Foundations of Internet-enabled Democracy

Liquid Democracy—a More Rigorous Approach (part 1 of 2)

Orr Paradise

Seminar on the Foundations of Internet-enabled Democracy, December 2017
Organized by Prof. Ehud Shapiro and Dr. Nimrod Talmon

Outline

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- Representative democracy stands in contradiction with the ideal of a modern, Internet-enabled democracy
- Direct democracy is not without problems:
    - In large scales, not enough citizens are well informed
    - In online communities, 90-9-1 implies votes do not represent the general opinion

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    - Do we allow fractional delegations?
    - As we've seen, this is open to interpretation

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- We also assume that voters only delegate to friends
- And that friendships are symmetric
    - Our first of several optimistic assumptions

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- For now, assume that G is connected

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- Our model can deal both with elections and motion-passing votes
- Can we simulate a voting process with our model?
  - Not yet, many details are still unspecified
  - Most importantly, the question of dealing with *delegation cycles* (a cycle in the delegation graph)

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- The k winners are particicpants (voters) with the highest voting scores

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- Is r_v always finite?

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Proof.

If v participates in a cycle $c \in P(v)$, denote by c^m the repetition of the cycle $m \in \mathbb{N}$ times. Then $c^m \in P(v)$ for all $m \in \mathbb{N}$, and for all $m \neq k$ clearly $c^m \neq c^k$. We found an injection of $\mathbb{N}$ to $P(v)$.

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Also $|P(v)| \leq |V^*| = \aleph_0$
On the other hand, if $P(v)$ is infinite then it must hold a cycle, as there are at most

$$\underbrace{2^n}_{\text{subsets of } V} \cdot \underbrace{n!}_{\text{orderings on a subset}}$$

simple paths in D □

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  - We need insight on the growth of $\left| P(v) \cap V^{m+1} \right|$ with m...
- It can be shown that

$$r_v = \sum_{p \in P(v)} \alpha^{|p|} \leq c(v) \sum_{m=1}^{\infty} \alpha^m \leq \frac{c(v)}{(1-\alpha)} < \infty$$

  where $c(v)$ is the number of paths leading to $[v]_\sim$ in the condensation graph of D (which is a DAG, hence $c(v)$ is finite)

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  - This yields some insight about its behavior
- Instead, let's prove some concrete results
  - Specfically, fix G, D, and study its behavior with large/small α

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- An interesting observation (perhaps), but does our model allow us to prove it rigorously?

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$$r_t = \frac{(1-\alpha)}{n} Q(\alpha)$$

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Since t delegates to v we have

$$r_v \geq \frac{(1-\alpha)}{n}\left(\underbrace{\alpha}_{t\text{'s delegation}} + \underbrace{\alpha Q(\alpha)}_{\text{votes delegated from } t}\right) = \frac{(1-\alpha)}{n} \cdot \alpha(1 + Q(\alpha))$$

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So for large enough α the relation holds$\blacksquare$

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- Large enough damping makes only direct votes count
  - When α is small enough the system resembles a direct democracy, since delegated votes are negligible.

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- But is liquid democracy necessarily better than direct democracy?

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- Specifically, assume there are two possible outcomes, one correct and one incorrect
- And each voter votes for the correct answer with some fixed probability
- The social network is now the labled graph $G = (V, E, p)$
  - $p : V \to [0, 1]$, each $v \in V$ votes correctly w.p p_v, incorrectly w.p $(1 - p_v)$

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    - Let $A_G(u) = \{v \in V | u \text{ approves of } v\}$
- Since the inequality is strict there are no approval cycles, hence no delegation cycles

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Outline

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  - Only for the specific network G...
- We now finally define what "better" means

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- A mechanism M satisfies the *Do No Harm (DNH)* property if for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for all graphs G_n on $n \geq N$ vertices, $\mathrm{gain}(M, G_n) \geq -\varepsilon$

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- Again, not interested in small instances
- But eventually (for large enough instances) there should always be instances on which the delegative mechanism is better than the direct one

DNH vs. PG

- *DNH*

$$\forall \varepsilon > 0 \; \exists N \; \forall n \geq N \; \forall G_n \; \mathrm{gain}\left(M, G_n\right) \geq -\varepsilon$$

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- Can any mechanism satisfy both *DNH* and *PG*?

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- Thank you!

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