

## CS 70 SPRING 2008 — DISCUSSION #10

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### 1. GREEN AND BLUE BALLS

**Exercise 1.** A box has 10 balls, 6 of which are green and 4 of which are blue. Three balls are removed from the box, their color unnoted. Find the probability that a fourth ball removed from the box is blue. Assume that the 10 balls are equally likely to be drawn from the box.

**Exercise 2.** With the same box composition as in the previous exercise, find the probability that all three of the removed balls will be green if it is known that at least one of the removed balls is green.

### 2. RANDOM SAMPLING

**Exercise 3.** Professor Wagner would like to poll  $m$  students from a class of  $n$  students. In his first experience, Professor Wagner picks a student from class uniformly at random (i.e. every student has the same probability of being picked), writes down his/her name, and marks the student as the first one chosen. He then randomly and independently picks a student from the class WITH the possibility of picking the same student as before, writes down his/her name, and marks the student as the second one chosen. He repeats this process  $m$  times. We denote this sample space by  $\Omega$ . Describe what the sample points are and the probability of each sample point.

**Exercise 4.** Let  $A$  be the event that Alice is picked,  $B$  be the event that Bob is picked,  $C$  be the event that Cathy is picked. What is  $Pr_{\Omega}[A]$ ? What is  $Pr_{\Omega}[A \cap B]$ ? Are  $A$  and  $B$  independent in  $\Omega$ ? (Note: Assume that exactly one student in the class is named Alice, exactly one is named Bob, and exactly one is named Cathy. These names are meant to be fictional.)

**Exercise 5.** Let  $A_1$  be the event that the first pick was Alice. Let  $C_3$  be the event that the third pick is Cathy. Let  $\Omega$  be the sample space. Are  $A_1$  and  $C_3$  disjoint? are they independent? Are  $\neg C, C_3$  independent? disjoint?

**Exercise 6.** Let  $A_2$  be the event that the second pick was Alice. Let  $\Omega$  be the sample space. Are  $A_1, A_2$  disjoint? Independent?

**Exercise 7.** What is the probability in  $\Omega$  that the  $m$  students Professor Wagner picked has no repetitions?

**Exercise 8.** Let  $E$  be the event that Professor Wagner picks Alice only once. What is the probability in  $\Omega$  that Professor Wagner picks Alice only once? What is  $Pr_{\Omega}[E | \text{no repetition}]$ ?

**Exercise 9.** About how many students does Professor Wagner have to pick so that the probability in  $\Omega$  of getting a repetition becomes approximately  $1/2$ ? Where have you seen this problem before? (Describe how you would get the answer. You do not need to give the details of approximations used.)