*; (b) *relaxing the rim into a 3D figure-8 shape*, (c) *key features: Möbius rim plus bottom funnel*; (d) *resulting single-sided surface*.

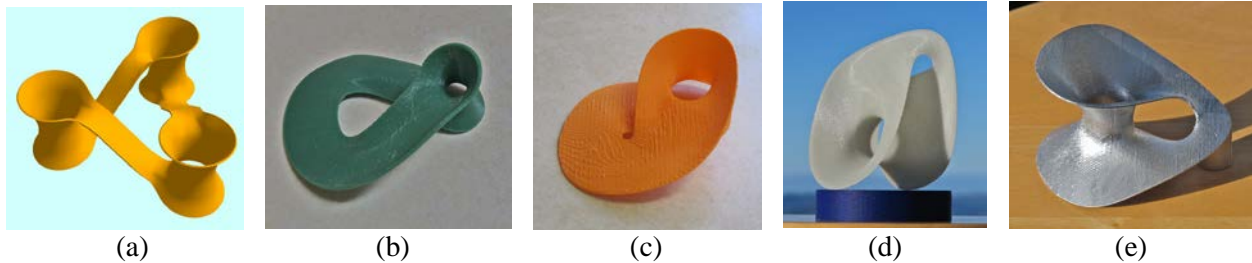
In a related experiment, I tried to make something like a “*Single-Sided Interruption*.” With this goal in mind, I replaced the bent oval border loop of *Interruption* with a stretched version of the Möbius rim (Fig.9c), and used the same internal combination of three tunnels as in Figure 4a. Figure 10a shows the skeleton formed by the key features and a few initial connecting faces (shown in red). NOME made it easy to add all the other facets to form the complete connecting mesh (Fig.10b). Two steps of subdivision already form a nice, smooth surface (Fig.10c). Figure 10d shows the resulting 3D print.



**Figure 10:** *Modeling “Single-Sided Interruption”*: (a) *Skeleton with Möbius ribbon on top*; (b) *connecting mesh*; (c) *subdivision surface*; (d) *resulting 3D print*.

### Tunnels and Tongues

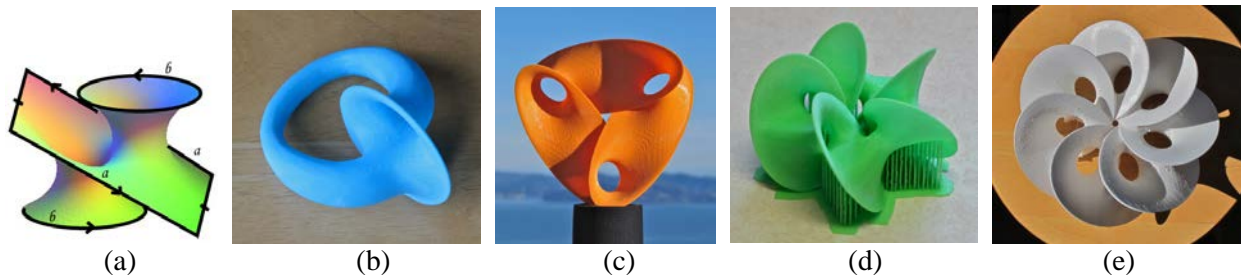
My second approach to producing non-orientable Hild-like surfaces starts with an odd number of *tunnels* in the form of hyperbolic “cooling towers.” A “*tongue*” extends from the top of one tower to the bottom of the next one, thus connecting the inside of one tower to the outside of the subsequent one (Fig.11a). The minimalistic version of this approach is a single tower with a helical tongue connecting the top to the bottom rim. I experimented with a few different geometries, changing the length, width and steepness of that ribbon (Figs.11b–11e); my favorite geometry is shown in Figure 11e.



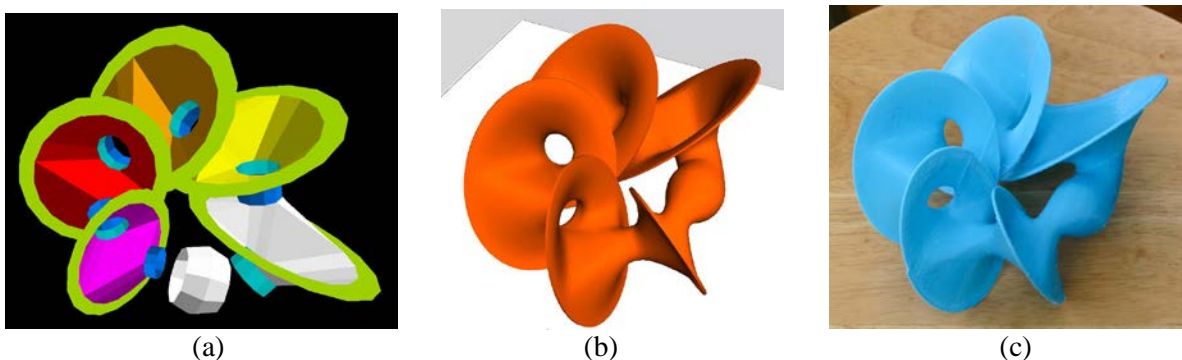
**Figure 11:** (a) 3 connected “cooling towers.” – (b-e) A single tower with a connecting tongue that is: (b) too long, (c) too wide, (d) too small, (e) “just right.”

