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Impulsive synchronization of chaotic Lur'e systems: state feedback case [1]

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Abstract

In this paper we consider impulsive control of master-slave synchronization schemes that consist of identical Lur'e systems. Impulsive control laws are investigated which make use of linear full static state feedback. A sufficient condition for global asymptotic stability is presented which is characterized by a set of matrix inequalities. The method is illustrated on Chua's circuit.
Keywords. impulsive control, synchronization, Lur'e systems, matrix inequalities, Chua's circuit.

1 Introduction

In [18, 19, 21, 9] methods for synchronization of nonlinear systems have been proposed which make use of impulsive control laws. In this way the error system of the synchronization scheme is stabilized using small control impulses. These methods are offering a direct method for modulating digital information onto a chaotic carrier signal for spread spectrum applications [17] and has been applied to chaotic digital code-division multiple access (CDMA) systems in [20]. The method discussed in [18, 19, 21, 9] is based on a theory of impulsive differential equations described in [7]. At discrete time instants, jumps in the system's state are caused by a control input. Global asymptotic stability of the error system is proven by means of a Lyapunov function and is characterized by a set of conditions related to the time instants, the time intervals in between and a coupling condition between these.

However, so far the method has been applied ad hoc to the special cases of Chua's circuit [18, 19] and the Lorenz system [21]. In this paper we present a systematic design procedure for a class of nonlinear systems, namely Lur'e systems. A sufficient condition for synchronization with linear full static state feedback is presented which is expressed in terms of matrix inequalities [1]. The feedback matrix is designed then by solving an optimization problem. Matrix inequality conditions for synchronization of Lur'e systems with a continuous control signal have been proposed in [14, 15]. The impulsive control method is illustrated here on Chua's circuit [2, 3, 8]. Other examples of chaotic Lur'e systems are e.g. n-scroll attractors [12] and arrays with unidirectional or diffusive coupling between such cells leading e.g. to double-double scroll attractors [5] and n-double scroll hypercube attractors [13], which exhibit hyperchaotic behaviour.

This paper is organized as follows. In Section II we discuss the impulsive synchronization scheme. In Section III we derive the stability condition, expressed in terms of matrix inequalities. In Section IV the method is illustrated on Chua's circuit.

2 Synchronization scheme and error system

We consider the following master-slave synchronization scheme

$$\left\{ \begin{array}{llcl} \mathcal{M}: & \dot{x} & = & Ax + B\sigma(Cx) \\ \mathcal{S}: & \dot{z} & = & Az + B\sigma(Cz)\,, \; t \neq \tau_i \\ \mathcal{C}: & \Delta z & = & K(x - z)\,, \quad t = \tau_i \end{array} \right. \tag{1}$$

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which consists of master system $\mathcal{M}$, slave system $\mathcal{S}$ and controller $\mathcal{C}$. $\mathcal{M}$ and $\mathcal{C}$ are identical Lur'e system with state vectors $x, z \in \mathbb{R}^n$ and matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_h}$, $C \in \mathbb{R}^{n_h\times n}$. A Lur'e system is a linear dynamical system, feedback interconnected to a static nonlinearity $\sigma(.)$ that satisfies a sector condition [5, 16] (here it has been represented as a recurrent neural network with one hidden layer, activation function $\sigma(.)$ and n_h hidden units [11]). We assume that $\sigma(.) : \mathbb{R}^{n_h} \mapsto \mathbb{R}^{n_h}$ is a diagonal nonlinearity with $\sigma_i(.)$ belonging to sector $[0, k]$ for $i = 1, ..., n_h$.

For the impulsive control law $\mathcal{C}$, a set of discrete time instants τ_i is considered where $0 < \tau_1 < \tau_2 < ... < \tau_i < \tau_{i+1} < ...$ with $\tau_i \to \infty$ as $i \to \infty$ [7, 18, 19, 21]. At the time instants τ_i, jumps in the state variable z are imposed

$$\Delta z|_{t=\tau_i} = z(\tau_i^+) - z(\tau_i^-). \quad (2)$$

Given the synchronization scheme (1), the synchronization error is defined as $e = x - z$. One has the error system

$$\mathcal{E}: \quad \begin{cases} \dot{e} = Ae + B\,\eta(Ce; z), & t \neq \tau_i \\ \Delta e = -K\,(x - z), & t = \tau_i \end{cases} \quad (3)$$

where $\eta(Ce; z) = \sigma(Ce + Cz) - \sigma(Cz)$ and $\Delta e = \Delta x - \Delta z$ with $\Delta x = 0$ for the master system.

