EE 147/247A Prof. Pister
Fall 2020

Homework Assignment #5

Due on bcourses Thursday 10/8/2020 (zero credit after 9 AM Friday)

1. [12 pts total] Fall 2013 Midterm question 6
   2 pts for each answer. 1 pt for getting the formula, 1 pt for getting the right answer
   a. x(1kHz)=(resonant amplitude divided by Q at low frequency) = 10nm
   b. x(100kHz)=(low frequency amplitude divided by (ω/ωn)^2 above resonance)=10nm/100=0.1nm
   Also acceptable to say 10nm because that's the DC deflection, and the question was ambiguous
   c. k∝ $\frac{1}{L^3}$ $so\ k * \frac{1}{8}$

   $$k * \frac{1}{8} \rightarrow \omega_n \sqrt{\frac{1}{8}}$$

   d. If 2(1.5V)(150V) gives us F0, then ½(150V)^2 will give ¼(150V/1.5V)F0
   and ½(1.5V)^2 will give ¼ (1.5/150)F0

$25\ F_0\ DC\ force\ at\ 0Hz$	$\frac{1}{400} F_0\ 2\omega\ force\ at\ 2Hz$

2. [12 pts total, 1 pt formula, 1 pt right answer] Fall 2013 Midterm question 7
   If you get one part wrong and that error cascades, you should only subtract the points for the first error.

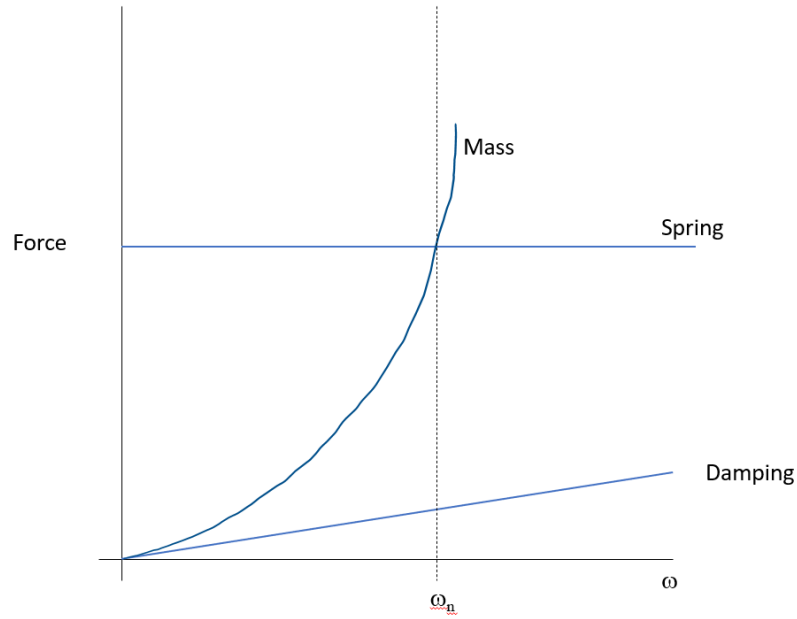
$k_{in\ plane} = \frac{Etw^3}{4L^3}$ increase by 2	b=$\frac{\mu A}{g}$ so b unchanged	m=$ptwL\ so\ increase\ by\ 2$
$f_n = \frac{1}{2\pi}\omega_n$ $so\ unchanged\ since \frac{k}{m} unchanged$	Q = $\frac{k}{\omega_n b}$ $so\ Q * 1/2$	$xn\ increases\ by\ 3/2$

Resonant deflection xn = F/(bωn). F has increased by the change in (t+g)/g, which was (2+2)/2=2, and is now (4+2)/2=3. So F has increased by 3/2.

