First-order logic

Chapter 7

Outline

◇ Syntax and semantics of FOL

◇ Fun with sentences

◇ Wumpus world in FOL

Syntax of FOL: Basic elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\land \quad \lor \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall \ \exists$

Atomic sentences

Atomic sentence $= predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $= function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \land S_2, \quad S_1 \lor S_2, \quad S_1 \;\Rightarrow\; S_2, \quad S_1 \;\Leftrightarrow\; S_2$$

E.g. $Sibling(KingJohn, Richard) \;\Rightarrow\; Sibling(Richard, KingJohn)$
$>(1,2) \lor \leq(1,2)$
$>(1,2) \land \neg{>}(1,2)$

Truth in first-order logic

Sentences are true with respect to a <u>model</u> and an <u>interpretation</u>

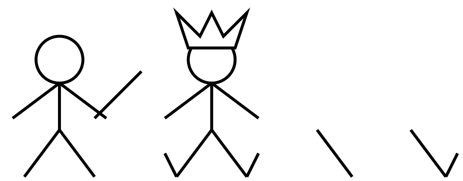
Model contains objects and relations among them

Interpretation specifies referents for
$constant\ symbols$ → <u>objects</u>
$predicate\ symbols$ → <u>relations</u>
$function\ symbols$ → <u>functional relations</u>

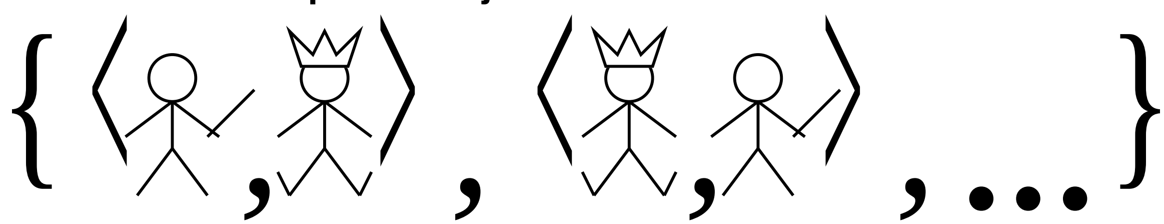
An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the <u>objects</u> referred to by $term_1, \ldots, term_n$
are in the <u>relation</u> referred to by $predicate$

Models for FOL: Example

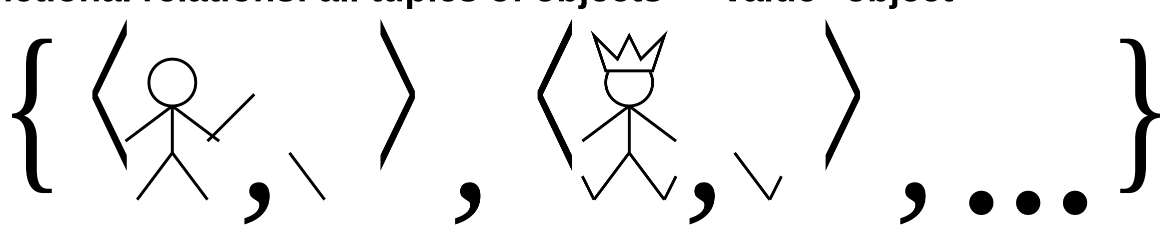
objects



relations: sets of tuples of objects



functional relations: all tuples of objects + "value" object



Universal quantification

$\forall \langle variables \rangle \;\; \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \;\; At(x, Berkeley) \;\Rightarrow\; Smart(x)$

$\forall x \;\; P$ is equivalent to the <u>conjunction</u> of <u>instantiations</u> of P

$$
\begin{aligned}
& At(KingJohn, Berkeley) \;\Rightarrow\; Smart(KingJohn) \\
\land\;& At(Richard, Berkeley) \;\Rightarrow\; Smart(Richard) \\
\land\;& At(Berkeley, Berkeley) \;\Rightarrow\; Smart(Berkeley) \\
\land\;& \ldots
\end{aligned}
$$

