Homework Assignment #5
Due on bcourses Thursday 10/8/2020 (zero credit after 9 AM Friday)

1. Fall 2013 Midterm question 6
   a. \( x(1\text{kHz}) = \text{(resonant amplitude divided by } Q \text{ at low frequency)} = 10\text{nm} \)
   b. \( x(100\text{kHz}) = \text{(low frequency amplitude divided by (} \omega/\omega_n)^2 \text{ above resonance)} = 10\text{nm}/100 = 0.1\text{nm} \)
      Also acceptable to say 10nm because that’s the DC deflection, and the question was ambiguous
   c. \( k \propto \frac{1}{L^3} \text{ so } k * \frac{1}{8} \)
      \( k * \frac{1}{8} \rightarrow \omega_n \sqrt{\frac{1}{8}} \)
   d. If \( 2(1.5\text{V})(150\text{V}) \) gives us \( F_0 \), then \( \frac{1}{2}(150\text{V})^2 \) will give \( \frac{1}{4}(150\text{V}/1.5\text{V})F_0 \)
      and \( \frac{1}{2}(1.5\text{V})^2 \) will give \( \frac{1}{4} (1.5/150)F_0 \)

<table>
<thead>
<tr>
<th>25 ( F_0 ) DC force at 0Hz</th>
<th>( \frac{1}{400}F_0 ) 2( \omega ) force at 2Hz</th>
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</table>

2. Fall 2013 Midterm question 7
   \( k_{\text{in-plane}} = \frac{Etw^3}{4L^2} \text{ increase by 2} \)
   \( b = \frac{\mu A}{g} \text{ so } b \text{ unchanged} \)
   \( m = ptwL \text{ so increase by 2} \)
   \( f_n = \frac{1}{2\pi} \omega_n \text{ so unchanged since } k/m \text{ unchanged} \)
   \( Q = \frac{k}{\omega_n b} \text{ so } Q \text{ * 1/2} \)
   \( x_n \text{ increases by 3/2} \)

Resonant deflection \( x_n = F/(b\omega_n) \). \( F \) has increased by the change in \((t+g)/g\), which was \( (2+2)/2=2 \), and is now \((4+2)/2=3 \). So \( F \) has increased by 3/2.

3. Fall 2013 Midterm question 8
   a. \( V_{\text{out}} = \frac{\Delta R}{4R} Vx = \frac{1}{4} (0.01)(10\text{V}) = 0.025\text{V} \)
   b. \( \frac{\Delta R}{R} = Ge = -20*0.001 = -0.02 = -2\% \)
   c. \( \frac{\Delta R}{R} = TRC * \Delta T = 0.1 \)

4. A resonator with a spring constant \( k = 1 \text{ N/m} \), a mass of 10 micro-grams, and a \( Q \) of 10 is driven by an external force to have an amplitude of motion of 1 um.
   a. Carefully plot the spring, mass, and damping forces vs. frequency, showing clearly the frequency at which spring and mass forces cross, and the magnitude of the damping force at this frequency. Axis labels should be clear.
   b. At 1 Hz, estimate the phase of the position, velocity, and acceleration of the mass relative to the drive force. Multiples of 90 degrees (or \( \pi/2 \)) are fine. I’m not looking for lots of digits of precision.
   c. Estimate the frequencies at which the magnitude of the sine terms (sum of spring and mass) is equal to the magnitude of the cosine term (damping). The difference between these is the 3dB bandwidth. Is it roughly the resonant frequency divided by the \( Q \)?
      a. Magnitude of the damping force at the resonant frequency should be exactly ten times lower than the mass and spring forces. Vertical axis should have 0 and \( kx0 \) labeled.
      Horizontal axis should have 0 and \( \omega_n \) labeled.
b. position: $0^\circ$
velocity: $90^\circ$
acceleration: position: $180^\circ$
d. The red dashed lines denote approximately where the sin terms (spring and mass) sum to have the same magnitude as the cosine term (damping).

