## MIDTERM EXAM

Your Name:
Your Class Computer Account: $\qquad$

Room: $\qquad$ "Row": $\qquad$ Seat: $\qquad$ Your student ID \#: $\qquad$

## INSTRUCTIONS (Read carefully!) <br> DO NOT OPEN UNTIL TOLD TO DO SO!

TIME LIMIT: 75 minutes. Maximum number of points: 150.
CLEAN DESKS: No books; no calculators; only writing implements and ONE double-sided sheet of size 8.5 by 11 inches of your own personal notes to assist your memory.
NO QUESTIONS ! ( They are typically unnecessary and disturb the other students.) If any question on the exam appears unclear to you, write down what the difficulty is and what assumptions you made to try to solve the problem the way you understood it.
DO ALL WORK TO BE GRADED ON THESE SHEETS OR THEIR BACKFACES.
NO PEEKING; NO COLLABORATION OF ANY KIND!

I HAVE UNDERSTOOD THESE RULES:

Your Signature: $\qquad$

Get a few points up front:
What was the most difficult-to-understand concept in the course so far ? (2 pts)

## Problem \#1 - Circle the correct answer ( 10 pts.; 2each, -3 each wrong)

| TRUE |FALSE | 2D translations can be represented by homogeneous orthonormal 3x3 matrices.
| TRUE |FALSE | In 3-space, any sequence of non-uniform scalings can be applied in arbitrary order without affecting the result.
| TRUE |FALSE | The Gouraud shading model will produce a uniform apparent brightness when applied to an isolated, irregular, planar Lambert polygon, illuminated with a single parallel light source, and viewed (perspectively) from a close-by eye-point.
| TRUE |FALSE | The Gouraud shading technique produces a planar $\left\{a^{*} x+b^{*} y+c\right\}$ brightness distribution on triangular faces of a polyhedral object.
| TRUE |FALSE | The transpose of an orthonormal matrix is equal to its inverse.

## Problem \#2 - Circle the correct answers: ( 14 pts.)

(4) Circle all other operations with which a rotation around the $\mathbf{x}$-axis does commute:

Mirroring in x ; Translation in y ; Uniform scaling; Non-uniform scaling; Rotation around z .
(4) Which of the four directional vector diagrams below describes most appropriately the perceived brightness observed on an ideal Lambert surface when viewing the surface from a direction opposite to each of the small arrow directions?

(6) A multi-segment cubic B-spline curve with no cusps is defined by six control points. Circle all the degrees of continuity that exist at its parametric midpoint $(\mathrm{t}=1.5)$ :

## G0 $\quad$ C0 $\quad$ G1 $\quad$ C1 $1 \quad$ G2 $\quad$ C2 $\quad$ G3 $\quad$ C3 $3 \quad$ G4 $\quad$ C4

## Problem \# 3 - Parametric Representation (12 pts.)

(6) Give a parametric representation of a ray that starts at the eye E, passes through pixel center P , and then goes off to infinity:
(6) Give two reasons why a parametric curve representation: $\mathbf{x}=\mathbf{F}_{\mathbf{x}}(\mathbf{t}), \quad \mathbf{y}=\mathbf{F}_{\mathbf{y}}(\mathbf{t}), \quad \mathbf{z}=\mathbf{F}_{\mathbf{z}}(\mathbf{t})$ is preferable to the form: $\mathbf{y}=\mathbf{f}_{\mathbf{y}}(\mathbf{x}), \mathbf{z}=\mathbf{f}_{\mathbf{z}}(\mathbf{x})$.

## Problem \# 4 - Polygon-fill ( 8 pts.)

Paint the "inside" areas according to the NON-ZERO WINDING-NUMBER MODEL.