3. [6 pts. 1 pt formula, 1 pt right answer] Fall 2013 Midterm question 8
   a. $V_{out} = \frac{\Delta R}{4R} Vx = \frac{1}{4}(0.01)10V = 0.025V$
   b. $\frac{\Delta R}{R} = G\epsilon$ =-20*0.001=-0.02 = -2%
   c. $\frac{\Delta R}{R} = TCR * \Delta T = 0.1$

4. [16 pts total] A resonator with a spring constant k=1 N/m, a mass of 10 micro-grams, and a Q of 10 is driven by an external force to have an amplitude of motion of 1 um.
   a. Carefully plot the spring, mass, and damping forces vs. frequency, showing clearly the frequency at which spring and mass forces cross, and the magnitude of the damping force at this frequency. Axis labels should be clear.
   b. At 1 Hz, estimate the phase of the position, velocity, and acceleration of the mass relative to the drive force. Multiples of 90 degrees (or pi/2) are fine. I'm not looking for lots of digits of precision.
   c. Estimate the frequencies at which the magnitude of the sine terms (sum of spring and mass) is equal to the magnitude of the cosine term (damping). The difference between these is the 3dB bandwidth. Is it roughly the resonant frequency divided by the Q?
      a. [8 pts total for part a]
         [3 pts] Inertial force should be quadratic, damping linear, spring constant
         [1 pt] Magnitude of the damping force at the resonant frequency should be exactly ten times lower than the mass and spring forces.
         [2 pts] Vertical axis should have 0 and kx0 labeled.

[2 pts] Horizontal axis should have 0 and wn labeled. wn should be at the point where the spring and mass lines cross.



b. [3 pts total for part b] Getting the sign wrong is ok, but think about it!

position: 0°

velocity: 90°

acceleration: position: 180°

c. [5 pts total for part c. 2 pts for getting the right formula, 2 pts for getting roughly the right answer. 1 pt for noting that it is the resonant frequency over Q]

The red dashed lines denote approximately where the sin terms (spring and mass) sum to have the same magnitude as the cosine term (damping).

$$+/-(k - m\omega^2) = b\omega$$

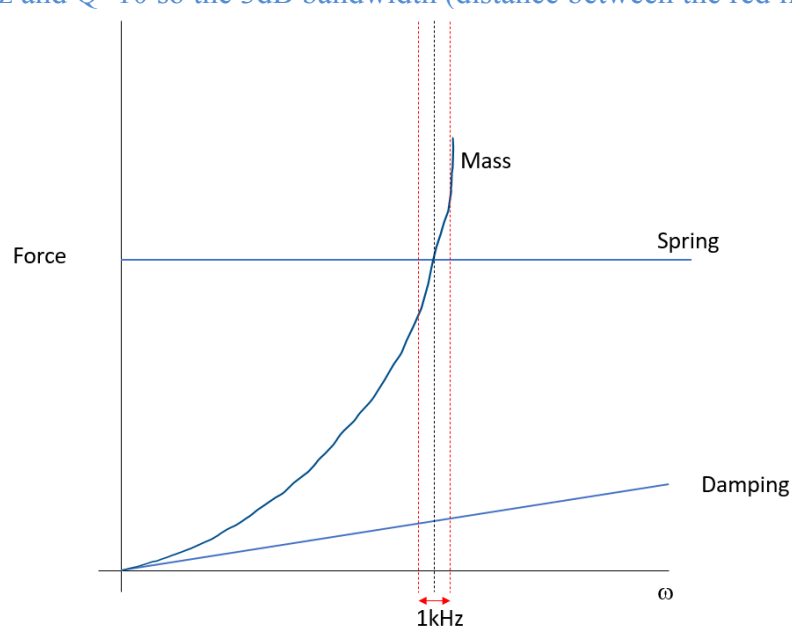
$$m\omega^2 +/- b\omega - k = 0$$

$$(10\text{nkg})\ \omega^2 +/- (10^{-5}\frac{Ns}{m})\omega - 1\frac{N}{m} = 0$$

$$\omega = 9500\text{Hz and } -10{,}500\text{Hz}$$

$$\Delta\omega = 1kHz$$

ω_n =10,000Hz and Q=10 so the 3dB bandwidth (distance between the red lines) = 1kHz



5. [11 pts total] The same resonator in the previous problem is driven with a sinusoidal force of 1 uN.

a. [6 pts total] Carefully plot the amplitude of the resulting motion on a linear plot, with particular attention to the amplitude and phase at 0, $\omega_n/2$, ω_n, $2\omega_n$

