## TAKE HOME EXAM

Your Name:
Your Class Computer Account: $\qquad$

Your student ID \#: $\qquad$

## DO NOT OPEN THIS INSIDE SODA HALL -- WAIT UNTIL YOU ARE ALONE! INSTRUCTIONS ( Read carefully!)

YOU MAY USE LECTURE NOTES, ANY TEXT BOOK, PAPERS ON THE INTERNET, -- BUT NOT THE HELP OF ANY OTHER HUMAN BEING -- TO DO THIS QUIZ.

On the in-class exams, you will be allowed ONE double-sided sheet of size 8.5 by 11 inches of your own personal notes to assist your memory. It might be worthwhile to start preparing and "test-driving" such a sheet now. It should take you about 2 hours ( $1 \mathrm{pt} . / \mathrm{min}$.) to complete this Exam; but this does not include any study time you might need to catch up on various topics.

## YOU MUST DO THIS WORK ENTIRELY BY YOURSELF !

## COLLABORATION IS CONSIDERED CHEATING.

There must be no discussion of any of the problems with anybody until after the deadline.
DO ALL WORK TO BE GRADED ON THESE SHEETS OR THEIR BACKFACES.
If any question on the exam appears unclear to you, write down what the difficulty was and what assumptions you made trying to solve the problem the way you understood it.

## I HAVE UNDERSTOOD THESE RULES AND WILL OBEY THEM:

Your Signature: $\qquad$

## DO NOT OPEN THIS UNTIL YOU HAVE SIGNED ABOVE AND ARE ALONE!

HAND IN THE EXAM TO PROF. SEQUIN BEFORE 9:40am, Wednesday, Feb. 16, 2011.

This Quiz serves as a "study guide" to help you ascertain that you are absorbing the important material covered in this course and that you understand how to apply it to novel situations. This is also good training for the Midterm and the Final Exams -- except that there will then be the added pressure of limited time.

## Problem \#1 -- Circle the correct answer: (6 points)

| TRUE | FALSE | In 2D, translations of homogeneous coordinate triples are described by asymmetrical $3 \times 3$ matrices.
$\mid$ TRUE $\mid$ FALSE $\mid$ If points $P_{0}, Q_{0}, R_{0}$ are carried into points $P_{1}, Q_{1}, R_{1}$, respectively, by a transformation described by an orthogonal matrix, then the angle $P_{0} Q_{0} R_{0}$ is equal to the angle $\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{R}_{1}$.
| TRUE | FALSE | In 2-space, "translation in x" and "translation in y" are commutative.
| TRUE | FALSE \| The bounding-box calculation in the rendering pipeline starts at the root node of the scene hierarchy.
| TRUE | FALSE \| A sphere with a surface that acts as a Lambertian diffuse reflector will look to an observer like a uniformly lit flat circular disk when illuminated with a uniform parallel illumination coming from the direction of the observer.

## Problem \#2 -- Circle the correct answers: (8 points)

(A) Circle all other 2D operations with which a rotation around the z-axis does commute:

Translation in x ; Mirroring in y; Uniform scaling; Non-uniform scaling; Rotation around z .
(B) In what direction does the $\{+\mathrm{x}\}$-axis point after $\mathrm{a}+90^{\circ}$ degree rotation around the y -axis (in a right-handed coordinate system) ?

$$
\{-\mathrm{x}\} \quad\{+\mathrm{x}\} \quad\{-\mathrm{y}\} \quad\{+\mathrm{y}\} \quad\{-\mathrm{z}\} \quad\{+\mathrm{z}\} \quad\{\text { other }\}
$$

(C) Circle the Calligraphic Display Devices in the following list:
| Pen Plotter | Ink-jet Printer | Cathode Ray Tube | Plasma Panel | Projector in Laser-Lightshow
(D) Which of the following homogeneous 2D points lies farthest from the origin? (Circle one).
$|P(7,-8,1) \quad| \quad P(2,-2,0.2) \quad|\quad P(-30,21,3) \quad| \quad P(56,72,8) \mid$

## Problem \#3: History (3 points)

What is the name of the "Father of Interactive Computer Graphics"?

