CS174 **Sample Midterm 1** **Spring 99**
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This is a closed-book exam with X questions. You are allowed to use the formula sheet that will be handed out with the exam. No other notes are allowed. Calculators are OK. Write all your answers in this booklet.

1. Let a biased coin have $Pr[Heads] = p$, where p is not necessarily 0.5. This coin can still be used to simulate fair coin tosses by tossing it twice, where if the actual sequence of tosses is HT, the output is "H", while if the actual sequence of tosses is TH, the output is "T". If the actual tosses are both the same, repeat the process. Let an "event" be a pair of tosses, and let the random variable X count the number of events until "H" or "T" is output, including the last, so $X \geq 1$.

   (a) What kind of distribution does X have (give a name)?

   (b) What is $E[X]$ in terms of p?

   (c) How could you use the same coin to simulate a fair 3-sided dice, i.e. to output 1, 2, or 3 with equal probability?

2. Let X be a random variable with $E[X] = 3$, $Var[X] = 4$, and let Y be a random variable with $E[Y] = 5$.

   (a) What is $E[3X]$?

   (b) What is $E[X + Y]$?

   (c) What is $E[XY]$?

   (d) What is $E[X^2]$?

3. Pizza Palace has implemented a new electronic dispatching system but are having trouble with it. Because of that, m pizzas are dispatched independently and uniformly at random to n customers.

   (a) How large should m be (in terms of n) to be confident that everyone gets a pizza?

   (b) How large should m be to be confident that no-one gets more than one pizza?

   (c) Assume $m = n$, that everyone ordered a pizza, and that one pizza gets delivered to each person but in a random permutation of the orders. What is the probability that no-one got the pizza that they ordered?

4. Let a fair coin be tossed 16 times. Let X be the number of heads, then $E[X] = 8$ and $Var[X] = 4$

   (a) Give a Markov bound for $Pr[X \geq 12]$.

   (b) Give a Chebyshev bound for $Pr[X \geq 12]$.

   (c) Give a Chernoff bound for $Pr[X \geq 12]$.