[1pt] Amplitude at wn is 10um

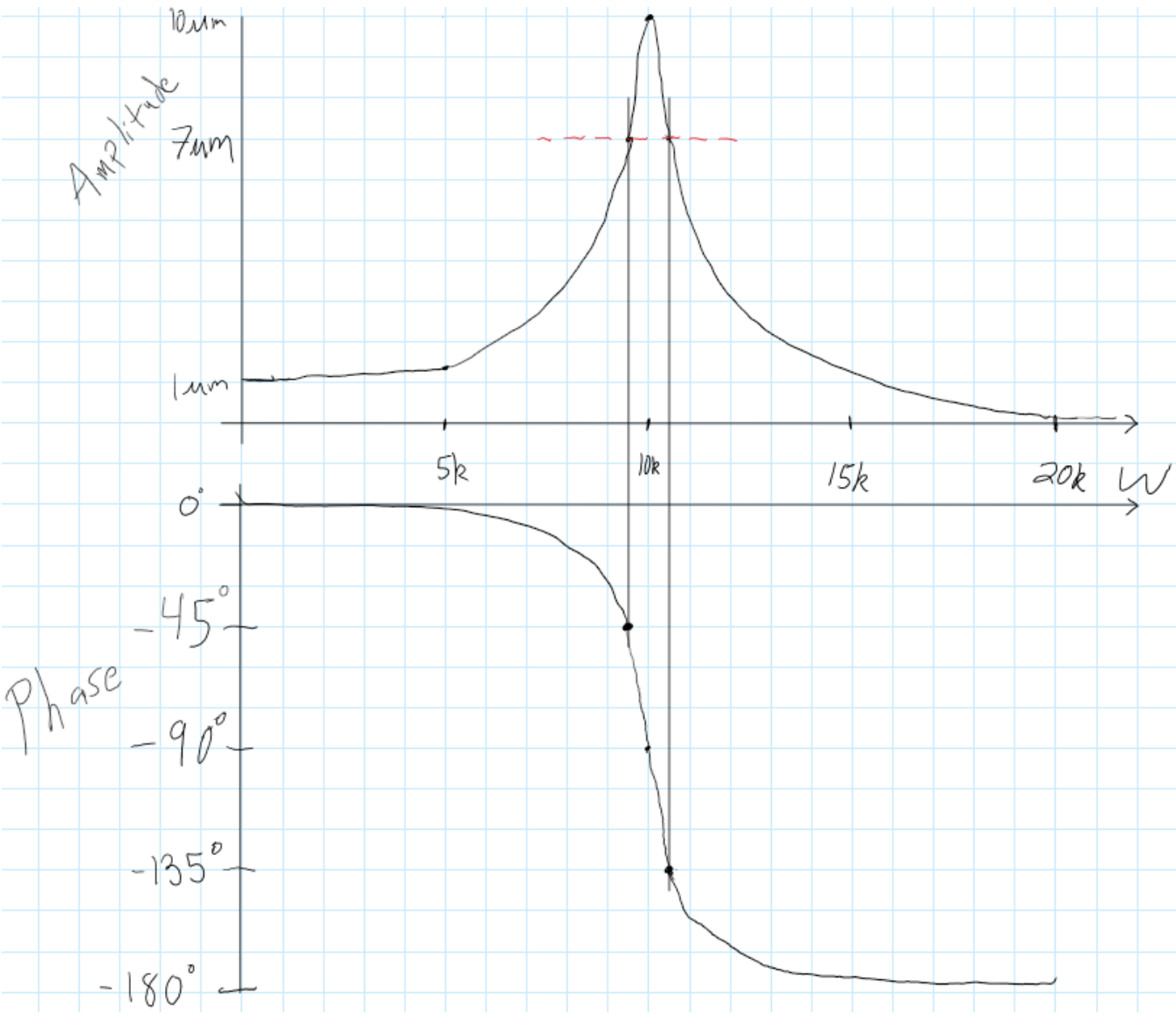
[1pt] Calculate the actual deflection at wn/2 (it's 4/3 times bigger than F0/k) =4/3um

[1pt] at 2 wn (it's 4/3 times bigger than F0/[m(2wn)^2]) = 0.33nm

3dB down (i.e. 10 um /sqrt(2) = 7um) make sure that the curve goes through those points at +/-0.5kHz

(I didn't ask you about these, so no points off if you missed them)