## Problem \#4: Rasterization (9 points)

You want to draw (approximate) a roughly one-pixel wide line (without using anti-aliasing techniques) on a raster device with square pixels. The line starts at the center of pixel [0, 0] and ends at the pixel $[\mathbf{H}, \mathbf{V}]$, where $\mathbf{H}$ is in the range $[50,70]$ and $\mathbf{V}$ is in the range $[-100,-80]$.

How many pixels "roughly" (i.e., +/-5 pixels) will be turned on, expressed in variables $\mathbf{H}$ and $\mathbf{V}$, if we use each of the following algorithms. Look at the relevant section of the lecture notes, textbook (and possibly the Internet, if you don't have a textbook).
a) Sampling the area of the perfect 1-pixel-wide band along the line will turn on $\qquad$ pixels.
b) Turning on all pixels touched (crossed) by the ideal thin line will turn on $\qquad$ pixels.
c) The Bresenham or Digital Differential Analyzer (DDA) algorithm will turn on $\qquad$ pixels.

## Problem \#5: CRT Display (8 points)

You have a raster graphics color display that has 9-bit DAC's (digital to analog converters) at the three guns of the display CRT. The display size is 512 by 256 pixels.

How many possible colors can you generate on this display? $\qquad$

How many different colors can you show simultaneously on this screen? $\qquad$
Now we want to introduce a color map so that we can display only 16384 different colors simultaneously; how many digital input bits and how many digital output bits should this look-up table have ?
\# of input bits: $\qquad$ \# of output bits: $\qquad$

## Problem \#6: Linear Transformations (5 points)

Prove (in a mathematically clean way) that for the transformations discussed so far in this course, we can transform all the points of a straight line segment by transforming its endpoints and then constructing a new straight line segment between the two transformed endpoints.

## Problem \#7: Matrix Multiplication (5 points)

Write down the homogeneous 2D transformation matrix for the following (multiplied out) transformation: translate $(3,4)$ followed by a rotation around $z$ of -30 degrees:

## Problem \#8: Intersection Test (6 points)

Suppose you want to find the intersection between a parabola and a plane in 3D space.
The parabola is defined parametrically by the expression $\mathbf{e}+\mathbf{d}^{*} t+\mathbf{f}^{*} t^{2}$, with $\mathbf{e}, \mathbf{d}, \mathbf{f}$ as vectors and $t$ as the free parameter.
The plane is defined implicitly by all points $\mathbf{p}$ satisfying $(\mathbf{p}-\mathbf{a}) \cdot \mathbf{n}=0$, where $\mathbf{a}$ and $\mathbf{n}$ are vectors. A) Write an expression for $t$ that you would evaluate to solve for the intersection.
B) In what cases are there NO solutions? Write a conditional statement that is TRUE in these cases, and false otherwise.

## Problem \#9: AABB (4 points)

A 2D scene consists of just two circles: one around point [-10,10] with radius 17 , and the other around point [20,20] with radius 26 . What are the four coordinate pairs for the four corners of the axis-aligned bounding box of this scene?

$$
\mathrm{P} 1=(\quad, \quad), \quad \mathrm{P} 2=(\quad, \quad), \quad \mathrm{P} 3=(\quad, \quad), \quad \mathrm{P} 4=(\mathrm{l}, \mathrm{e} \text {. }
$$

## Problem \#10: Hierarchical Bounding Boxes (8 points)

Sketch all the bounding boxes (as in assignment \#3) for frame $\mathbf{t}=\mathbf{3}$ for the following scene:
Remember: \{expressions in curly parens \} are time-dependent; t denotes the frame number.
(Include fish "fish.obj" ) \#\# this fish has this $\mathrm{AABB}: x \min =0 ; x \max =4 ; y \min =0 ; y \max =2$. (G schoolA
(I inst1 fish (S 0.50 .5 ) (T 105 ) ) \#\# small fish
(I inst2 fish (S 2.0 2.0) ) \#\# large fish
)
(G myScene
(I i1 schoolA (R $\left.\left\{t^{*} 15^{\circ}\right\}\right)$ )
(I i2 schoolA (T $\{\mathrm{t} * 2\}\left\{\mathrm{t}^{*} 2\right\}$ ))
)
(Render myScene )