\[ +/- (k - m\omega^2) = b\omega \]
\[ m\omega^2 +/- b\omega - k = 0 \]
\[ (10\text{kg})\omega^2 +/- (10^{-5}\frac{N_s}{m})\omega - 1\frac{N}{m} = 0 \]
\[ \omega = 9500\text{Hz and } -10,500\text{Hz} \]
\[ \Delta\omega = 1\text{kHz} \]
\[ \omega_n=10,000\text{Hz and } Q=10 \text{ so the 3dB bandwidth (distance between the red lines) } = 1\text{kHz} \]

5. The same resonator in the previous problem is driven with a sinusoidal force of 1 uN.
a. Carefully plot the amplitude of the resulting motion on a linear plot, with particular attention to the amplitude and phase at 0, $\omega_n/2$, $\omega_n$, 2$\omega_n$
Calculate the actual deflection at $\omega_n/2$ (it's 4/3 times bigger than $F_0/k$) = 4/3um
and at 2 $\omega_n$ (it's 4/3 times bigger than $F_0/[m(2\omega_n)^2]$ ) = 0.25nm
3dB down (i.e. 10 um /sqrt(2) = 7um) make sure that the curve goes through those points at +/-0.5kHz
b. What is the amplitude at 10 \( \omega_n \),

\[
x_{DC} = \frac{F_{applied}}{k} = 1\text{um} \\
x_{high\omega} = \frac{x_{DC}}{(\omega_n)^2} = \frac{x_{DC}}{1(10\omega_n)^2} = \frac{x_{DC}}{100} = 10\text{nm}
\]

c. What is the phase of the motion at 0, \( \omega_n \), 10\( \omega_n \)

DC(\sim 0\text{Hz}): 0^\circ \\
\omega_n: -90^\circ \\
10\omega_n: -180^\circ 

6. The same resonator is run in vacuum, and you measure the Q as 10,000

a. Estimate \( b \)

\[
b = \frac{k}{\omega_n Q} = \frac{1N}{10k \text{rad/sec} + 10,000} = 10^{-8} \text{ rad/kg/sec}
\]

b. On a log/log scale, plot the transfer function, showing clearly the magnitude of the low frequency response and resonant response.
c. What is the slope of the response well above the resonant frequency (on your log/log scale)?

*Well above resonance the curve goes as $\frac{F_0}{m\cdot w^2}$, so when we take a log we get a slope of $-2$. This is also $-40\text{ dB dec}$.*

d. What is the 3dB bandwidth of the resonator?

1Hz

e. On a linear/log plot, sketch the phase near resonance. Clearly indicate the frequencies at which the phase is -45, -90, and -135. Ponder whether this is related to problem 4c in any way.

Zoomed in plot of phase to show that the difference in frequencies when the phase is -45 and -135 is the 3dB bandwidth.

7. In the SOIMUMPs process, you fabricate a beam of length $L=1\text{ mm}$ with a gap-closing electrostatic actuator on the end. Assuming $E=170\text{ GPa}$, the beam is 2um wide and the gap is 1um wide and 100um long.

a. what is the pull-in voltage?
b. If there are gap-stops at 0.5 microns, what is the pull-out voltage?

\[
\begin{align*}
\text{Assumed } t &= 10 \text{um but using } 25 \text{um is fine too.} \\
k_y &= \frac{Etw^3}{4L^3} = \frac{170GP_a(10\text{um})^3(2\text{um})^3}{4(1000\text{um})^3} = 0.0034 \frac{N}{m} \\
\text{a. } V_{PI} &= \sqrt{\frac{8kg_0^3}{27\epsilon A}} = \sqrt{\frac{8(0.0034)(1\text{um})^3}{27(8.854\times10^{-12} \frac{F}{m})(100\text{um}+40\text{um})}} = 0.17V \\
\text{b. } V_{PO} &= \sqrt{\frac{2kg_d^2(g_0-g_d)}{\epsilon A}} = \sqrt{\frac{2(0.014N/m)(0.5\text{um})^2(1\text{um}-0.5\text{um})}{(8.854\times10^{-12} \frac{F}{m})(100\text{um}+40\text{um})}} = 0.15V
\end{align*}
\]

8. If you were to apply a sinusoidal voltage to this structure, could you get it to pull in at a lower voltage? Why/how? What frequency would you use if you were applying a pure sine wave with no DC bias?