[3pts] phase at wn/2, wn, and 2wn should be close to 0, -90, and -180



b. [2 pts] What is the amplitude at 10 ω_n,

$$x_{DC} = \frac{F_{applied}}{k} = 1\text{um}$$

$$x_{high\ \omega} = \frac{x_{DC}}{(\frac{\omega}{\omega_n})^2} = \frac{x_{DC}}{(\frac{10\omega_n}{\omega_n})^2} = \frac{x_{DC}}{100} = 10\text{nm}$$

c. [3 pts] What is the phase of the motion at 0, ω_n, $10\omega_n$

DC(~0Hz): 0°

ω_n: -90°

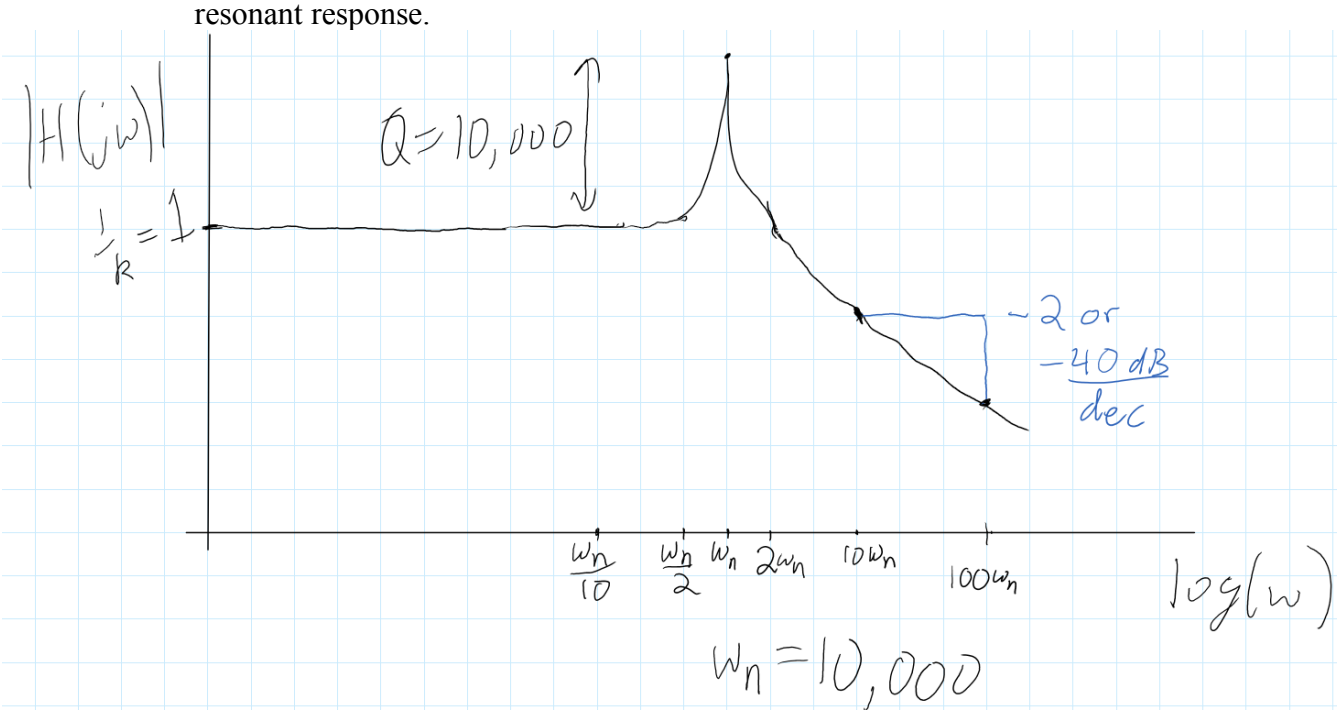
$10\omega_n$: -180°

6. [14 pts total] The same resonator is run in vacuum, and you measure the Q as 10,000

a. [2 pts] Estimate b

$$b = \frac{k}{\omega_n Q} = \frac{1\frac{N}{m}}{10k\frac{rad}{sec} * 10{,}000} = 10^{-8}\,\frac{N}{m/s}$$

b. [5 pts total] 3 pts for shape: 1 each for flat, pointy, and sloped regions. 2 pts for showing that the peak is 10,000 times higher than low freq response. On a log/log scale, plot the transfer function, showing clearly the magnitude of the low frequency response and resonant response.



c. [1 pt. The answer is -2, but give yourself credit if you said -40dB/dec] What is the slope of the response well above the resonant frequency (on your log/log scale)?