### Dyck’s Surface

A third approach starts with a single circular disk from which two tubular stubs emerge in opposite directions (Fig.12a); this is known as Dyck’s surface [3]. If these two stubs are joined together with a toroidal loop, one obtains a single-sided 2-manifold (Fig.12b). However, such a prominent toroidal handle is not a typical occurrence in Hild’s artwork. Thus, I replaced the toroidal handle by adding two more Dyck disks to obtain the desired loop closure (Fig.12c). Any such loop formed by an odd number of disks will result in a single-sided manifold; Figure 12d and 12e shows loops with five and seven such disks.



**Figure 12:** (a) Dyck’s surface; (b) closed with toroidal loop; (c) three elliptical disks in a circuit; (d) five Dyck disks in a circuit; (e) seven Dyck disks in a circuit.



**Figure 13:** Five Dyck disks of different sizes connected into a loop: (a) the placement of the defining key features and part of the connecting mesh; (b) the smooth subdivision surface; (c) a 3D print.



Eva Hild typically avoids rigid symmetry and makes the tunnels and lobes in a sculpture of somewhat different sizes (Fig.6a). Thus, in Figure 13, I scaled subsequent instances of Dyck's disks by 10% and let this logarithmic spiral sweep through only 300°. The remaining 60° are then filled in with a bulbous element (as found in Fig.5d); this is a convenient way to connect two tubular stubs of rather different diameters.

## 6. Summary and Conclusions

NOME [14] makes the design process for 2-manifold sculptures in the style of Eva Hild much easier. Still, it is unlikely that with my approach I will ever create something approximating the fluid, natural beauty of her sculptures. In a first e-mail exchange, in which I had sent Eva a draft of this paper, she mentioned in her response that her pieces grow slowly and organically. She has an initial idea and a starting point, but this will then change and develop gradually during a process, which may take months or even years. Often she is surprised when she looks at the resulting final sculpture and wonders, where that shape might have come from.

In contrast, my own computer-based approach is much more “top-down.” I start with a well-defined plan, and specify the overall symmetry that I want to maintain. The use of symmetry significantly reduces the amount of detailed design work that I have to do. On the other hand, the use of computer-aided procedures allows me to create structures of a complexity that would be difficult to achieve in a gradual, bottom-up approach. While my own creations may have a quite different look and feel to them, I still would like to thank Eva Hild – and many other “intuitive” artists – for the inspiration they provide.

## Acknowledgements

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## References

- [1] D. Asimov, D. Lerner, *Sudanese Möbius Band*, SIGGRAPH '84 Electronic Theater, Issue 17, (1984).
- [2] E. Catmull and J. Clark, *Recursively generated B-spline surfaces on arbitrary topological meshes*. Computer-Aided Design 10 (1978), pp 350-355.
- [3] *Dyck surface*. – <http://www.mathcurve.com/surfaces/dyck/dyck.shtml>
- [4] Google, *Street View*. – [https://en.wikipedia.org/wiki/Google\\_Street\\_View](https://en.wikipedia.org/wiki/Google_Street_View)
- [5] E. Hild, *Homepage*, (2017). – <http://evahild.com/>
- [6] E. Hild, “*Hollow*” (2006). – [http://evahild.com/?page\\_id=366](http://evahild.com/?page_id=366)
- [7] E. Hild, “*Interruption*” (2002). – <https://www.bukowskis.com/en/auctions/H043/67-an-eva-hild-stoneware-sculpture-interruption-2002>
- [8] E. Hild, “*Whole*” (2007). – [http://evahild.com/?page\\_id=365](http://evahild.com/?page_id=365)
- [9] Microsoft, *Kinect: Motion sensing input device*. – <https://en.wikipedia.org/wiki/Kinect>
- [10] C. H. Séquin, *Shape Representation: “Gabo curve.”* (slide 36). – <http://slideplayer.com/slide/9035370/>
- [11] C. H. Séquin, *2-Manifold Sculptures*, Bridges Conf. Proc., pp 17-26, Baltimore, July 29-August 2, (2015). – [http://people.eecs.berkeley.edu/~sequin/PAPERS/2015\\_Bridges\\_2manifolds.pdf](http://people.eecs.berkeley.edu/~sequin/PAPERS/2015_Bridges_2manifolds.pdf)
- [12] J. Smith, *SLIDE design environment*, (2003). – <http://www.cs.berkeley.edu/~ug/slide/>
- [13] Type A machines. – <https://www.typeamachines.com/>
- [14] Y. Wang, *Robust Geometry Kernel and UI for Handling Non-orientable 2-Manifolds* (EECS-2016-65). – <https://www2.eecs.berkeley.edu/Pubs/TechRpts/2016/EECS-2016-65.html>