Typically, $\Rightarrow$ is the main connective with $\forall$.
Common mistake: using $\land$ as the main connective with $\forall$:

$$\forall x \;\; At(x, Berkeley) \land Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Someone at Stanford is smart:
$\exists x \ \ At(x, Stanford) \land Smart(x)$

$\exists x \ \ P$ is equivalent to the disjunction of instantiations of P

$$\begin{array}{ll} & At(KingJohn, Stanford) \land Smart(KingJohn) \\ \lor & At(Richard, Stanford) \land Smart(Richard) \\ \lor & At(Stanford, Stanford) \land Smart(Stanford) \\ \lor & \ldots \end{array}$$

Typically, $\land$ is the main connective with $\exists$.
Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \ \ At(x, Stanford) \ \Rightarrow \ Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall x \ \forall y$ is the same as $\forall y \ \forall x$ (why??)

$\exists x \ \exists y$ is the same as $\exists y \ \exists x$ (why??)

$\exists x \ \forall y$ is not the same as $\forall y \ \exists x$

$\exists x \ \forall y \ \ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y \ \exists x \ \ Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x \ \ Likes(x, IceCream) \qquad \neg\exists x \ \ \neg Likes(x, IceCream)$

$\exists x \ \ Likes(x, Broccoli) \qquad \neg\forall x \ \ \neg Likes(x, Broccoli)$

Fun with sentences

Brothers are siblings

.

"Sibling" is reflexive

.

One's mother is one's female parent

.

A first cousin is a child of a parent's sibling

.

.

$$\forall\, x,y \;\; Brother(x,y) \;\Leftrightarrow\; Sibling(x,y).$$

.

$$\forall\, x,y \;\; Sibling(x,y) \;\Leftrightarrow\; Sibling(y,x)$$

.

$$\forall\, x,y \;\; Mother(x,y) \;\Leftrightarrow\; (Female(x) and Parent(x,y))$$

.

$$\forall\, x,y \;\; FirstCousin(x,y) \;\Leftrightarrow\; \exists\, p,ps \;\; Parent(p,x) \wedge Sibling(ps,p) \wedge Parent(ps,y)$$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall\, x \;\; \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid

E.g., definition of (full) $Sibling$ in terms of $Parent$:

$$\forall\, x, y \;\; Sibling(x,y) \;\Leftrightarrow\; [\neg(x = y) \land \exists\, m, f \;\; \neg(m = f) \land Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$$

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$\textsc{Tell}(KB, Percept([Smell, Breeze, None], 5))$
$\textsc{Ask}(KB, \exists a \;\; Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: Yes, $\{a/Shoot\}$ $\leftarrow$ <u>substitution</u> (binding list)

Given a sentence S and a substitution σ,
$S\sigma$ denotes the result of plugging σ into S; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$\textsc{Ask}(\mathrm{KB}, \mathrm{S})$ returns some/all σ such that $KB \models S\sigma$

Knowledge base for the wumpus world

<u>"Perception"</u>

$\forall b, g, t \;\; Percept([Smell, b, g], t) \Rightarrow Smelt(t)$

$\forall s, b, t \;\; Percept([s, b, Glitter], t) \Rightarrow AtGold(t)$

<u>Reflex</u>: $\forall t \;\; AtGold(t) \Rightarrow Action(Grab, t)$

<u>Reflex with internal state</u>: do we have the gold already?

$\forall t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \Rightarrow Action(Grab, t)$

$Holding(Gold, t)$ cannot be observed

$\Rightarrow$ keeping track of change is essential

Deducing hidden properties

Properties of locations:
$\forall\, l, t \;\; At(Agent, l, t) \land Smelt(t) \;\Rightarrow\; Smelly(l)$
$\forall\, l, t \;\; At(Agent, l, t) \land Breeze(t) \;\Rightarrow\; Breezy(l)$

Squares are breezy near a pit:

Diagnostic rule—infer cause from effect

$$\forall\, y \;\; Breezy(y) \;\Rightarrow\; \exists\, x \;\; Pit(x) \land Adjacent(x, y)$$

