

Electricity Demand Shaping via Randomized Rewards: A Mean Field Game Approach

Galina A. Schwartz, Hamidou Tembine, Saurabh Amin, and S. Shankar Sastry

Abstract—In this paper, we develop a novel mechanism for reducing volatility of residential demand for electricity. We construct a reward-based (rebate) mechanism that provides consumers with incentives to shift their demand to off-peak time. In contrast to most other mechanisms proposed in the literature, the key feature of our mechanism is its modest requirements on user preferences, i.e., it does not require information of user responsiveness about shifting their demand from the peak to off-peak time. Specifically, our mechanism utilizes a probabilistic reward structure for users who shift their demand to the off-peak time, and is robust to incomplete information about user demand and/or risk preferences. We approach the problem from the public good perspective, and demonstrate that the mechanism can be implemented via lottery-like schemes. Our mechanism permits to reduce the distribution losses, and thus improve efficiency of electricity distribution. Finally, the mechanism can be readily incorporated into the emerging demand response schemes (e.g., the time-of-day pricing, and critical peak pricing schemes), and has security and privacy-preserving properties.

I. INTRODUCTION

In recent years, *demand response (DR) in smart grid* infrastructures has emerged as an important topic of research. These schemes aim to control/flatten the aggregate demand curves by shifting the users' consumption times to improve efficiency. For e.g., time-of-day variations of residential demand for electricity posit a considerable problem for stability and efficiency of electric grid. Indeed, higher volatility of user demand results in higher average distributor's expenses on network maintenance and electricity provision which creates upward pressure on retail prices. Altogether, these reasons have resulted in a considerable interest in mechanisms for reducing demand volatility. The ongoing deployments of Advanced metering infrastructures (AMI) and Smart Utility Networks (SUN) present new opportunities for the deployment of DR schemes.

Real-time pricing may sound as an attractive theoretical choice for DR via AMI/SUN. However, numerous complications arise in any practical implementation of real-

time pricing. Firstly, various studies on risk preferences of users indicate that they prefer flat-rate prices, and are even willing to pay a premium to avoid being charged a non-flat price for their electricity consumption. Moreover, residential users show low responsiveness to price signals. Secondly, the communication of disaggregated user demand data to a distribution utility (distributor) may cause substantial privacy concerns. The bi-directional real-time communication between the distributor and the users also introduces numerous insecurities [1]. For e.g., the demand data from AMIs to the distributor could be falsified, corrupted, or suppressed. Even when the actual demand data reaches the distributor uncorrupted, the pricing information could be maliciously altered (e.g., by fraudulent users). Systematic exploitation of such insecurities could even induce network instability [8].

Yet, with increasing wholesale price(s), and predictions of further price escalation, distributors are experimenting with tiered pricing schemes. Essentially, the distributors' considerations about DR schemes in smart infrastructures include the following trade-off: On one hand, in order to increase the efficiency of electricity provision, user prices should reflect the scarcity of electricity. On the other hand, fairness, privacy and security considerations limit the usability and attractiveness of real-time pricing for retail electricity.

In this paper, we develop a DR scheme of reducing the volatility of residential energy demand by building on Morgan's paper [10] on economics of public goods. We view the user actions that contribute to the reduction of demand volatility as means of public good provision. Thus, while we employ Morgan's technique [10], in our model, the users contribute not the money, but *negawatts* [16], i.e., the amount of demand they shift from peak to off-peak time. Our goal here is to design a reward-based scheme that incentivizes users to shift their demand to off-peak times. The key feature our scheme is its relative simplicity. Specifically, our scheme does not require the knowledge of how responsive the users are in shifting their demand. Essentially, we employ a probabilistic reward structure for users who shift their demand to the off-peak times. This feature makes our scheme robust to incomplete information about user demand and risk preferences, and introduces privacy-preserving features. In addition, our scheme is advantageous from the perspective of cyber-security. Lastly, our incentive scheme could be used in conjunction with other existing demand response mechanisms (e.g., the time-of-day pricing [5]).