Tricky! What happens if the frequency of the sinusoid was near resonance? We get a larger displacement! This larger displacement makes movable plate closer to the stationary plate, which means the gap is smaller.

\[V_{PI} \propto \sqrt{g_0^3}\] so if the gap decreases, then \(V_{PI}\) decreases.

9. In Spencer et al. (reference [1]), an electrostatic gap-closing relay is made with a gate/body overlap area \(A_{ov}\), and initial gap \(g_0\), a displacement \(g_d\) (the amount that it moves before contact), and a spring constant \(K\) (see table 1 for specific values of the "current devices" reported in the paper).

a. Checkout the pictures of the people on the last page – you may recognize some of them
b. calculate the expected pull-in voltage based on the specs for "current devices"
c. calculate the expected value for the pull-out voltage based on the same specs
d. Compare your results from parts b and c to the measured results in Figure 11. Are their results close to what the theory says that should be? What do you think might explain any differences between theory and experiment?
e. How fast is the mechanical response of these devices today, and how fast do they hope to make them in the “scaled model” future? (see Table 1)


a. Our Dean!

For b and c use equations from #7 in this assignment and values from Table1 in the linked paper.

\[A_{ov} = 450 \text{ um}^2\]
\[g_0 = 180\text{nm}\]
\[g_d = 90\text{nm}\]
\[k = 62.5 \frac{N}{m}\]

b. \(V_{PI} = 5V\) (Not what’s listed in Table1!)
c. \(V_{PO} = 4.75V\)
d. Pretty good approximation, especially for quick formula but not exact in part due to parasitic capacitance and resistance.
e. “Current”: 34us but scaled is only 0.02us-0.08us! Super fast for mechanical devices, but not very fast compared to transistors (which switch in a few picoseconds these days)

10. [247] As the beam gets shorter in the problem above, the parallel plate approximation for the capacitor becomes an increasingly poor model for the actuator, and the linear spring becomes a poor model for the beam, because both ignore rotation and torque. How would you go about calculating the actual pull-in voltage for this structure? Would the pull-out voltage be easier to calculate (assuming gap stops on both sides)?

11. The pictures below are of the ADXL202, which is now obsolete but looks a lot like the newer parts that replaced it. Die photo (CMOS+MEMS), SEM of the MEMS device, detail SEM, and simulation of response to a vertical acceleration.
   a. Using the scale bar, estimate the mass of the proof mass, assuming a 3um thick polysilicon film.
   b. Using the sensor resonant frequency quoted on the datasheet, estimate the combined spring constant of all of the support springs.
   c. In the detailed SEM, identify the proof mass, support springs, proof mass anchor, over-acceleration mechanical stops, differential sense capacitors (big), actuation capacitors (small).
   d. Assuming a 3um thick film with 1um gaps, estimate the voltage needed across the actuator gap to support the weight of the proof mass (at 1 gravity).

   a. mass=$\frac{2300\text{kg}}{m^3} \times 3\text{um} \times (4 \times 75\mu m \times 75\mu m + 200\mu m \times 200\mu m) = 450\text{p kg (450 pico-kilo grams)}$
   b. $k=m \times \omega^2 = 450\text{p kg} \times (14\text{kHz})^2 = 0.09\text{N/m}$
c.

There are four drive capacitors for each in-plane direction.

\[ F = mg = 4 \times F_{\text{cap}} \]

\[ F_{\text{cap}} = 0.5 \varepsilon_0 V^2 \frac{A}{g^2} = 0.5 \left( 8.85 \times 10^{-12} \frac{F}{m} \right) V^2 \frac{3 \text{um} \times 40 \text{um}}{(1 \text{um})^2} = 450 \text{p kg} \times \frac{10 \text{N}}{\text{m}^2} \]

\[ V = \sqrt{\frac{450 \text{p kg} \times \frac{10 \text{N}}{\text{m}^2} \times (1 \text{um})^2}{0.5 \left( 8.85 \times 10^{-12} \frac{F}{m} \right) \times 3 \text{um} \times 40 \text{um}}} = 3V \]