EE227 BT 9/15/16

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Quiz #1

NAME:
SID:

The quiz lasts 1 hour. Notes are allowed.

1. *Least-squares.*

   (a) Let $A \in \mathbf{R}^{m \times n}$ and $y \in \mathbf{R}^m$ be given. Express the *optimal set* of the least-squares problem

   $$\min_x \|Ax - y\|_2$$

   in terms of the SVD of A, given as

   $$A = [U_r, U_{n-r}] \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} [V_r, V_{m-r}]^T.$$

   Here, $U = [U_r, U_{n-r}]$ and $V = [V_r, V_{n-r}]$ are unitary, and decomposed into blocks consistent with the rank r of A, and $\Sigma \succ 0$ is the diagonal matrix containing the singular values.

   (b) Explain why we cannot do the same derivation when an l_1-norm penalty is added to the objective. Be as precise as you can.

2. *Optimal loss as a function of the regularization parameter.* For $A \in \mathbf{R}^{m \times n}$ and $y \in \mathbf{R}^m$ given, we consider the function $p^* : \mathbf{R}_+ \to \mathbf{R}_+$, with values

$$p^*(\lambda) := \min_x \ \|Ax - y\|_2 + \lambda \|x\|_1.$$

(a) Express the function value for a given $\lambda \geq 0$, as the optimal value of an SOCP in standard form. How many variables and constraints do you end up with?

(b) Is the function p^* convex, concave, both, neither? Explain, with a proof or a counterexample.