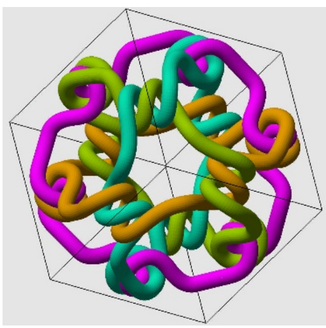
Cuboctahedral Edge Knots

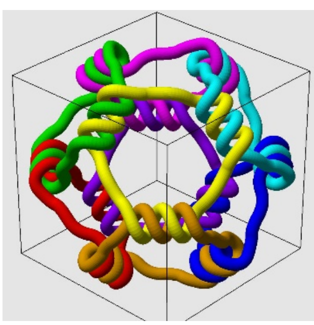
I am interested in mathematical knots as aesthetic, constructivist sculptures. Prime knots cannot exhibit mirror symmetry, nor can they assume any of the higher-order symmetries of the regular or semi-regular polyhedra.

Still, it is possible to make tubular sculptures based on such knots that overall display the structure of some Platonic or Archimedean solid. My approach is to run the knot strand along all the edges of such a polyhedron, forming a closed Eulerian circuit. This can readily be done on polyhedra where all the vertices are of even valence, such as the octahedron or the cuboctahedron; here I focus on the latter geometry.

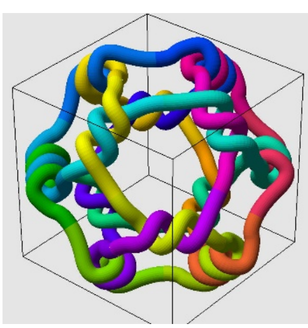
First, I try to compose an Euler circuit that exhibits as much symmetry as possible. Then the knot strand must be deformed slightly near the vertices to avoid any direct strand intersections. These deformations, necessary to obtain a true knot, unfortunately will further reduce the overall symmetry. My three art submissions explore this trade-off.



CASE 1:
The overall shape of the Euler circuit depends on how the knot strand passes through the valence-4 vertices. A first symmetricaql possibility is to let the two passes through each vertex cross each other. To make the sculpture more interesting, displaying a "truly knotted" character, I implemented these crossings as small helices, in which the two strands wind around each other with 1.5 turns. However, this results in a link with four components, which roughly follow equatorial circles around the cuboctahedron.



CASE 2:
In a second attempt I tried to "link" the pair of strands passing throuhg each vertex. Again, for aesthetic reasons, to make this a "knotted-looking" constructivist sculpture, I implemented these linkings with short helices that let the two strands make two full turns around each other. However, again this does not result in a mathematical prime knot, but in an 8-component link of eight roughly hexagonal loops that lie in the faces of the underlying octahedron.



CASE 3:
To shape this strand into a single prime knot, three of the helices need to make full twists, while the other nine helices make an odd number of half-twists. This then results in a single alternating knot, with D_3 symmetry. I have given this single knot strand an overall rain-bow coloring, running around the hue-circle in RGB space, tho make it easier to follow the strand going twice through all twelce helices. Note, that we may interpret these helices as short, fat "double-edges," which then results in an overall structure that can be seen as a truncated octahedron, in which twelve edges have been split and twisted into helices.