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Quality Factor Control for Micromechanical Resonators

Clark T.-C. Nguyen and Roger T. Howe

Berkeley Sensor and Actuator Center
Department of Electrical Engineering and Computer Sciences
and the Electronics Research Laboratory
University of California at Berkeley
Berkeley, California 94720

ABSTRACT

The implementation of very high Q microelectromechanical filters, constructed of spring-coupled or parallel resonators, requires strict control over the quality factor of the constituent resonators. This report details electrostatic feedback techniques which allow precise control of the quality factor of a micromechanical resonator device, independent of the ambient operating pressure of the micromechanical system. Theoretical formulas governing Q-control are derived and experimentally verified.

I. INTRODUCTION

Micromachined mechanical resonators [1] have recently emerged as integrated electromechanical devices with frequency selectivity superior to integrated RC active filtering techniques based upon traditional electron devices. Using such integrated micromechanical resonators—which have quality factors (Q's) in the tens of thousands—microelectromechanical filters with selectivity comparable to macroscopic mechanical and crystal filters may potentially be fabricated on-chip. Prototypes of parallel micromechanical filters, which operate through addition of properly phased output currents from distinct resonators, and series filters, consisting of resonators coupled by compliant mechanical springs, have been reported [2].

Since the passband shape of these filter designs depends strongly upon the Q of the constituent resonators, a precise technique for controlling resonator Q is required to optimize the filter passband. Such a Q-control technique would be most convenient and effective if the Q was controllable through a single voltage or an element value, e.g. a resistor, and if the controlled value of Q were independent of the original Q.

This paper demonstrates, for the first time, electrostatic feedback techniques which allow precise control of the quality factor of a micromechanical resonator, independent of the ambient operating pressure of the micromechanical system. The basic Q-control architecture is summarized in Fig. 1. Here, the motional current output from a dc-biased oscillating three-port micromechanical resonator is electronically sensed and converted to a voltage, and then fed back to another electrode of the microresonator. The microresonator sums the input and negative feedback signals, closing the loop and reducing its own original Q. Alternatively, positive feedback can be applied to increase the original device Q. The Q is effectively controlled by the gain of the transresistance amplifier, which can be made voltage or resistor controllable.

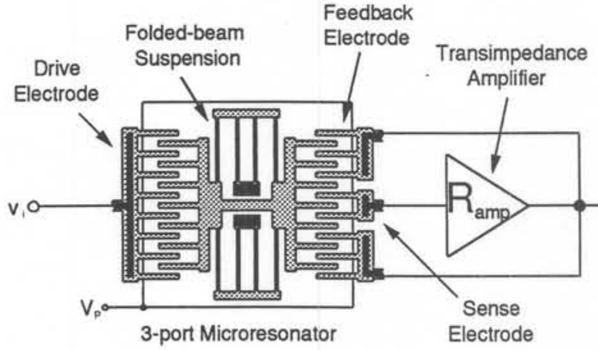


Fig 1. Schematic showing the implementation of Q-control for a multi-port electrostatic-comb driven micromechanical resonator anchored at the darkly shaded regions. R_{amp} represents a transresistance amplifier which senses microresonator motional current. The microresonator serves as both the high-Q element and the summer in the closed-loop system.

Section II establishes the theoretical foundation for the proposed Q-control method. Design formulas for practical implementation are then developed in Section III, leading to verification that the controlled Q is independent of ambient pressure in Section IV. Experimental demonstration of Q-control is presented in Section V, followed by concluding remarks concerning further applications for Q-control.

II. Q-CONTROL VIA ELECTROSTATIC FEEDBACK

The equivalent system block diagram for the schematic of Fig. 1 is shown in Fig. 2, where $Y_{i\cdot o}(j\omega)$ and $Y_{fb\cdot o}(j\omega)$ correspond to the μresonator input port-to-output and feedback port-to-output transfer functions, respectively. Using Fig. 2, and modelling the resonator n port to m port transfer functions

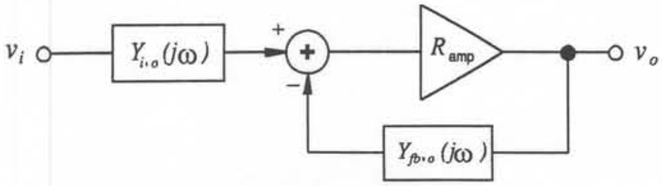


Fig 2. System block diagram for the circuit of Fig. 1.

