Symmetry plays a very important role in all facets of music. It gives ideas structure, defines musical motifs, and gives a composition structure and direction. However, symmetry is less well defined in jazz: the main focus of jazz is improvisation over set chord changes. When a jazz musician improvises a symmetrical idea, they are not reading a symmetrical line of music; rather, they are playing these symmetrical ideas spontaneously. The purpose of this research is two fold: to investigate how jazz musicians use symmetrical ideas by analyzing their transcribed solos, and to investigate if it is possible to write an entirely symmetrical jazz solo from scratch that is pleasing to the ear. For the purpose of this research, the applied symmetries will be the seven frieze symmetries, and it is requisite that the written solo exhibit all seven of these symmetries.

Due to the large amount of jazz solo transcriptions available and the constraints of the research, only one solo was analyzed. This is Coltrane’s improvised solo in the original recording of his chart entitled “Mr. P.C.” In this solo, the first moment of symmetry is exhibited in what is jazz musicians call the “head” or “top” of the chart. Here, D_n rotational symmetry is used as Coltrane traverses through the chords B flat seven, A seven, and then finally ending at the D minor chord. Here, Coltrane is cleverly working with the interplay of the chords: he plays with the seventh chord tones of the chords Bb and A7. During Bb, he plays Ab, the seventh of Bb, but also plays G, the seventh of A, allowing for a very fluid transition from chord to chord. Then, he layers symmetry on to this delicate interplay, giving the melody more structure while revealing the intent of the idea. By giving the idea symmetry, it makes the seventh transitions more noticeable to the listener, and the idea more pleasing to the ear. The visualization is depicted below (the rotation centers are highlighted in red).

The second, and perhaps most important, usage of symmetry gives Coltrane’s solo structure and direction. The shape of ideas, specially how they begin and end, is an important aspect of jazz improvisation. Without a sense of continuity and direction, the listener would be lost. Here, Coltrane asserts the intent of an idea. Then, eight bars later, he reasserts the same structure of idea with subtle changes. This change in structure is based upon a Cnh frieze symmetry. Coltrane takes an ascending line on the D minor chord, and then eight bars later plays a descending idea, as if he flipped the idea over a horizontal symmetry line in the staff. This a nuance that the average listener would not notice explicitly, but would subconsciously register as more pleasing to the ear. As Coltrane was playing the first eight bars of the idea, he was in all
probability considering how to connect this idea to the next eight bar idea, and he chose to do so symmetrically. Below, the first and second iterations of the idea are depicted.

The third and fourth instances of symmetry showcase possible applications of Cnv (vertical symmetry) and S2h (glide symmetry). Not only are these instances symmetrically different from the previous example, but also the ideas differ contextually: these lines are not played at the beginning of an idea, but rather in the middle of one. Instead of providing structure, they serve as fluid connectors. It is important that a jazz solo is continuous and fluid, otherwise it may not be pleasing to the ear. Large leaps between notes, such as playing two consecutive notes that are a large distance apart on the staff, make the solo sound jagged and disconnected. Thus, Coltrane uses Cnv summery to descend and then ascend again, as he wants to play a diverse mix of notes of low and high timbre without taking large and sudden leaps. Similarly, he uses S2h glide symmetry to ascend and descend in a separate idea. Below, the Cnv and S2h ideas are depicted respectively:

Writing a symmetrical jazz solo under the restrictions of using all even frieze symmetries and using only chord tones was a challenge, but was possible with some exceptions. One such exception was using the second tone of the relevant scale to create symmetrical ideas over a horizontal symmetry line. This is not a chord tone, as a seventh chord would be composed of the first, third, fifth, and seventh tones of the relevant scale. Another issue encountered was the usage of the root of the chord. The tone symmetrical to the root of the chord was more often than not outside of the chord tones, and thus incorporating the root, one of the more important chord
tones, was a difficult task. The way the solo is structured is the first 6 bars are symmetrical to the last six, for example the first bar would be symmetrical with the seventh. Below is an example of two pairs of symmetrical bars in the solo.

Two symmetries are depicted above, Cnh symmetry, which includes a horizontal mirror line, and Dnd symmetry, which includes a glide axis, vertical mirror lines, and rotation points. Cnh symmetry was easier to implement at this point since the two bars have different chords. As a generality, if the chords were different, there were more notes to work with to create symmetry. For example, above I used the third of the G minor chord (Bb) to create horizontal mirror symmetry with D of the D minor chord. An interesting continuation of this research would be to investigate which chords pairings can create the most symmetrical ideas over a horizontal mirror axis.

Another discovery while writing the solo was that triplets were the most versatile rhythm, as they could be used for many different symmetries. I used triplets in the Dnd symmetrical idea above to create glide, rotation, and vertical symmetry all in one rhythm. Not only did they offer symmetry, but also they were probably the best sounding part of the solo. The triplet rhythms served a smooth connector of ideas, and the triplet idea itself is integral to the swing rhythms of a jazz blues chart.

Despite being able to complete the symmetrical solo, it had its flaws. As aforementioned, exceptions to the rules had to be made, such as using notes outside the chord tones. However, its most outstanding flaw was not in the symmetry, but rather in how it sounded to the ear. It sounded decidedly contrived, and justifiably so. To accommodate the symmetry, unnaturally large steps (leaps) between notes had to be made on a few occasions. Also, the inability to use the root of the chord in certain situations made the solo sound like it was “outside” of the chord changes. Playing "outside” the changes means non-chord tones, or less important chord tones, were given emphasis.

In conclusion, symmetry in jazz is more nuanced and less explicit than in other genres of music. Writing an entirely symmetrical solo demonstrates that in jazz improvisation, symmetry should not be a requirement that pigeonholes an idea. Instead, symmetry is a natural evolution of an idea, and is one factor among many that contributes to the greater arc of a solo.
provides contextual and structural significance to jazz improvisation, allowing for the soloist to build upon and create interplay between ideas, and making the solo easier to follow for the listener. Thus, the study of how jazz greats use symmetry in their solos would be beneficial to jazz education. Studying symmetry would emphasize that a solo is equal parts playing the right notes and the right rhythms. After all, playing a jazz solo is telling a story, and the presentation of the story, its structure and flow, is just important as the content.