We model the interactions of the distributor with customers (here assumed to be residential or household users) as a game. Practically, electricity consumption of each individual

G. A. Schwartz and S. Sastry are with department of Electrical Engineering and Computer Science, University of California, Berkeley, CA, USA. Their research was supported by the TRUST (Team for Research in Ubiquitous Secure Technology), which receives support from the National Science Foundation (NSF award number CCF-0424422). Email: {schwartz, sastry}@eecs.berkeley.edu

S. Amin is with department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA. His research was partially supported by the MIT faculty start-up grant. Email: amins@mit.edu

H. Tembine is with the Département de Télécommunications, SUPÉLEC, France. His research has been partially supported by the ERC Starting Grant 305123 MORE (Advanced Mathematical Tools for Complex Network Engineering). Email: tembine@ieee.org

user has negligible effect on the aggregate demand faced by the distributor. Still, the electricity price faced by each user depends on the aggregate demand. This feature is exactly what is considered in so-called *aggregative games*. In aggregative games, the payoff function of each player depends on his own action and an aggregative term reflecting the effect of other players' actions; but not on the individual actions of others, as in the standard non-aggregative games. A generic payoff function of player i in such a game with an additive aggregative term is given by

$$u_i \left(x_i, \sum_{j \neq i} x_j \right).$$

Aggregative games in large-scale systems are often called *mean-field games* [13], [14], [15]. To develop our reward-based DR scheme, we model the provision of megawatts as an aggregative game, where the reward is chosen by the distributor, based on his reduction of the network maintenance costs, which we assume as a decreasing and convex function of the maximum (peak) and variance of the demand. In our setup, the users who shift their demand to the off-peak time obtain rewards via a lottery-like scheme. We characterize the equilibrium of the mean-field game and compare it with the social optima. We demonstrate that the users and the distributor are better off in the presence of our reward-based mechanism. Specifically, in equilibrium, individual users will produce more megawatts, and therefore receive higher utility relative to the case when such mechanism is not offered.

A. Outline

The rest of the paper is organized as follows. In Section II we overview of lottery-based approach to public good problem, and discuss technical problems that emerge in adopting lotteries to reduce variability of electric demand. We introduce our model in Section III. In Section IV, we formulate the corresponding mean field game. We prove the existence and the uniqueness of the Nash equilibrium and show that interior equilibrium (i.e., equilibrium in which random rebates are offered) exists, and is welfare superior to the no-rebate case. In Section V we conclude with a short discussion of our results and future extensions.

II. BACKGROUND

A. Literature Review

Below we briefly review three different threads of research relevant for our paper.

I. In the first part, we cover the literature about demand shaping, which naturally divides into two approaches,

II. The second part of our review covers mean field-like game models in electricity market. The aggregate response of several consumers (or producers) to a price signal may be modeled as a function of individual contribution and an aggregative term [2], [3], [4].

In [5] Jiang and Low analyzed demand management in smart grid. They considered the case user demand is not time-correlated and provided an independent optimization

scheme per time period. In [6] studied real-time demand response with uncertain renewable energy in smart grid. They designed and evaluated distributed algorithms for optimal energy procurement and demand response in the presence of uncertain renewable supply and time-correlated demand. When random renewable energy is realized at delivery time, it actively manages user load through real-time demand response and purchases balancing power on the spot market to meet the aggregate demand. A summary of recent progress on demand response can be found in [9].

The authors in [7] proposed a decentralized charging control strategy for large populations of plug-in electric vehicles (PEVs). They have proposed aggregative and mean-field games in situation where PEV agents are rational and weakly coupled via their operation costs. Each of the PEV agents reacts optimally with respect to the average charging strategy of all the PEV agents.

III. Lastly, in the third part, we discuss the papers which apply lottery-like mechanisms to address public good provision.

We are building on the insights of a famous paper about the economics of public goods [10], i.e., by viewing user actions that are reducing demand volatility as a tool of public good provision. However, the specifics of demand management for residential electricity usage requires substantial changes of Morgan's approach in [10].

The idea of lotteries have also been used in other contexts, e.g., for the raffle scheduling technique applied in computer operating systems. Recent interest in application of lotteries to congestion management was facilitated by Mergu, Prabhakar, and Rama who demonstrated with a field study that lottery-based mechanisms can be used to decongest transportation systems. In contrast, our focus is similar to [12], and it is predominantly methodological. These papers approach lotteries as a technical tool of congestion management, and also compare their setup(s) to Morgan's scheme of using lotteries for funding public goods. We refer to [12] (and the references therein) for an overview of the driving forces behind the lottery-based schemes usage to reduce congestion in different domains (internet and traffic).