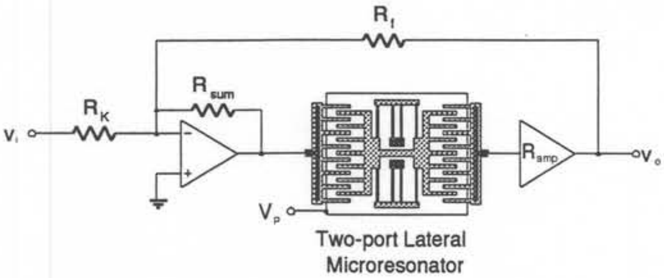


Fig 3. Q-control schematic for a two-port microresonator. Here, two amplifiers are utilized, one for transresistance amplification and one for input summing.

with the form

$$Y_{m\cdot n}(j\omega)=\frac{1}{R_{xm\cdot n}}\frac{1}{1+2jQ(\Delta\omega/\omega_o)}, \tag{1}$$

where $R_{xm\cdot n}$ is the equivalent series resistance of the resonator from port m to port n, and ω_o is the natural resonance frequency; direct analysis yields

$$\frac{v_o}{v_i}(j\omega)=\frac{(R_{amp}/R_{xi\cdot o})}{1+(R_{amp}/R_{xfb\cdot o})}\frac{1}{1+2jQ'(\Delta\omega/\omega_o)} \tag{2}$$

where

$$Q'=\frac{Q}{1+(R_{amp}/R_{xfb\cdot o})} \tag{3}$$

is the controlled value of quality factor. For large loop gain, the gain of Eq. (2) reduces to $(R_{xfb\cdot o}/R_{xi\cdot o})$, which, as will be seen in the following section, is determined by the number of input and feedback fingers, and stays constant as Q is varied. The Q can be changed by adjusting R_{amp}.

Although the above discussion of Q-control has utilized resonators with more than two ports, electronic control of Q can also be achieved for two-port resonators using additional electronics. A schematic of the Q-control architecture for two-port resonators is shown in Fig. 3. Here, a summing amplifier is introduced to sum the input and feedback signals (which, in Fig. 1, are summed by the multi-port resonator).

This discussion of Q-control has so far concentrated on the lowering of Q through the application of negative feedback. By using positive feedback, however, the Q of a resonator can be raised. Positive feedback implementations of Q-control can be realized by merely changing R_{amp} from positive to negative in Figs. 1 and 3, or more conveniently, by interchanging finger connections, as in Fig. 4. The equation for controlled Q under positive feedback becomes

$$Q'=\frac{Q}{1-(R_{amp}/R_{xfb\cdot o})}. \tag{4}$$

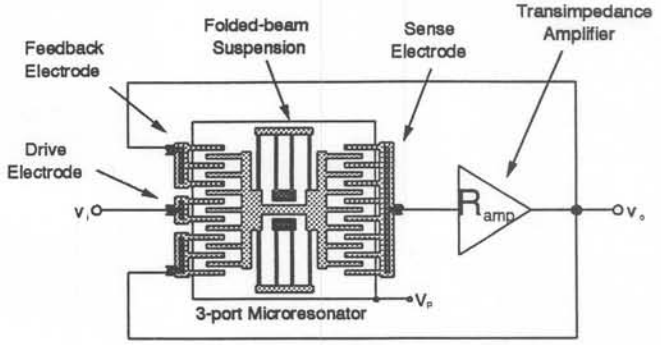


Fig 4. Circuit for raising the Q of a three-port μresonator.

III. Q-CONTROL DESIGN: EQUIVALENT PORT-TO-PORT RESISTANCE

To design for a specific Q and voltage gain $\frac{v_o}{v_i}$ for the circuit of Fig. 1, we require the equivalent input-to-output and feedback-to-output series resistances, $R_{xi\cdot o}$ and $R_{xfb\cdot o}$, of the resonator. To this end, we introduce the equivalent circuit for a three-port micromechanical resonator in Fig. 5. This circuit was obtained through straightforward extension of a previously reported circuit for two-port resonators [3, 4].

