

TAKE HOME EXAM

Your Name: _____ Your Class Computer Account: _____

Your student ID #: _____

DO NOT OPEN THIS INSIDE SODA HALL -- WAIT UNTIL YOU ARE ALONE !

INSTRUCTIONS (Read carefully !)

YOU MAY USE LECTURE NOTES, ANY TEXT BOOK, PAPERS ON THE INTERNET,
-- BUT NOT THE HELP OF ANY OTHER HUMAN BEING -- TO DO THIS QUIZ.

On the in-class exams, you will be allowed ONE double-sided sheet of size 8.5 by 11 inches of your own personal notes to assist your memory. It might be worthwhile to start preparing and “test-driving” such a sheet now. It should take you about 2 hours (1pt./min.) to complete this Exam (this does not include any study time you might need to brush up on various topics).

YOU MUST DO THIS WORK ENTIRELY BY YOURSELF !

COLLABORATION IS CONSIDERED CHEATING.

There must be no discussion of any of the problems with anybody until after the deadline.

DO ALL WORK TO BE GRADED ON THESE SHEETS OR THEIR BACKFACES.

If any question on the exam appears unclear to you, write down what the difficulty was and what assumptions you made trying to solve the problem the way you understood it.

I HAVE UNDERSTOOD THESE RULES AND WILL OBEY THEM:

Your Signature: _____

DO NOT OPEN THIS UNTIL YOU HAVE SIGNED ABOVE AND ARE ALONE !

HAND IN THE EXAM TO PROF. SEQUIN BEFORE 2:40pm, Wednesday, March 11, 2009.

This Quiz serves as a “study guide” to help you ascertain that you are absorbing the important material covered in this course and that you understand how to apply it to novel situations. This is also good training for the Midterm and the Final Exams -- except that there will then be the added pressure of limited time.

Problem #1 -- Circle the correct answer: (6 points)

- | **TRUE** | **FALSE** | The bounding-box calculation in the rendering pipeline starts at the root node of the scene hierarchy.
- | **TRUE** | **FALSE** | In 2D, translations of homogeneous coordinate triples are described by asymmetrical 3x3 matrices.
- | **TRUE** | **FALSE** | If points P_0, Q_0, R_0 are carried into points P_1, Q_1, R_1 , respectively, by a transformation described by an orthogonal matrix, then the angle $P_0Q_0R_0$ is equal to the angle $P_1Q_1R_1$.
- | **TRUE** | **FALSE** | In 2-space, “translation in x” and “translation in y” are commutative.
- | **TRUE** | **FALSE** | A sphere with a surface that acts as a Lambertian diffuse reflector will look to an observer like a uniformly lit flat circular disk when illuminated with a uniform parallel illumination coming from the direction of the observer.

Problem #2 -- Circle the correct answers: (8 points)

(A) Circle all other 2D operations with which a **rotation around the z-axis** does commute:
Translation in x; Rotation around z; Mirroring in y; Uniform scaling; Non-uniform scaling.

(B) In what direction does the $\{+x\}$ -axis point after a $+90^\circ$ degree rotation around the y-axis (in a right-handed coordinate system) ?

$\{+x\}$ $\{-x\}$ $\{+y\}$ $\{-y\}$ $\{+z\}$ $\{-z\}$ $\{\text{other}\}$

(C) Circle the **Calligraphic Display Devices** in the following list:

| Pen Plotter | Ink-jet Printer | Cathode Ray Tube | Plasma Panel | Projector in Laser-Lightshow

(D) Which of the following homogeneous 2D points lies farthest from the origin ? (Circle one).

| $P(7, -8, 1)$ | $P(2, -2, 0.2)$ | $P(-30, 21, 3)$ | $P(56, 72, 8)$ |

Problem #3: Modeling (5 points)

Name THREE different ways to model (represent in a graphics program) a 3D brick-like object:

Problem #4: Rasterization (9 points)

You want to draw (approximate) a roughly one-pixel wide line (without using anti-aliasing techniques) on a raster device with square pixels. The line starts at the center of pixel $[0, 0]$ and ends at the pixel $[\mathbf{H}, \mathbf{V}]$, where \mathbf{H} is in the range $[50,70]$ and \mathbf{V} is in the range $[-100,-80]$.