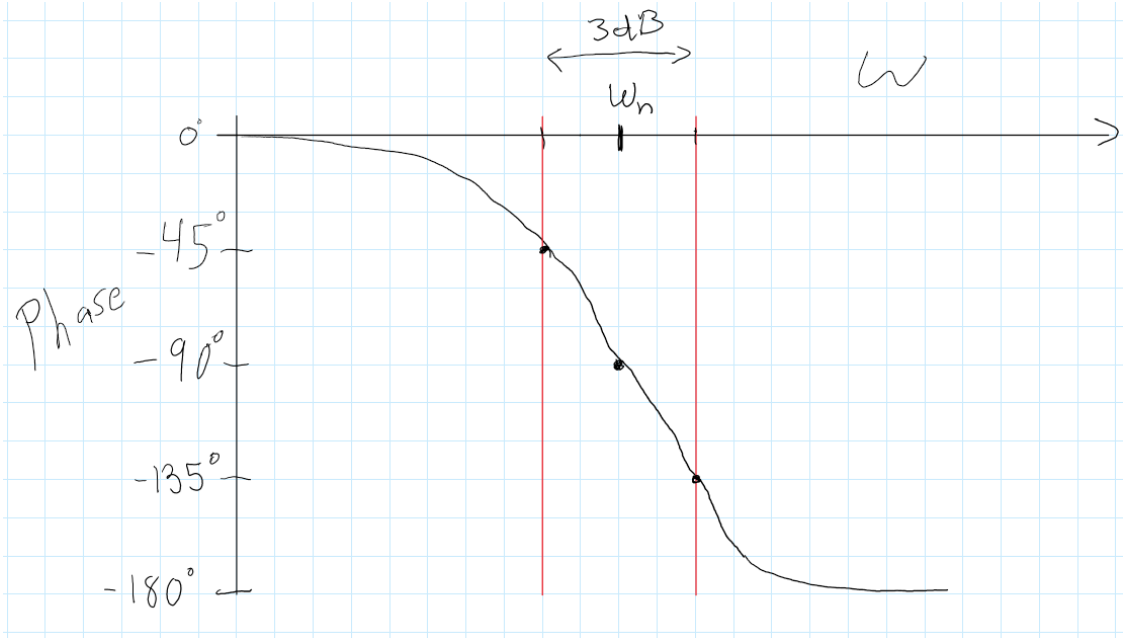
Well above resonance the curve goes as $\frac{F0}{mw^2}$, *so when we take a* log *we get a slope of* -2. *This is also* $\frac{-40dB}{dec}$

d. [2 pts]What is the 3dB bandwidth of the resonator?

wn/Q=1Hz

e. [4 pts total. 1 for straight line with negative slope, 1 each for frequencies as requested] On a linear/log plot, sketch the phase near resonance. Clearly indicate the frequencies at which the phase is -45, -90, and -135. Ponder whether this is related to problem 4c in any way.

Zoomed in plot of phase to show that the difference in frequencies when the phase is -45 and -135 is the 3dB bandwidth. The plot below would only get 2 pts, since it does not indicate the frequencies at which the phase is -45 and -135 (9,999.5 and 10,000.5 Hz)



7. [4 pts total. 1 pt each for formulas, 1 for right answer] In the SOIMUMPs process, you fabricate a beam of length L=1 mm with a gap-closing electrostatic actuator on the end. Assuming E=170GPa, the beam is 2um wide and the gap is 1um wide and 100um long,
   a. what is the pull-in voltage?
   b. If there are gap-stops at 0.5 microns, what is the pull-out voltage?