We first determine the input-to-output resistance, given by (refer to Fig. 5)

$$R_{xi\cdot o}=\frac{v_i}{i_o}. \tag{5}$$

Driving the equivalent circuit of Fig. 5 at the input port i and grounding the other ports, we have (at resonance)

$$i_o=\phi_{oi}i_{xi} \text{ and } i_{xi}=\frac{v_i}{R_{xi}}. \tag{6}$$

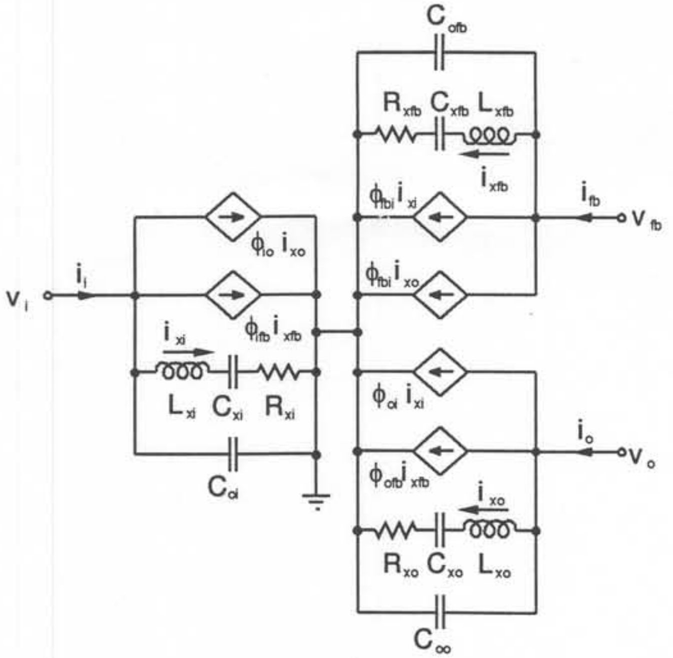
Applying (6) to (5), we have

$$R_{xi\cdot o}=\frac{R_{xi}}{\phi_{oi}}. \tag{7}$$

A similar analysis yields

$$R_{xfb\cdot o}=\frac{R_{xfb}}{\phi_{ofb}}. \tag{8}$$

To maximize the range of Q-control afforded by a given transresistance amplifier, the loop gain of the circuit, $A_l=(R_{amp}/R_{xfb\cdot o})$, should have a wide range. Thus, $R_{xfb\cdot o}$ should be minimized, which in turn requires that R_{xfb} be minimized and ϕ_{ofb} be maximized. Reduction in R_{xfb} can be achieved by increasing the number of feedback fingers, decreasing the gaps



$$R_{xn} = \frac{k_{sys}}{\omega_o Q V_P^2 (\partial C/\partial x)_n^2} \quad C_{xn} = \frac{V_P^2}{k_{sys}} (\partial C/\partial x)_n^2 \quad C_{on} = C_{dc\cdot overlapn}$$

$$L_{xn} = \frac{k_{sys}}{\omega_o^2 V_P^2 (\partial C/\partial x)_n^2} \quad \phi_{mn} = \frac{(\partial C/\partial x)_m}{(\partial C/\partial x)_n}$$

Fig 5. Equivalent circuit for a three-port μresonator, biased and excited as in Fig. 1, along with equations for the elements. In the equations, k_{sys} is the system spring constant and $(\frac{\partial C}{\partial x})_n$ is the change in capacitance per displacement at port n of the μresonator. Typical element values for high-Q (Q=50,000) operation of a μresonator with $f_o = 20\,\text{kHz}$ are $C_o = 15\,\text{fF}$, $C_x = 0.3\,\text{fF}$, $L_x = 100\,\text{kH}$, and $R_x = 500$ kΩ.

between these fingers, and increasing finger thickness. ϕ_{ofb} is increased with similar modifications to the output fingers.

The number of input and feedback fingers also determines the gain of the Q-control circuit. Using (7) and (8), the equation for gain at resonance is

$$\left.\frac{v_o}{v_i}\right|_{\omega=\omega_o} = \frac{R_{xfb\cdot o}}{R_{xi\cdot o}} = \frac{R_{xfb}}{\phi_{ofb}}\frac{\phi_{oi}}{R_{xi}} = \frac{(\partial C/\partial x)_i}{(\partial C/\partial x)_{fb}} = \frac{N_i}{N_{fb}}, \qquad (9)$$

where N_i and N_{fb} are the number of input and feedback fingers, respectively. The last equality assumes identical finger gaps and thicknesses for both ports. Thus, the gain is determined by resonator geometry and is independent of variables which determine the controlled Q.