Causal rule—infer effect from cause

$$\forall\, x, y \;\; Pit(x) \land Adjacent(x, y) \;\Rightarrow\; Breezy(y)$$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

Definition for the $Breezy$ predicate:

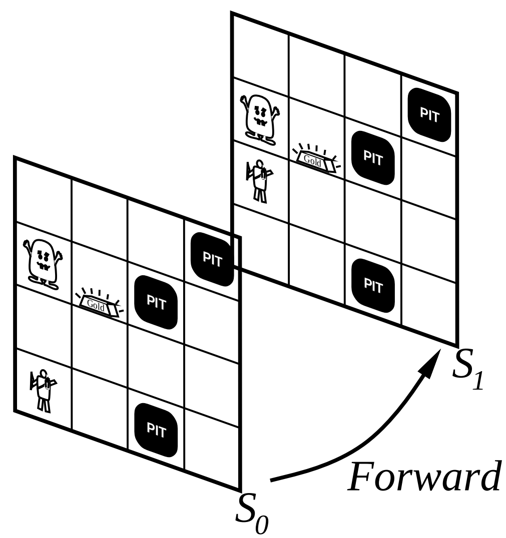
$$\forall\, y \;\; Breezy(y) \;\Leftrightarrow\; [\exists\, x \;\; Pit(x) \land Adjacent(x, y)]$$

Keeping track of change

Facts hold in situations, rather than eternally
E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

Situation calculus is one way to represent change in FOL:
Adds a situation argument to each non-eternal predicate
E.g., Now in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing a is s



Describing actions I

"Effect" axiom—describe changes due to action
$\forall s \;\; AtGold(s) \;\Rightarrow\; Holding(Gold, Result(Grab, s))$

"Frame" axiom—describe <u>non-changes</u> due to action
$\forall s \;\; HaveArrow(s) \;\Rightarrow\; HaveArrow(Result(Grab, s))$

<u>Frame problem</u>: find an elegant way to handle non-change
(a) representation—avoid frame axioms
(b) inference—avoid repeated "copy-overs" to keep track of state

<u>Qualification problem</u>: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or ...

<u>Ramification problem</u>: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, ...

Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is "about" a predicate (not an action per se):

$$\begin{array}{lcl} \text{P true afterwards} & \Leftrightarrow & [\text{an action made P true} \\ & \lor & \text{P true already and no action made P false}] \end{array}$$

For holding the gold:

$$\begin{array}{l} \forall\, a, s \;\; Holding(Gold, Result(a,s)) \;\Leftrightarrow \\ \quad [(a = Grab \land AtGold(s)) \\ \quad \lor (Holding(Gold, s) \land a \neq Release)] \end{array}$$

Making plans

Initial condition in KB:

$At(Agent, [1,1], S_0)$
$At(Gold, [1,2], S_0)$

Query: $\textsc{Ask}(KB, \exists\, s\ \ Holding(Gold, s))$
i.e., in what situation will I be holding the gold?

Answer: $\{s/Result(Grab, Result(Forward, S_0))\}$
i.e., go forward and then grab the gold

This assumes that the agent is interested in plans starting at S_0 and that S_0 is the only situation described in the KB

Making plans: A better way

Represent plans as action sequences $[a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing p in s

Then the query $\textsc{Ask}(KB, \exists\, p\;\; Holding(Gold, PlanResult(p, S_0)))$ has the solution $\{p/[Forward, Grab]\}$

Definition of $PlanResult$ in terms of $Result$:

$$\forall\, s\;\; PlanResult([\,], s) = s$$

$$\forall\, a, p, s\;\; PlanResult([a|p], s) = PlanResult(p, Result(a, s))$$

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner

Summary

First-order logic:
- objects and relations are semantic primitives
- syntax: constants, functions, predicates, equality, quantifiers

Increased expressive power: sufficient to define wumpus world

Situation calculus:
- conventions for describing actions and change in FOL
- can formulate planning as inference on a situation calculus KB