B. Morgan's model

Game-theoretic analysis in Morgan's paper [10] shows that lotteries are attractive tools for financing of public goods. We now present an overview of Morgan's results; also see [18]. Let $N = \{1, \dots, n\}$ denote the set of users. Each user $i \in N$ has a wealth w_i , and chooses an amount $x_i \in [0, w_i]$ as his contribution to the level of public good G° , which is defined as the sum of individual contributions, i.e., $G^\circ = \sum_{i=1}^n x_i$. Under voluntary contributions (i.e., no incentive scheme), the expected utility of i -th user is:

$$U_i(x_i, x_{-i}) = w_i + h_i \left(\sum_{i=1}^n x_i \right) - x_i,$$

where x_{-i} denotes the vector of contributions of all users but i , and $h_i(\cdot)$ is a strictly increasing and strictly concave function, and reflects the user's valuation of public good.

The *raffle-based scheme* in [10] gives a reward $R > 0$ to one or more users, and each user's expected reward is proportional to his contribution to the public good. The scheme is financed by deducting the reward R from the total contributions G° of all users. Then, the expected utility of i -th user becomes

$$U_i(x_i, x_{-i}) = w_i + h_i(G) - x_i + R \cdot \frac{x_i}{\sum_{i=1}^n x_i}, \quad (1)$$

where the level of public good is now $G = \sum_{i=1}^n x_i - R$. An underlying assumption in [10] is that the lottery is conducted by a charity. If the total contributions are insufficient to cover the prize up to an arbitrarily small amount $\delta > 0$ (i.e., $\sum_{i=1}^n x_i < R - \delta$), the raffle is canceled and each contributor gets his contribution back from the charity. If the denominator term $\sum_{i=1}^n x_i$ is zero, the payoff is the initial wealth. This creates a discontinuity in the payoff function. The aggregate welfare W is

$$W = \sum_{i=1}^n U_i. \quad (2)$$

Let social optimum be defined as an allocation with maximal aggregate (total) user welfare. In the following, the superscript ^(eq) (resp. *) denotes the respective quantities corresponding to Nash equilibrium (resp. social optimum).

Theorem 1 (Main results of Morgan [10]):

- (i) For any $R > 0$, there exists a unique Nash equilibrium, whereas for $R = 0$, multiple equilibria can exist, all with the same level of public good.
- (ii) There exists a unique level of public good G^* , which maximizes the aggregate welfare, and for any $R > 0$, we have $G^{(\text{eq})}(0) < G^{(\text{eq})}(R) < G^*$.
- (iii) The equilibrium level of public good with the raffle-scheme $G^{(\text{eq})}(R)$ can be made arbitrarily close to the socially optimal level G^* by choosing a sufficiently large reward R .
- (iv) For any $R > 0$, $G^{(\text{eq})}(R) > 0$ if and only if $G^* > 0$.
- (v) A fixed-prize raffle with a prize R is outcome equivalent to a game in which each individual receives a rebate share that is proportional to his or her contributions to the public good relative to total contributions. The charity financing the public good stipulates a rebate amount R which will be set aside from total contributions (provided that these exceed R).

From Theorem 1, we know that a probabilistic reward scheme could be implemented as a raffle (see (v)). However, to implement such a scheme with a goal of shaping electricity demand, a number of issues must be addressed. First, we have to decide who finances the reward R and how. Here we introduce the distributor as the player who organizes the lottery. Second, in [10], the users participate in the lottery via monetary contributions. However, in our model, the electricity users contribute to demand shaping via negawatts. Third, we need to explicitly consider the case when the lottery might be canceled in equilibrium. In general, it is hard to "return" the negawatts to the users. In [10], there is a possibility of raffle cancelation in the case when users

believe that others would not contribute enough, and there is no super-rich player who has sufficient funds to finance the raffle. Lastly, since each electricity user has a negligible effect on the aggregate demand faced by the distributor, a mean-field game model is appropriate for our environment.