IV. INDEPENDENCE OF CONTROLLED Q ON AMBIENT PRESSURE

Because of squeeze-film damping, Couette flow, or similar fluid-based damping mechanisms, the quality factor of a micromechanical resonator is strongly dependent upon the ambient pressure in which it operates. In addition, the intrinsic Q of a μresonator is a function of the anchor and is also temperature dependent [1, 5, 6] For lateral electrostatic-comb driven resonators, the Q ranges from under 50 in atmosphere to over 50,000 in 10 mTorr vacuum. Since the operation pressure for a micromechanical resonator is not easily controlled, a Q-control method independent of the original Q of the resonator is desirable.

The controlled Q in the proposed scheme can be shown to be independent of the original resonator Q (and thus, of ambient pressure) using the equivalent series resistance derived in the previous section. Inserting (8) in (3) and assuming sufficient loop gain (i.e. $(R_{amp}/R_{xfb\cdot o}) \gg 1$) yields

$$\begin{aligned} Q' = \frac{QR_{xfb\cdot o}}{R_{amp}} &= \frac{k_{sys}}{\omega_o V_P^2 \left(\frac{\partial C}{\partial x}\right)_{fb}^2 \phi_{ofb} R_{amp}} \\ &= \frac{[M_{eff} k_{sys}]^{\frac{1}{2}}}{V_P^2 \left(\frac{\partial C}{\partial x}\right)_{fb} \left(\frac{\partial C}{\partial x}\right)_o R_{amp}}, \end{aligned} \qquad (10)$$

where the equation for the first mode resonance frequency $\omega_o = \sqrt{k_{sys}/M_{eff}}$ has been inserted. In the above equations, M_{eff} is an effective mass of the resonator (includes support beams and folding truss). Note that the controlled quality factor Q' depends only upon the transresistance amplification R_{amp}, the bias voltage V_P, and μresonator geometry, and has no dependence on the original Q provided there is sufficient loop gain.

Note that the above discussion applies to the case of Q lowering, where negative feedback is used. For positive feedback, the controlled Q is directly dependent upon the original Q, and thus, is directly dependent upon ambient pressure.

V. EXPERIMENTAL RESULTS

Multi-port microresonators and both series and parallel filters have been fabricated to demonstrate the above concepts. Figure 6 presents a scanning electron micrograph (SEM) of a multi-port micromechanical resonator, equipped with Q-control ports. Experimental demonstration of Q-control via the above techniques has been accomplished using the circuit of Fig. 7 under a vacuum probe station (pressure = 10 mTorr). The results are presented in Fig. 8, where resonator transconductance spectra are plotted for different values of R_{amp}.

The dependence of controlled Q on ambient pressure was also investigated by varying the pressure in the vacuum probe station. Figure 9 presents experimental verification that the value of the controlled Q is invariant under changing ambient pressures, being dependent only on the Q-controlling feedback set by R_{amp}.

VI. CONCLUSIONS

Utilizing additional ports on a micromechanical resonator,

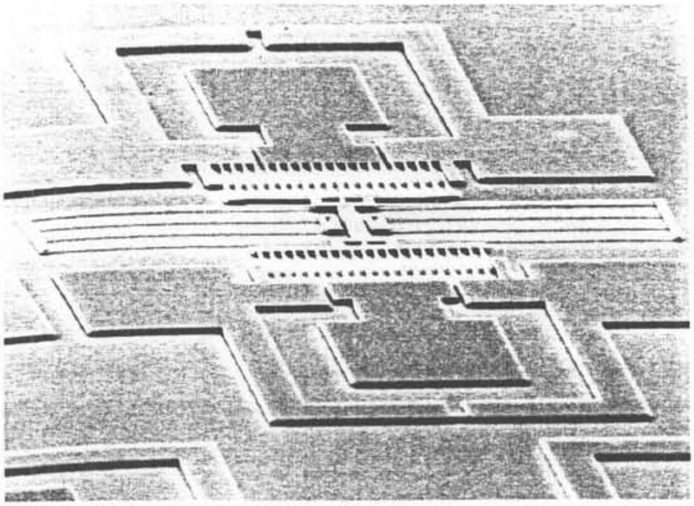


Fig 6. SEM of a 4-port electrostatic-comb driven lateral micromechanical resonator.

