CS174 Sp2001 **Homework 4** **due: Feb 15, 2001**

This homework is due by 5pm on Thursday Feb 15th. Please hand it to the CS174 homework box on the second floor of Soda Hall.

1. Show that if all the males have the same preference ordering, then the proposal algorithm runs for $\Omega(n^2)$ steps. Your argument should work for any set of preferences for the females. Hint: in a final stable marriage, consider the spouse of the female who is k^{th} on the males preference list. How many proposals did he make?

2. The notes for lecture 7 contains an alternative form of the definition of expected value that works for random variables X with range $\{0, \ldots, n\}$:

$$\mathrm{E}[X] = \sum_{k=1}^{n} \Pr[X \geq k]$$

   Prove this from the usual definition of expected value.

3. When we randomly distribute m balls into n bins, the outcome can be represented as an "m-permutation" of $1, \ldots, n$ which allows repetition. That is, as a sequence $(b_1, \ldots, b_m)$ where $b_i \in \{1, \ldots, n\}$ is the number of the bin that ball i goes into. What is the probability that a random n-permutation of $1, \ldots, n$ is also a true permutation? Use Stirling's approximation to simplify.

4. If m balls are randomly dropped into n bins, compute:

   (a) For $n = 2$, the probability that bin 1 contains m_1 balls and bin 2 contains m_2 balls such that $m_1 + m_2 = m$. Your answer will depend on m_1 and m_2.

   (b) For $n = 3$, the probability that bin i contains m_i balls, and such that $m_1 + m_2 + m_3 = m$. Hint: "glue" bins 2 and 3 together first and use part (a), then separate them.

   (c) Using induction, derive the general probability that the distribution of m balls into n bins gives $m_1, \ldots, m_n$ balls in bin 1 through bin n, where $m_1 + \cdots + m_n = m$.