How many pixels “roughly” (i.e., ± 5) will be turned on, expressed in variables \mathbf{H} and \mathbf{V} , if we use each of the following algorithms. Look at the relevant section of the lecture notes, textbook (and possibly the Internet, if you don’t have a textbook).

- a) Sampling the area of the perfect 1-pixel-wide band along the line will turn on ___ pixels.
- b) Turning on all pixels touched (crossed) by the ideal thin line will turn on ___ pixels.
- c) The Bresenham or Digital Differential Analyzer (DDA) algorithm will turn on ___ pixels.

Problem #5: CRT Display (8 points)

You have a raster graphics color display that has 10-bit DAC’s (digital to analog converters) at the three guns of the display CRT. The display size is 1024 by 512 pixels.

How many possible colors can you generate on this display ? _____

How many different colors can you show simultaneously on this screen ? _____

Now we want to introduce a color map so that we can display only 65536 different colors simultaneously; how many digital input bits and how many digital output bits should this look-up table have ?

of input bits: _____ # of output bits: _____

Problem #6: DOF (9 points)

(A) How many degrees of freedom (DOF) are associated with all possible finite-size cylinders in R^3 in arbitrary positions ? Give a brief explanation:

DOF = _____

(B) How many degrees of freedom (DOF) are associated with all possible pizza slices (an arbitrary sector of a circular disk) in arbitrary positions in R^3 ? Give a brief explanation:

DOF = _____

(C) How many degrees of freedom (DOF) are associated with the set of all infinitely long straight lines in 3D space ? Give a brief explanation:

DOF = _____

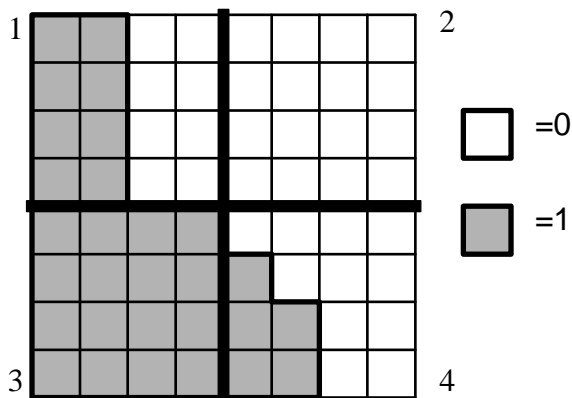
Problem #7: Bboxes (4 points)

A 2D scene consists of just two circles: one around point [-10,10] with radius 17, and the other around point [20,20] with radius 26. What are the four coordinate pairs for the four corners of the axis-aligned bounding box of this scene?

P1=(,), P2=(,), P3=(,), P4=(,).

Problem #8: Quad Tree (6 points)

Show a QUAD-TREE representing the geometry in the Figure below. Draw the tree with the children of each node appearing in order { 1,2,3,4 } from left to right; show the leaf-node values.



Problem #9: Modeling (6 points)

For each of the objects below, list the representation that most naturally fits the structure of the object. Choose from: Boundary Repr. (BRep) -- Constr. Solid Geometry (CSG) -- Volume Repr. (Octree) -- Proced. generation by sweeping (Sweep) -- Prototype instantiation (Inst.):

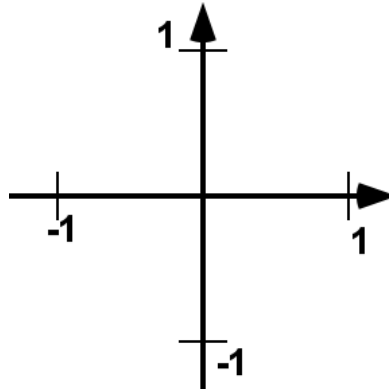
- The surface (height-field) $z = \cos(x*x+y*y) / (x*x+y*y)$ for $|x|, |y| < 10$. _____
- A geometrical sculpture made of 6 interpenetrating spheres. _____
- An intricate kitchen floor with two different types of interlocking tiles. _____
- An idealized wedge of Swiss Cheese. _____
- The porous inner structure of a bone. _____

Problem #10: Transformations (6 points)

Prove (in a mathematically clean way) that for the transformations discussed so far in this course, we can transform all the points of a straight line segment by transforming its endpoints and then constructing a new straight line segment between the two transformed endpoints.

