## **Carlo Séquin's Sculpture Generator 1**

Nat Friedman artmath@albany.edu



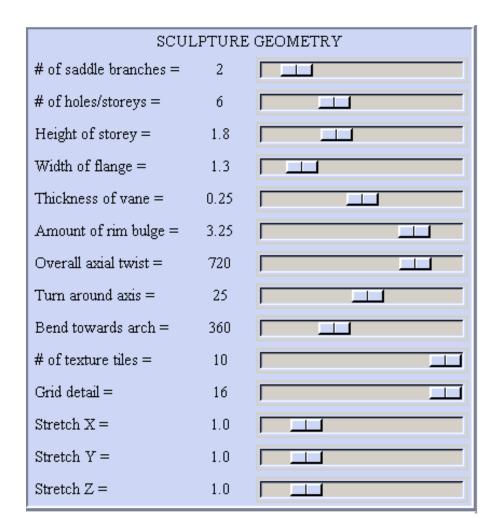
Figure 1. Carlo Séquin, Trefoil Sculpture Models, 2000.

#### **Sculpture Generator 1**

The Sculpture Generator 1 (SG1) is a program for computer-designing sculpture created by Professor Carlo Séquin of the Department of Computer Science, University of California, Berkeley. It was motivated by the sculptures of Brent Collins that can be seen to relate to Scherk minimal surfaces [1,2]. I first experienced working with SG1 in 1997 at the Art and Mathematics Conference AM97 at the University at Albany-SUNY. I thought it was fantastic then and I think it is fantastic now. Carlo has been generous in making the program available on-line and the purpose of this short article is to present some of examples to whet your appetite to try it yourself. A set of sculptures by Carlo Sèquin, designed with SG1, are the Trefoil Sculpture Models, 2000, shown in Figure 1. These models were displayed in the exhibit Art and Mathematics 2000 at Cooper Union College, New York City. The models were also displayed at Art and Mathematics 2001 at Berkshire Community College, Pittsfield, Ma.

#### **Design Parameters for SG1.**

The design parameters as they appear in the computer program for SG1 are shown in Figure 2. We will later discuss the sculpture corresponding to the particular values of the design parameters in Figure 2. The parameters are set with sliders controlled with your mouse. We will discuss the parameters below.





### Examples

We begin with a simple rectangle in Figure 3. The parameters are 1, 1, 5, 0.7, .09, 2.01, 0, 0, 0, 5, 16, 1, 1, 1. The relevant parameters for the rectangle are the **height** of 5 and **width of flange** of 0.7. The background has been set as **blue**. We now give the rectangle a half-twist by increasing the **overall axial twist** to 180° as in Figure 4.

# of saddle branches =	1	
# of holes/storeys =	1	
Height of storey =	5.0	
Width of flange =	0.7	
Thickness of vane =	0.09	
Amount of rim bulge =	2.01	
Overall axial twist =	0	
Turn around axis =	0	
Bend towards arch =	0	
# of texture tiles =	5	
Grid detail =	16	
Stretch X =	1.0	
Stretch Y =	1.0	
Stretch Z =	1.0	
CC	DLOR SI	ELECTION
<ul> <li>Set Color for Backgr</li> </ul>	ound C	Set Color for Sculpture
		Rim 1 🗖 Rim 2 🗖 Rim 3
Red = 0		
Greeen = 0		
Blue = 255		
200 - 200		

•



# of saddle branches =	1		
# of holes/storeys =	1		
Height of storey =	5.0		
Width of flange =	0.7		
Thickness of vane =	0.09		
Amount of rim bulge =	2.01		
Overall axial twist =	180		
Turn around axis =	0		
Bend towards arch =	0		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	DLOR S	ELECTION	
<ul> <li>Set Color for Backgr</li> </ul>	ound C	Set Color for Sculpture	
🗖 Face 1 🦵 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0			
Blue = 255	<b>F</b>		

Figure 4. Overall axial twist of 180°.

We next increase the **bend towards arch** to 180° to obtain the twisted arch in Figure 5. In Figure 6 the **bend towards arch** has been increased to 360° to obtain a Möbius band.

# of saddle branches =	1		
# of holes/storeys =	1		
Height of storey =	5.0		
Width of flange =	0.7		
Thickness of vane =	0.09		
Amount of rim bulge =	2.01		
Overall axial twist =	180		
Turn around axis =	0		
Bend towards arch =	180		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	DLOR S	ELECTION	
€ Set Color for Backgr	ound C	) Set Color for Sculpture	
🗖 Face 1 🗖 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0			
Blue = 255			

Figure 5. Bend towards arch of 180°.

# of saddle branches =	1		
# of holes/storeys =	1		
Height of storey =	5.0		
Width of flange =	0.7		
Thickness of vane =	0.09		
Amount of rim bulge =	2.01		
Overall axial twist =	180		
Turn around axis =	0		
Bend towards arch =	360		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	DLOR S	ELECTION	
<ul> <li>Set Color for Backgroup</li> </ul>	ound C	`Set Color for Sculpture	
🗖 Face 1 🧖 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0			
Blue = 255	1		

Figure 6. Möbius band, bend towards arch of 360°.