Assumed t=10um but using 25um is fine too.

$$k_y = \frac{Etw^3}{4L^3} = \frac{170GPa(10um)(2um)^3}{4(1000um)^3} = 0.0034\frac{N}{m}$$

$$\text{a.}\quad V_{PI} = \sqrt{\frac{8kg_0{}^3}{27\epsilon A}} = \sqrt{\frac{8(0.0034)(1um)^3}{27(8.854*10^{-12}\frac{F}{m})(100um*40um)}} = 0.17V$$

$$\text{b. } V_{PO} = \sqrt{\frac{2kg_d{}^2(g_o-g_d)}{\epsilon A}} = \sqrt{\frac{2(0.014\frac{N}{m})(0.5um)^2(1um-0.5um)}{(8.854*10^{-12}\frac{F}{m})(100um*40um)}} = 0.15V$$

8. [3 pts total. 1 pt for "yes", 1 pt for saying something about resonance. 1 pt if you said that you should apply the sinusoid at $\omega_n/2$] If you were to apply a sinusoidal voltage to this structure, could you get it to pull in at a lower voltage? Why/how? What frequency would you use if you were applying a pure sine wave with no DC bias?

Tricky! What happens if the frequency of the sinusoid was near resonance? We get a larger displacement! This larger displacement makes movable plate closer to the stationary plate, which means the gap is smaller.

$V_{PI}\ \alpha\ \sqrt{g_0{}^3}$ so if the gap decreases, then V_{PI} decreases.

To get a sinusoidal force at the resonant frequency you need to apply a pure sine wave at half that frequency, since it gets squared → frequency doubles

9. [7 pts] In Spencer et al. (reference [1]), an electrostatic gap-closing relay is made with a gate/body overlap area A_{ov}, and initial gap g_o, a displacement g_d (the amount that it moves before contact), and a spring constant K (see table 1 for specific values of the "current devices" reported in the paper).
   a. Checkout the pictures of the people on the last page – you may recognize some of them
   b. calculate the expected pull-in voltage based on the specs for "current devices"
   c. calculate the expected value for the pull-out voltage based on the same specs
   d. Compare your results from parts b and c to the measured results in Figure 11. Are their results close to what the theory says that should be? What do you think might explain any differences between theory and experiment?
   e. How fast is the mechanical response of these devices today, and how fast do they hope to make them in the "scaled model" future? (see Table 1)

   [1] Spencer, et al., "Demonstration of Integrated Micro-Electro-Mechanical Relay Circuits for VLSI Applications", IEEE JSSCV46N1, 2011.

a. Our Dean!

For b and c use equations from #7 in this assignment and values from Table1 in the linked paper.

$$A_{ov} = 450\ um^2$$
$$g_0 = 180nm$$
$$g_d = 90nm$$
$$k = 62.5\frac{N}{m}$$

b. [2pt, 1 formula, 1 answer] $V_{PI} = 5V$ (Not what's listed in Table1!)

c. [2pt, 1 formula, 1 answer] $V_{PO} = 4.75V$

d. [1 pt for any thoughtful answer] Pretty good approximation, especially for quick formula but not exact in part due to variations in device geometry, material properties, and parasitic capacitance.

e. [2 pts for the correct values] "Current": 34us but scaled is only 0.02us-0.08us! Super fast for mechanical devices, but not very fast compared to transistors (which switch in a few picoseconds these days)

10. [247] [10 pts for some discussion of each question] As the beam gets shorter in the problem above, the parallel plate approximation for the capacitor becomes an increasingly poor model for the actuator, and the linear spring becomes a poor model for the beam, because both ignore rotation and torque. How would you go about calculating the actual pull-in voltage for this structure? Would the pull-out voltage be easier to calculate (assuming gap stops on both sides)?

11. The pictures below are of the ADXL202, which is now obsolete but looks a lot like the newer parts that replaced it. Die photo (CMOS+MEMS), SEM of the MEMS device, detail SEM, and simulation of response to a vertical acceleration.
   a. Using the scale bar, estimate the mass of the proof mass, assuming a 3um thick polysilicon film.
   b. Using the sensor resonant frequency quoted on the datasheet, estimate the combined spring constant of all of the support springs.

http://www.analog.com/media/en/technical-documentation/obsolete-data-sheets/ADXL202_210.pdf

