EE227BT Discussion Section #3

Exercise 1 (Linear Term And Translation of Argument) Let $f : \mathbb{R}^n \to \mathbb{R}, b, c \in \mathbb{R}^n$ and define $g : \mathbb{R}^n \to \mathbb{R}$ as $g(x) = c^\top x + f(x+b)$. Show that

$$g^*(y) = -(y-c)^\top b + f^*(y-c),\ \mathrm{dom}(g^*) = c + \mathrm{dom}(f^*)$$

Exercise 2 (Conjugate of Log Moment Generating Function) Let X be a random n-dimensional vector. The *moment generating function* associated with X is defined as:

$$M_X(\lambda) \doteq \mathbb{E}[e^{\lambda^\top X}]$$

The *logarithmic moment generating function* associated with X is defined as:

$$\Lambda_X(\lambda) \doteq \log M_X(\lambda)$$

For each of the following random variables find the conjugate of Λ_X.

1. X is supported on the standard basis $\{e_1, \dots, e_n\}$ of $\mathbb{R}^n$ and $\mathbb{P}(X = e_i) = p_i$.
2. $X \sim N(\mu, \Sigma)$, where $\mu \in \mathbb{R}^n$ is the mean vector, and $\Sigma \in \mathbb{S}^n_{++}$ is the covariance matrix.