EE227BT Discussion Section #5

Exercise 1 (Almost Optimal Prefix Free Codes) Let $p \in \Delta_n$ be a probability distribution. Our goal is to approximately solve the integer program

$$\min_{l \in \mathbb{N}^n} p^\top l \ : \ \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

In order to so we relax the integrability constraints to

$$\min_{l \in \mathbb{R}_+^n} p^\top l \ : \ \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

1. Find the optimal solution of the convex relaxation of the integer program.
2. Suggest a rounding scheme for the optimal solution of the convex relaxation and give an approximation guarantee.

Exercise 2 (Dual Of Square-Root LASSO) Let $A = \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix} \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, \lambda \in \mathbb{R}_{++}$. Consider the square-root LASSO problem

$$p^* = \min_{x \in \mathbb{R}^n} \|Ax - b\|_2 + \lambda \|x\|_1$$

1. Derive the dual of the square-root LASSO problem.
2. Show that if $\|a_k\|_2 \leq \lambda$, then we can set $x_k^* = 0$ at optimum.