## Problem \#11: Inside/outside Test (6 pts.)

Paint the "inside" areas according to the POSITIVE WINDING NUMBER MODEL.


## Problem \# 12: Polygon Clipping ( 8 points)

Show the polygon contour(s) including the spurious double segments on the Window frame that will be output from the Sutherland-Hodgman polygon clipping algorithm for the polygons shown below. Assume that the clipping sequence is : a, b, c, d. Do your draft on the left, and show the final result in the right figure by strongly tracing out all output line segments.


## Problem \#13: Polygon Rasterization (10 points)

Using the paradigm discussed in class and used on Assignment \#8 (lower left pixel-corner sampling), rasterize the polygon below, i.e. paint in all the pixels that would get turned on in a scanline based fill algorithm. (Apparent coincidences are meant to be exact coincidences).


## Problem \#14: - Vector distributions (4 pts.)

Which of the four directional vector diagrams below describes most appropriately the photon escape probability from a Lambert surface?


## Problem \#15: Local Illumination (6 points)

Draw all the vectors used in the Phong reflectance model for calculating the color observed at point $P$, when that surface area is illuminated with ambient light $A$, directional light D , and spot light S .


## Problem \#16: Reflection (4 points)

A mirrored cube of side-length 2 is positioned centered and axis-aligned at the origin of the world coordinate system. A laser beam, originating from point $(10,4,-2)$ shines towards the origin, in the direction $(-5,-2,1)$. In what direction does the beam point after being reflected off the cube?

## Problem \#17: Brightness Profile (10 pts.)

Sketch observed brightness B, as seen from camera eye at ( $0,10,0$ ), along real face F (Phong model, $\mathrm{K}_{\mathrm{amb}}=\mathrm{K}_{\text {diff }}=\mathrm{K}_{\text {spec }}=0.5, \mathrm{~N}_{\text {phon }}=50$ ), illuminated by directional light D and spotlight S . Directional light D is of intensity 1 and shines from direction ( $-1000,1000,0$ ); Spotlight S is of intensity 10 and shines from point $(10,10,0)$ towards $(0,0,0)$; its angular falloff is 4 .


## Problem \#18: Apparent Brightness (10 points)

Given a yellowish $\left(\right.$ color $=\left(\begin{array}{lll}1.0 & 1.0 & 0.5\end{array}\right)$ ) directional light source sending 5 trillion photons per second per $\mathrm{mm}^{2}$ shining at an angle of 40 degrees (measured against the normal vector) on to a cyan-colored lacquered surface with those specifications: color $=\left(\begin{array}{lll}0.2 & 1.0 & 0.7\end{array}\right), \mathrm{k}_{\mathrm{a}}=\mathrm{k}_{\mathrm{d}}=0.3$, $\mathrm{k}_{\mathrm{s}}=0.7, \mathrm{k}_{\mathrm{sp}}=30, \mathrm{k}_{\mathrm{sm}}=0.6$; the surface is in an environment of greenish ambient light: color $=\left(\begin{array}{lll}0.5 & 1.0 & 0.5\end{array}\right), \quad \mathrm{I}=0.4 * 10^{12}$ photons $/ \mathrm{s} / \mathrm{mm}^{2}$. Calculate the intensities of the $(\mathrm{r}, \mathrm{g}, \mathrm{b})$ components sent towards an eye located in direction of the ideally reflected rays.

$$
I_{e y e, r}=\ldots ; \quad I_{e y e, g}=\ldots \quad I_{e y e, b}=
$$