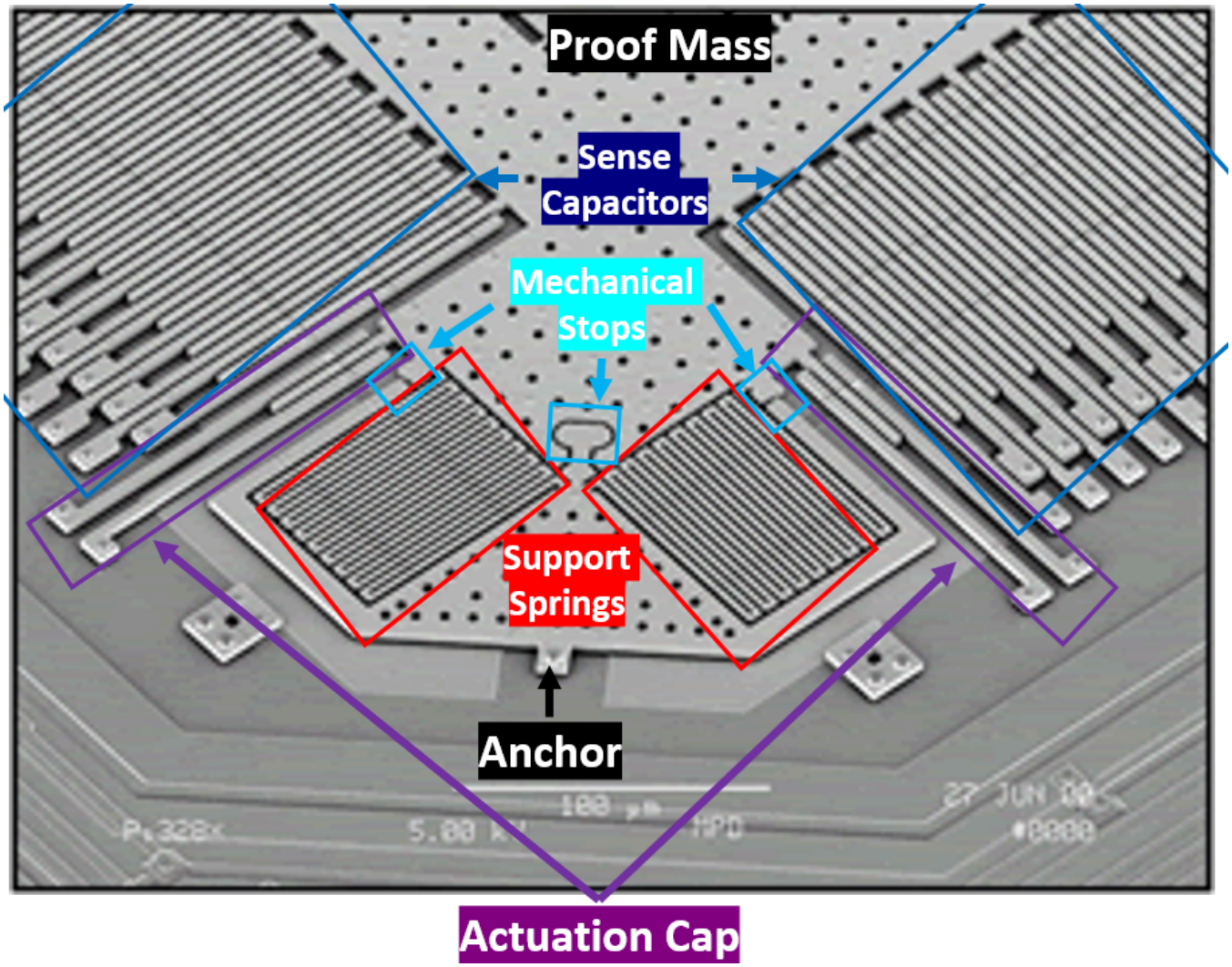
   c. In the detailed SEM, identify the proof mass, support springs, proof mass anchor, over-acceleration mechanical stops, differential sense capacitors (big), actuation capacitors (small).
   d. Assuming a 3um thick film with 1um gaps, estimate the voltage needed across the actuator gap to support the weight of the proof mass (at 1 gravity).

a. [2 pts] mass=$2300\frac{kg}{m^3} * 3um * (4 * 75um * 75um + 200um * 200um)$ =450p kg (450 pico-kilo grams)

b. [2 pts] k=$m * \omega_n{}^2$=450p kg* (2 pi 14kHz)^2 = $(2\ pi)2\ 0.09\frac{N}{m}$

c. [6 pts – 1 pt each]

c.