Problem #11 (8 points)

Suppose you use homogeneous coordinates to define a 3x3 matrix which translates a 2D fish along the X axis by 0.4 units. What happens if you apply the same matrix to a unit cube in 3D? Draw the transformed cube in an orthographic (parallel) projection gazing down the Y axis (i.e., in the direction $\langle 0, -1, 0 \rangle$) into the right-handed coordinate system below – **label the axes correctly!**



Problem #12: Intersection Test (6 points)

Suppose you want to find the intersection between a parabola and a plane in 3D space. The parabola is defined parametrically by the expression $\mathbf{e} + \mathbf{d} * t + \mathbf{f} * t^2$, with $\mathbf{e}, \mathbf{d}, \mathbf{f}$ as vectors and t as the free parameter.

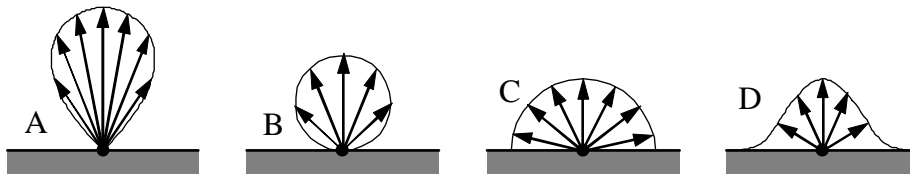
The plane is defined implicitly by all points \mathbf{p} satisfying $(\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$, where \mathbf{a} and \mathbf{n} are vectors.

A) Write an expression for t that you would evaluate to solve for the intersection.

B) In what cases are there no solutions? Write a conditional statement which is true in these cases, and false otherwise.

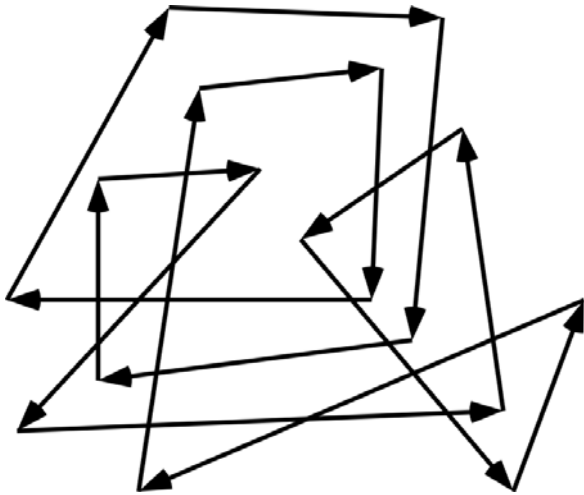
Problem #13: — Vector distributions (3 pts.)

Which of the four directional vector diagrams below describes most appropriately the photon escape probability from a Lambert surface ?



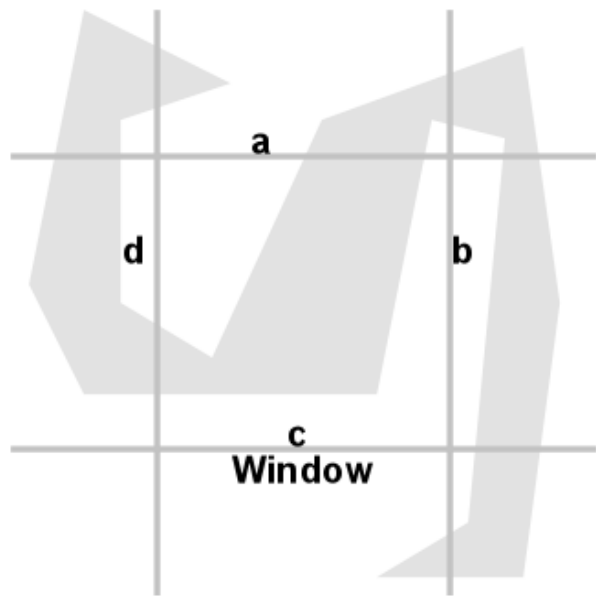
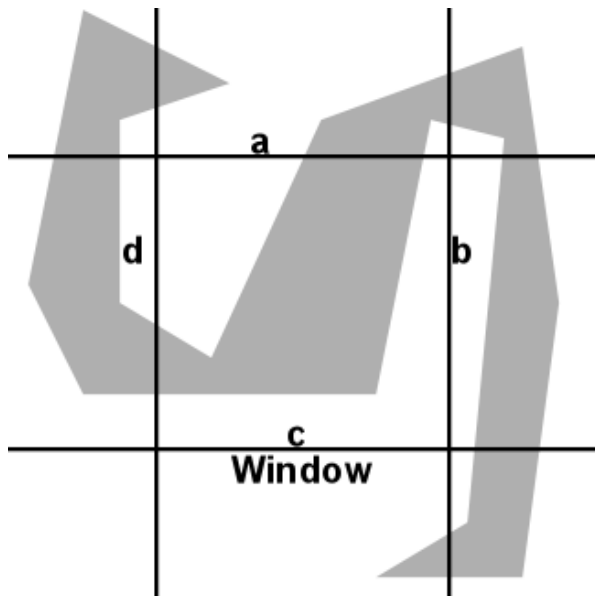
Problem #14: Inside/outside Test (6 pts.)