In Figure 7 we increase the overall axial twist to 540° to obtain a triple-twist Möbius band.

# of saddle branches =	1		
# of holes/storeys =	1		
Height of storey =	5.0		
Width of flange =	0.7		
Thickness of vane =	0.09		
Amount of rim bulge =	1.01		
Overall axial twist =	540		
Turn around axis =	0		
Bend towards arch =	360		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	LOR SE	LECTION	
• Set Color for Backgro	ound C	Set Color for Sculpture	
🗖 Face 1 🗖 Fac	e 2 Г	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0		1	
Blue = 255	-		

Figure 7. Triple-twist Möbius band, overall axial twist of 540°.

We shall now introduce replace the rectangle by Scherk towers, which are towers of saddle surfaces. A tower of three saddle surfaces is shown in Figure 8. We say each saddle surface has two saddle branches since there are two branches going up and two branches going down. For convenience, we will refer to a saddle of two branches as a 2-saddle. Thus there are three 2-saddles in the Scherk tower in Figure 8.

When this Scherk tower is given an overall axial twist of  $270^{\circ}$  and a bend towards arch of  $360^{\circ}$ , the ends come together and the result is the sculpture in Figure 9. Note that the number of "outer holes" is 3 corresponding to the three saddles. The **thickness of vane** has also been increased to .15 and the **amount of rim bulge** has been increased to 1.5 to give the sculpture more mass. The ends would also come together with an axial twist of 90°, but an axial twist of 270° results in a more interesting sculpture.

The result of twisting a Scherk tower and bending it 360° to form a closed circular form is referred to as the **toroidal closure of a Scherk tower**. This was a break-through idea of Carlo Séquin. The idea of forming the toroidal closure of a Scherk tower was motivated by a sculpture of Brent Collins, which can also be viewed as the central cross-section of a Costa minimal surface. This relates Scherk minimal surfaces and Costa minimal surfaces.

<pre># of saddle branches =</pre>	2		
# of holes/storeys =	3		
Height of storey =	4.5		
Width of flange =	3.4		
Thickness of vane =	0.09		
Amount of rim bulge =	1.01		
Overall axial twist =	0		
Turn around axis =	0		
Bend towards arch =	0		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	DLOR S	ELECTION	
← Set Color for Backgr	ound 🤇	Set Color for Sculpture	
🗖 Face 1 🗖 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0		1	
Greeen = 0			
Blue = 255	1		

Figure 8. Scherk tower with three 2-saddles.

<pre># of saddle branches =</pre>	2		
# of holes/storeys =	3	r 💷 r	
Height of storey =	4.5		
Width of flange =	3.4		
Thickness of vane =	0.15		
Amount of rim bulge =	1.5		
Overall axial twist =	270		
Turn around axis =	0		
Bend towards arch =	360		
# of texture tiles =	1		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CO	LOR SE	LECTION	
• Set Color for Backgro	und C	Set Color for Sculpture	
🗖 Face 1 🗖 Face	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0		1	
Greeen = 0			
Blue = 255	<b></b>		

Figure 9. Toroidal closure of a Scherk tower with three 2-saddles.

One of Séquin's brilliant ideas was to let the bend towards arch increase past 360°. As this happens, the Scherck tower can be seen passing through itself and then closure is obtained again at 720°. An example is shown in Figure 10. We will refer to this as a **double toroidal closure**. Sèquin has also considered triple toroidal closures.

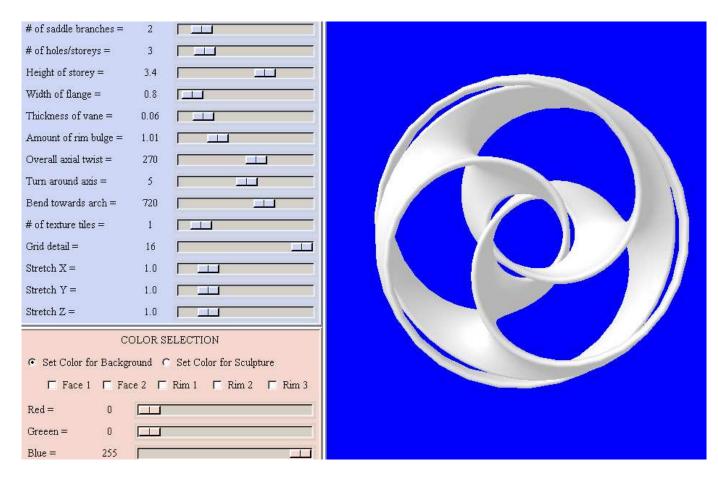
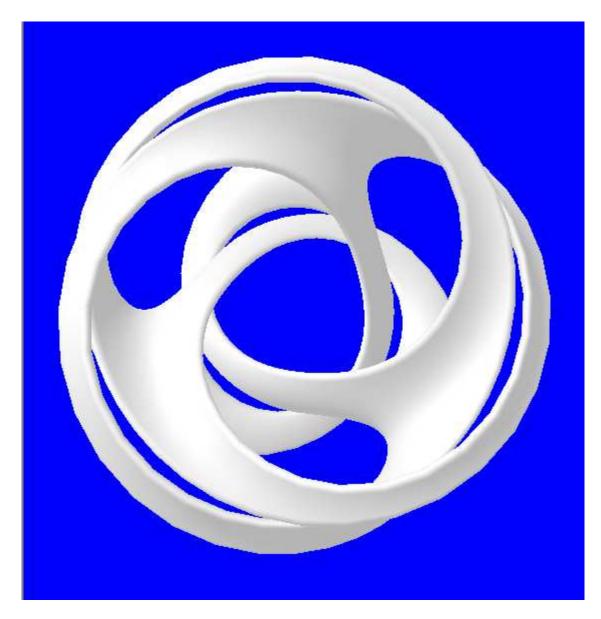