III. MODEL

Consider n users and one distributor. We model the distributor's payoff as follows

$$\pi_n(R_n) = \frac{1}{n}(Q_n + \hat{Q}_n)p - \frac{1}{n}R_n - c_0\left(\frac{1}{n}Q^{\max}, \sigma\right),$$

where where Q^{\max} is a peak demand, $\sigma = \frac{1}{n^2}[Q - \bar{Q}]^2$, and $c_0(\frac{1}{n}Q^{\max}, \sigma)$ is the cost of network maintenance, which is increasing and convex in both arguments.

Each consumer has a certain peak demand that we decompose as shiftable demand q_i and non-shiftable demand \hat{q}_i . The choice variable for a user i is an amount of energy consumption $x_i \in [0, q_i]$ that he shifts from the peak time. The payoff u_i of user i is equal to

$$u_i = w(s_i) - [q_i + \hat{q}_i]p + u_{2,i},$$

The second term $u_{2,i}$ reflects the incentive scheme:

$$u_{2,i} = \mathbb{1}_{\{\sum_{j=1}^n x_j \neq 0\}} \left[h(G_n) + R_n \frac{x_i}{\sum_{j=1}^n x_j} - d(x_i) \right]$$

where G_n denotes the aggregate term

$$G_n = \left(\frac{1}{n} \sum_{j=1}^n x_j \right) - r_n,$$

and to simplify, below we will illustrate how our model works with a linear the dis-utility of shifting $d(x_i) = x_i$. We use the indicator function in order to well-define the payoff at 0. If the denominator is zero the term $\frac{x_i}{\sum_j x_j} = \frac{0}{0}$ but we will replace the payoff by $-\alpha \leq 0$. This makes a discontinuity at 0.

We introduced a strategic decision-making problem with two-levels. At the first level, the utility proposes a probabilistic reward r_n in order to incentivize the consumers to shift their demand. Then, each consumer, knowing r_n , chooses x_i that maximizes his u_i .

IV. RESULTS

The standard method for solving bi-level games is the backward induction. A equilibrium of our hierarchical game is a profile $(r_n^*, x^*(r_n^*))$ such that $r_n^* \in \arg \max_{r_n} \pi_n(r_n, x^*(r_n))$ and $\forall i, x_i^*(r_n) \in \arg \max_{x_i} [u_i]$.

We analyze the equilibrium properties of the game in the asymptotic regime:

$$u(x, m, r) = w(s) - p(q + \hat{q}) + \left[h(m - \tilde{r}) - x_i + r \frac{x_i}{m} \right] \mathbb{1}_{\{m \geq \tilde{r} - \delta\}}$$

where $\tilde{r}(r)$ is the inverse quantity required to compensate the cost due to congestion with the reward r . In order to simplify the analysis, we choose $\tilde{r}(r) = r$.

A. Equilibrium analysis between consumers

For a fixed r which is an inferior limiting point of r_n , we examine the game between consumers. The best response to the mean m and r is given by

$$BR(s, m, r) = \begin{cases} 0 & \text{if } m > r \\ \text{any feasible action} & \text{if } m = r \\ \text{max. action} & \text{if } r > m \geq r - \bar{\delta} > 0 \\ \text{any feasible action} & m < r - \bar{\delta} \end{cases}$$

We define a mean field equilibrium between consumers for a fixed $r > 0$ as follows.

Definition 1 (Pure mean field equilibrium): (x, m) is a pure mean field equilibrium if $\forall s, x(s) > 0 \implies x(s) \in \arg \max_{x'} u(s, x'(s), m, r)$ and the mean of the actions $x(s)$ with the respect to the state should generate m by consistency.

From the definition, a pure mean field equilibrium satisfies $m \in \arg \max_x u(s, x(s), m, r)$. Suppose $r > \bar{\delta}$. Then $x(s) = m^*(r) = r$ provides a pure mean field equilibrium between consumers. The equilibrium payoff is greater than the one without incentives whenever $h(0) \geq 0$. Note that the equilibrium quantity increases with the reward.