## Problem \# 5 - Short Questions ( 20 pts.)

(4) Given the choices (voxels $\mid$ B-rep mesh $\mid$ CSG $\mid$ sweep $\mid$ instantiation ), which is the prefered way to model :
a) A perfect, rotationally symmetric ellipsoid?
b) A piece of sponge (e.g., to wash your car)?
(4) A cubic B -spline in the $\mathrm{x}, \mathrm{y}$ plane has the following control points: $\mathrm{A}(0,0), \mathrm{B}(0,1), \mathrm{C}(0,2), \mathrm{D}(0,3), \mathrm{E}(0,4), \mathrm{F}(0,5), \mathrm{G}(0,6)$;

How long is the drawn curve?
(4) A square cross section of area $1 \mathrm{~cm}^{2}$ is swept along a piecewise linear space path.

What is area of the cross-sectional "rib" at a properly mitered joint that makes a 90 degree turn?
(4) A rotation-minimizing frame (RMF) is swept around a planar circular path.

The RMF is initialized to the Frenet frame. How many degrees is it rotated relative to the Frenet frame after sweeping through a full revolution?
(4) Describe in one sentence the essence of a contribution that Mr. Phong has made to the field of computer graphics:

## Problem \# 5 - Clipping ( 8 pts.)

For the figure below list all the line segments that, based on their "outcodes," can be trivially eliminated from being subjected to a more detailed line clipping algorithm.


## These lines can be trivially eliminated:

## Problem \# 7 - Gouraud Shading (12 pts.)

You are scan-line processing (in the usual way) the polygon below using Gouraud interpolation. The rendering intensities at the vertices are shown. Write out the intensities at the labeled points.

$\mathbf{A}=$ $\qquad$

B = $\qquad$
$\mathrm{C}=$ $\qquad$

D = $\qquad$

Problem \# 8 - Polygon-fill ( 5 pts. each)
(A) Draw a curve with a turning number of -1 and a winding number of +2 around a point P .
(B) Draw a closed curve that has G1- and C1-contuinuity but not G2- or C2-continuity.

## Problem \# 9 - Bézier Curve ( 10 pts.)

For the given cubic Bezier segment $(P, Q, R, S)$, find the points at $\mathbf{t}=\mathbf{1 / 3}$ and $\mathbf{t}=\mathbf{2 / 3}$ and their tangent direction using the deCasteljau method. Then sketch the resulting curve.


## Problem \# 10 - Illumination (10 pts.)

Sketch apparent brightness B, as seen from camera C, along real face F (Phong model, $\mathrm{K}_{\mathrm{amb}}=\mathrm{K}_{\text {diff }}=\mathrm{K}_{\text {spec }}=0.5, \mathrm{~N}_{\text {phong }}=50$ ), illuminated by point-light P and directional light D . Follow example X , showing the brightness of an ideal Lambert surface L , illuminated by point-light P .


## Problem \# 11 - CSG ( 10 pts.)

Given the 2-dimensional bar-bell below and a 2-D computer graphics CSG system with only the primitives unit-square and unit-circle, draw a simple CSG tree that will model the bar-bell. Use a minimal number of elements and of Boolean operations (transformations do not count). Also show the transformed, instantiated leaf objects overlaid on the picture of the bar-bell.


Problem \# 12 - Refraction and Reflection (10 pts.)


## Problem \# 13 - Texture Mapping ( 8 pts.)

Use the texture map below and apply it to the rectangular surface on the right, carefully observing the given texture coordinates ( $\mathrm{u}, \mathrm{v}$ ).

| $(0,1)$ |  |  |
| :--- | :--- | :--- | :--- |
| $(0,0)$ |  |  |
|  |  |  |

$(0.5,0)$
$(0.5,1)$
$(1,1)$

## Problem \# 14 - Surface "Decoration" ( 6 pts.)

You should understanding the fundamental principles behind the following "decoration" techniques": Texture-mapping (T), Bump-mapping (B), Displacement-mapping (D), and Environment-mapping (E). Indicate with the proper labels $\left({ }^{*}\right)$ which of these four techniques do the following:
(a): Affect the surface normal used for the lighting calculation: $\qquad$
(b): Use the surface normal as an entry into a look-up table: $\qquad$