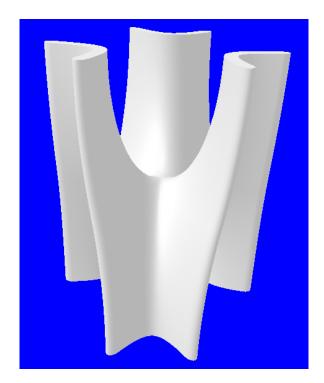
Figure 10. A double toroidal closure of a Scherk tower with three 2-saddles.

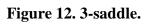
So far, we haven't mentioned the parameter **turn around axis**, which simply rotates a vertical Scherk tower around its central axis but does not change the overall shape of the Scherk tower. However, once the Scherk tower is closed, the turn around axis can dramatically change the shape of the resulting sculpture. In this case varying the nturn around axis parameter looks like "turning the toroidal closure inside-out" resulting in a variety of different sculptures. In Figure 11 we have only increased the turn around axis from  $5^{\circ}$  to  $60^{\circ}$  and left the other parameters the same. We also note that the double toroidal closure sculptures in Figures 10 and 11 do not self intersect. This can be seen by hyperseeing the sculptures by moving them around using the mouse.



# Figure 11. Double toroidal closure sculpture obtained from Figure 10 by increasing turn around axis from 5° to 60°.

We have now considered some examples with 2-saddles. A 3-branch saddle has three branches upward and three branches downward and is referred to as a 3-saddle. An example of a 3-saddle is shown in Figure 12. A 3-saddle is also referred to as a monkey saddle. The closure of a Scherk tower with three 3-saddles is shown in Figure 13. The double closure of a Scherk tower with three 3-saddles is shown in Figure 14.





# of saddle branches =	3		
# of holes/storeys =	3		
Height of storey =	4.5		
Width of flange =	3.4		
Thickness of vane =	0.07		
Amount of rim bulge =	1.01		
Overall axial twist =	180		
Turn around axis =	0		
Bend towards arch =	360		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	DLOR SE	LECTION	
• Set Color for Backgro	ound C	Set Color for Sculpture	
🗖 Face 1 🗖 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0			
Blue = 255	-		

Figure 13. Toroidal closure of a Scherk tower with three 3-saddles.

# of saddle branches =	3		
# of holes/storeys =	3		
Height of storey =	4.5		
Width of flange =	3.4		
Thickness of vane =	0.07		
Amount of rim bulge =	1.01		
Overall axial twist =	180		
Turn around axis =	90		
Bend towards arch =	360		
# of texture tiles =	5		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
со	LOR SE	LECTION	
• Set Color for Backgro	und C	Set Color for Sculpture	
🗖 Face 1 🗖 Face	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0			
Greeen = 0			
Rhue - 255	-		

Figure 14. Double Toroidal Closure with three 3-branches.

As promised, the sculpture corresponding to the parameters in Figure 2 is shown in Figure 15.

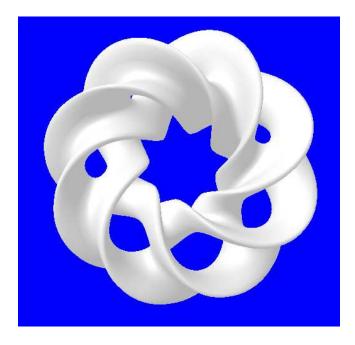


Figure 15. Figure 2 parameters.

## Warning

You can get addicted to SG1, leading to eyestrain, overexcitement, and late nights. So you have been warned. Its now 4 am and I just have to do one more!!

# of saddle branches =	2		
# of holes/storeys =	4		
Height of storey =	2.5		
Width of flange =	1.3		
Thickness of vane =	0.25		
Amount of rim bulge =	3.25		
Overall axial twist =	360		
Turn around axis =	20		
Bend towards arch =	360		
# of texture tiles =	10		
Grid detail =	16		
Stretch X =	1.0		
Stretch Y =	1.0		
Stretch Z =	1.0		
CC	LOR SE	ELECTION	
• Set Color for Backgro	ound 🤉	Set Color for Sculpture	
🗖 Face 1 🗖 Fac	e 2 🗖	Rim 1 🗖 Rim 2 🗖 Rim 3	
Red = 0		1	
Greeen = 0			
Blue = 255			

Figure 15. Last One.