**CS174** **Midterm 1** **Spring 2000**
**J. Canny** **Feb 29**

This is a closed-book exam with 4 questions. You have 80 minutes. All questions are worth equal points, so be sure to budget 20 minutes per question. You are allowed to use the formula sheet that will be handed out with the exam. No other notes are allowed. No Calculators please. Write all your answers in this booklet. Good luck!

NAME_____

SID Number_____

1. Suppose two fair 6-sided dice are tossed. Let $X$ be the number on one of the dice, and let $Y$ be the *sum* of the two dice.

   (a) What is $\mathrm{E}[X + Y]$?

   (b) What is $\mathrm{Var}[X]$?

   (c) What is $\mathrm{Cov}[X, Y]$? Hint: the answer to part (b) is useful here.

2. Suppose we begin with a graph $G$ with $n$ vertices and no edges, and then add edges $\{i, j\}$ one at a time by choosing $i, j$ at random from $\{1, \ldots, n\}$.

   (a) What is the expected number of edges $m$ added before some vertex is hit twice, i.e. before some vertex has two edges touching it? Give your answer as $\mathrm{E}[m] \approx f(n)$ where this means that $\lim_{n\to\infty} \mathrm{E}[m]/f(n) = 1$.

   (b) What is the expected number of edges $m$ before every vertex is hit at least once? Again give your answer as $\mathrm{E}[m] \approx f(n)$.

   (c) Suppose the graph $G$ has $m$ edges, $n$ vertices, and exactly two connected pieces. Let $p$ be the probability that the next random edge connects the two pieces. Give upper and lower bounds for $p$.

3. Give a name to the probability distributions for random variable $X$, and compute the mean and variance in each case:

   (a) Suppose 5 cards are drawn from a deck of 52 cards, one-at-a-time, with replacement. Let $X$ be the number of aces.

   (b) Suppose $n$ people seat themselves randomly around a meeting table with $n$ seats before lunch. Then after lunch, they sit down again in new random positions. Let $X$ be the number of people who sit in the same seat. Assume $n$ is large.

   (c) We draw cards again from a deck of 52 cards, this time in turns. On each turn, we pick *two* cards without replacement. Then we replace them, reshuffle and repeat. Let $X$ be the number of *turns* up to and including the first time we draw a pair.

4. Suppose we have a random variable $X$ such that $\mathrm{E}[X] = 2n$ and $\mathrm{Var}[X] = 4n$. Find a function $f(n)$ such that

$$\Pr[X > \mathrm{E}[X] + f(n)] < \frac{1}{\sqrt{n}}$$