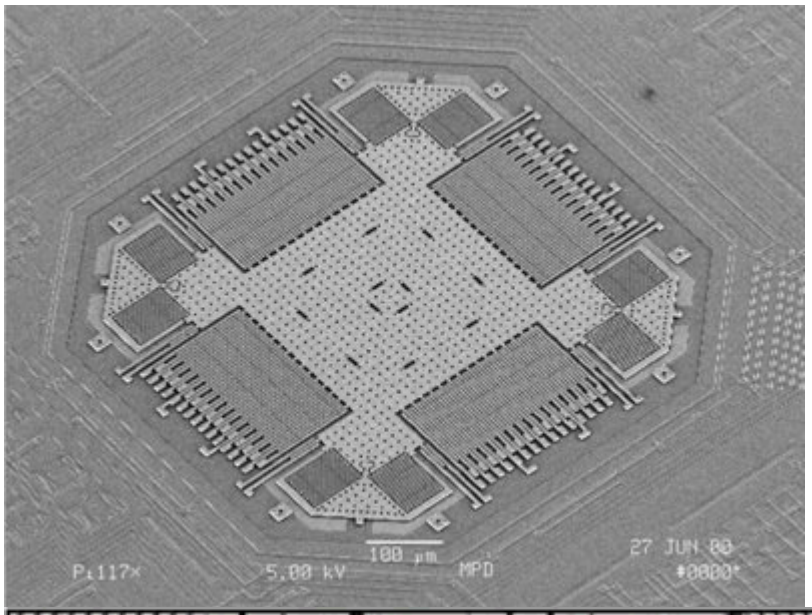


d. [3 pts. Basic setup, right equation, right answer]

F=mg=4*F_{cap} (There are four drive capacitors for each in plane direction)

$$F_{cap} = 0.5\epsilon_0 V^2 \frac{A}{g^2} = 0.5\left(8.854 * 10^{-12}\frac{F}{m}\right)V^2\frac{3um * 40um}{(1um)^2} = 450\text{p kg} * \frac{10\text{N}}{\text{m}^2}$$

$$V = \sqrt{\frac{450\text{p kg} * \frac{10\text{N}}{\text{m}^2} * (1um)^2}{0.5\left(8.854 * 10^{-12}\frac{F}{m}\right) * 3um * 40um}} = 3V$$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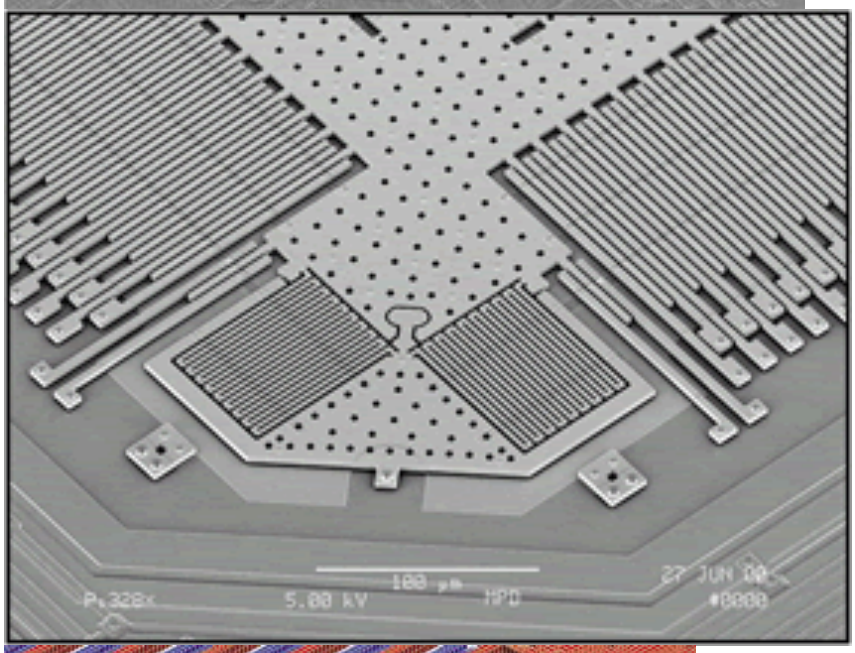
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