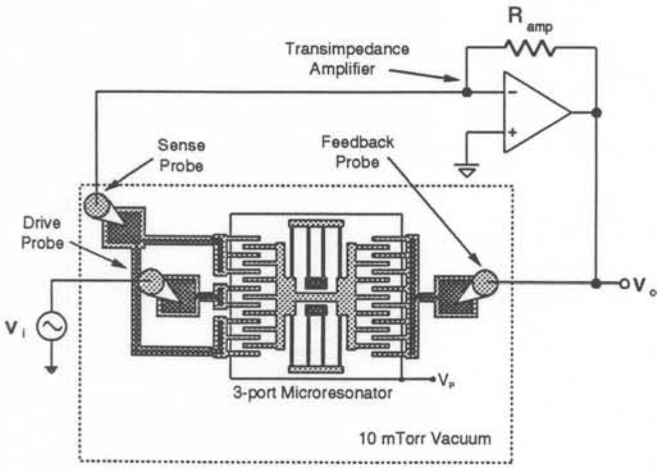


Fig 7. Schematic of the experimental set-up used to implement Q-control.

electrostatic feedback techniques which control the Q of the μresonator have been demonstrated. It has already been mentioned that such Q-control techniques can be applied to passband smoothing of micromechanical filters. However, the ability to control Q to the above precision has implications beyond this. For example, using the Q-control architecture of Fig. 3, changes in pressure can be quantified by measuring the feedback signal at the output of the summing amplifier (which adjusts to maintain constant Q under varying pressure). Such a Q-balanced resonant pressure sensor would have the advantage of automatic limiting of the resonator amplitude, and thus, would have a wide sensing range.

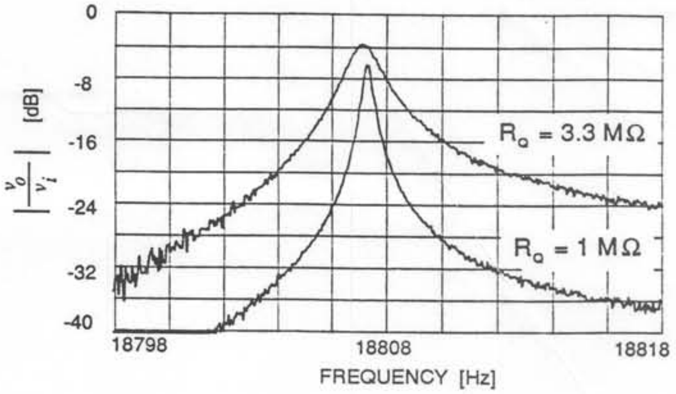


Fig 8. Experimental demonstration of Q-control using the set-up of Fig. 7. Each microresonator transconductance spectrum corresponds to a different value of R_{amp}. The measured values of Q are 53,000 for $R_{amp} = 1\ \mathrm{M}\Omega$ and 17,000 for $R_{amp} = 3.3\ \mathrm{M}\Omega$.

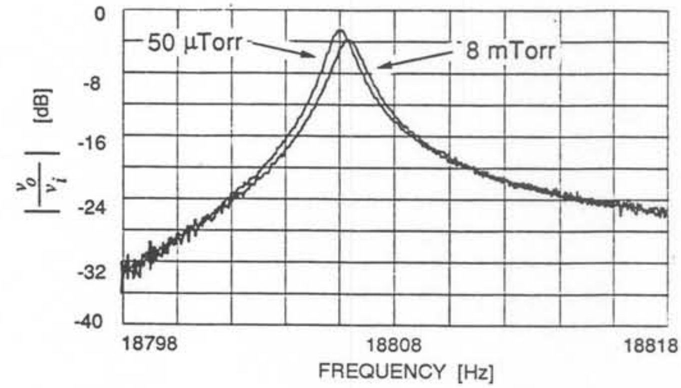


Fig 9. Transconductance spectrum for a micromechanical resonator subjected to Q-control with $R_Q = 3.3M\Omega$ and with varying ambient pressure. Without Q-control, the original Q at 8 mTorr is 53,000 and that at 50 μTorr is 84,000. With Q-control, the Q for both cases is 17,000.

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