Paint the “inside” areas according to the NON-ZERO WINDING NUMBER MODEL.



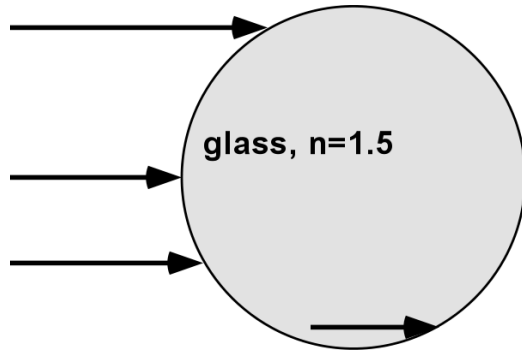
Problem # 15: Polygon Clipping (8 points)

Show the polygon contour(s) including the spurious double segments on the Window frame that will be output from the Sutherland-Hodgman polygon clipping algorithm for the polygons shown below. Assume that the clipping sequence is : a, b, c, d. Do your draft on the left, and show the final result in the right figure by strongly tracing out all output line segments.



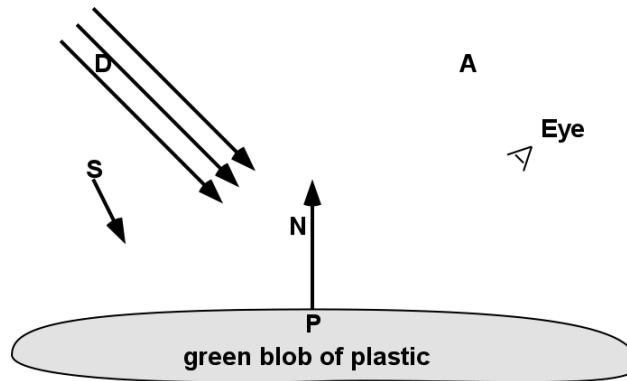
Problem #16: Refraction (6 points)

Complete drawing the following four rays as they move through this scene. Draw each ray as it enters, passes through, and/or exits the glass sphere.



Problem #17: Local Illumination (6 points)

Draw all the vectors used in the Phong reflectance model for calculating the color observed at point P, when that surface area is illuminated with ambient light A, directional light D, and spot light S.



Problem #18: Apparent Brightness (10 points)

Given a yellowish (color = (1.0 1.0 0.5)) directional light source sending 5 trillion photons per second per mm^2 shining at an angle of 40 degrees (measured against the normal vector) on to a cyan-colored lacquered surface with those specifications: color = (0.2 1.0 0.7), $k_a = k_d = 0.3$, $k_s = 0.7$, $k_{sp} = 30$, $k_{sm} = 0.6$; the surface is in an environment of greenish ambient light: color = (0.5 1.0 0.5), $I = 0.4 \cdot 10^{12}$ photons / s / mm^2 . Calculate the intensities of the (r,g,b) components sent towards an eye located in direction of the ideally reflected rays.

$I_{\text{eye, r}} = \underline{\hspace{2cm}}$; $I_{\text{eye, g}} = \underline{\hspace{2cm}}$; $I_{\text{eye, b}} = \underline{\hspace{2cm}}$.

Calculations below and on the back ...