B. Social welfare of consumers

Next, we examine the social welfare of consumers. The arithmetic average payoff of all the consumers is given by

$$-p(q + \hat{q}) + \frac{1}{n} \sum_{j=1}^n w(s_j) + \frac{1}{n} \sum_{j=1}^n h(\cdot) + r_n - \frac{1}{n} \sum_j x_j$$

which limiting behavior has the following form:

$$-p(q + \hat{q}) + \bar{w} + h(m - r) + r - m$$

Optimizing the above function yields in optimizing the function $m \mapsto h(m - r) - m$ which has a maximizer $m^{*,so} = m^{*so}(r)$.

Suppose that $h(\cdot)$ is a concave diffeomorphism. Then, any interior solution is given by $h'(m - r) = 1$ i.e.

$$m^{*so} = m^{*so}(r) = r + (h'^{-1}(1))$$

and the optimum social welfare of consumers is $-p(q + \hat{q}) + \bar{w} + h(h'^{-1}(1)) - h'^{-1}(1)$.

C. Operator's anticipation

Now we focus on the operator anticipation problem. The operator will anticipate the reaction of the consumers in his payoff function and compute the profit as

$$(q + \hat{q})p - r - c_0(q^{\max}, [m^*(r) - \bar{m}]^2).$$

The profit optimization of the large scale system is equivalent in minimizing the cost $r + c_0(q^{\max}, [r - \bar{m}]^2)$ in r . Assume that the highest operator's cost is greater than \bar{m} : $c_0(q^{\max}, \bar{m}^2) > \bar{m}$. Then, the maximizer of the profit is positive, i.e., $r^* > 0$ because of very high cost $c_0(q^{\max}, \bar{m}^2)$ which increases with $\bar{m}^2 > 0$, and $c_0(q^{\max}, \bar{m}^2) > \bar{m}$. To compute the interior point r^* , we use the first order condition $-1 = 2[r - \bar{m}] \partial_\sigma c_0(q^{\max}, [r - \bar{m}]^2)$. If there are multiple

maximizers, the operator will choose the minimum cost to organize the lottery. The minimum among the maximizers of $r + c_0(q^{\max}, [r - \bar{m}]^2)$ will be chosen. Thus,

$$0 < r^* < \bar{m}.$$

In other words, the operator will finance because it reduces its cost.

V. CONCLUDING REMARKS

Recent technological developments necessitate a reconsideration of how electricity distribution is done. Indeed, wide deployment of advanced metering infrastructure (AMI) gives utility operators and individual households ample new possibilities to interact. A budding literature investigates new mechanisms of demand management with both, technology-based and market-based features. While the primary focus of demand response mechanisms is to assure integrity and availability of electric infrastructure, in this paper we are also concerned by potential threats to privacy of individual users, as well as by security of electric grid, which could be compromised via real-time information flows that AMI permits.

Our main contribution is a novel demand response scheme, based on a probabilistic rebate mechanism. We adopted a lottery-based scheme of public goods financing to reduce the societal costs of variability of the residential electric demand. We developed a probabilistic rebate scheme based on a lottery mechanism, to incentivise household users to shift their electric consumption to the off-peak times. In this paper, we consider only the effect of our mechanism on the utility costs of its distribution network maintenance. Our setting is stylized and minimalist, to demonstrate the essence of the probabilistic rebate mechanism. In our future work, we will consider the following extensions.

First, we will address the effects of our scheme on the utility's costs of electricity acquisition. Our intuition is that with the inclusion of these costs in the model, cost savings of the utility would be even higher than now, due to its reduction of real-time (balancing) purchases at the highest prices (locational marginal prices (LMPs)). Second, the positive effects of flatter electric demand are not limited by the reduced utility costs. Other parties, such as regional transmission operators (RTOs) are also actively investing in demand response programs due to their concern about the overall robustness of the grid, and specifically transmission system. An interesting extension of our work will be to consider savings of transmission operators and, possibility of using some part of these savings to finance user rebates. Third, our paper lacks quantitative estimates. Thus, our next step will be to include explicit functional forms of user demand and for utility costs of network maintenance, and fit the existing household demand data to construct predictions about the magnitude of the effects of our mechanism. Lastly, we are working on extending our setting to a dynamic game, in which evolutionary learning technics are used by the utility to learn user preferences for demand shifting.

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