3 Stability, matrix inequalities and controller synthesis

In order to derive a sufficient condition for global asymptotic stability of the error system $\mathcal{E}$, we take the quadratic Lyapunov function

$$V(e) = e^T Pe, \; P = P^T > 0. \quad (4)$$

According to [7, 18, 19] it is sufficient then to prove that

$$\begin{aligned} &\dot{V} \leq \alpha V, && \alpha > 0, \quad t \neq \tau_i && (5.1)\\ &V(\zeta + \Delta\zeta) < \beta V, && \beta > 0, \quad t = \tau_i && (5.2)\\ &\|\zeta + \Delta\zeta\|_2 < \|\zeta\|_2, && \qquad\quad\; t = \tau_i && (5.3)\\ &\alpha\,(\tau_{i+1} - \tau_i) + \log\beta < 0 && && (5.4) \end{aligned}$$

are satisfied together. We will express the conditions (5.1)-(5.3) now as matrix inequalities. In the derivation we exploit the inequalities

$$\eta(Ce)^T \Lambda\,[\eta(Ce) - Ce] \leq 0, \; \forall e \in \mathbb{R}^n. \quad (6)$$

These are related to the sector condition on the nonlinearity $\eta(.)$, which is assumed to belong to sector $[0, 1]$.

By employing (6) in an application of the S-procedure [1] a matrix inequality is obtained by writing

$$\dot{V} - \alpha V - 2\eta(Ce)^T \Lambda\,[\eta(Ce) - Ce] \leq 0 \quad (7)$$

as a quadratic form $w^T Y w \leq 0$ in $w = [e; \eta]$. Imposing this for all w yields

$$Y = Y^T = \begin{bmatrix} A^T P + PA - \alpha P & PB + C^T\Lambda \\ B^T P + \Lambda C & -2\Lambda \end{bmatrix} \leq 0. \quad (8)$$

In order to express the other conditions (5.2) and (5.3) in terms of matrix inequalities, we write $e + \Delta e = (I - K)e$ such that

$$(I - K)^T P (I - K) < \beta P \quad (9)$$

for (5.2) and

$$(I - K)^T (I - K) < I \quad (10)$$

for (5.3).

The controller synthesis can be formulated then as the feasibility problem:

$$\begin{aligned} &\text{Find} \quad K, Q, \Lambda, \alpha, \beta \\ &\text{such that} \quad \begin{cases} Y \leq 0 \\ (I - K)^T P (I - K) < \beta P \\ (I - K)^T (I - K) < I \\ \alpha\,(\tau_{i+1} - \tau_i) + \log\beta < 0 \end{cases} \end{aligned} \quad (11)$$

with $P = Q^T Q$, which imposes that $P = P^T > 0$.

4 Example: Chua's circuit

We consider master-slave synchronization of two identical Chua's circuits. We take the following representation of Chua's circuit for the master system $\mathcal{M}$:

$$\begin{cases} \dot{x}_1 = a\,[x_2 - h(x_1)] \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -b\,x_2 \end{cases} \quad (12)$$

with nonlinear characteristic

$$h(x_1) = m_1 x_1 + \frac{1}{2}(m_0 - m_1)\,(|x_1 + c| - |x_1 - c|) \quad (13)$$

and parameters $a = 9$, $b = 14.286$, $m_0 = -1/7$, $m_1 = 2/7$ in order to obtain the double scroll attractor [2, 3, 8] (Fig.1). The nonlinearity $\phi(x_1) = \frac{1}{2}(|x_1+c| - |x_1-c|)$ (linear characteristic with saturation) belongs to sector $[0, 1]$. A Lur'e representation $\dot{x} = Ax + B\phi(Cx)$ of Chua's circuit is given by

$$A = \begin{bmatrix} -a\,m_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \; B = \begin{bmatrix} -a\,(m_0 - m_1) \\ 0 \\ 0 \end{bmatrix},$$
$$C = [1 \;\; 0 \;\; 0]. \quad (14)$$

A feedback controller K has been designed then by solving (11) with sequential quadratic programming

(SQP) (*constr* in Matlab), where the inequalities have been programmed as hard constraints. As starting point has been chosen: K random according to a Gaussian distribution with zero mean and standard deviation 1, $Q = I$, $\Lambda = 10I$, $\alpha = 1$, $\beta = 1$. We give the simulation results for a fixed time interval $\tau_{i+1} - \tau_i = 0.1$. The simulations have been done by means of a Runge-Kutta integration rule (*ode23* in Matlab) and are shown on Fig.2-3 for some randomly chosen initial state.

5 Conclusion

Impulsive control for master-slave synchronization of Lur'e systems with linear full static state feedback has been discussed. A sufficient condition for synchronization has been derived which is characterized by a set of matrix inequalities. This offers a systematic design procedure by solving a nonlinear optimization problem. The method is illustrated on Chua's circuit.

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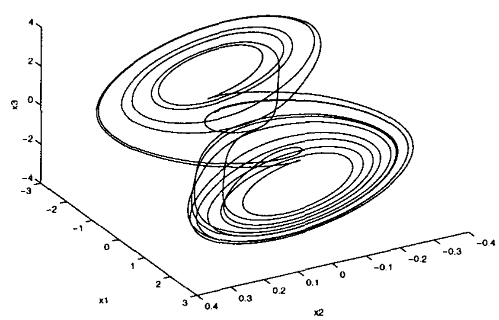

Figure 1: *Double scroll attractor as a master Lur'e system.*

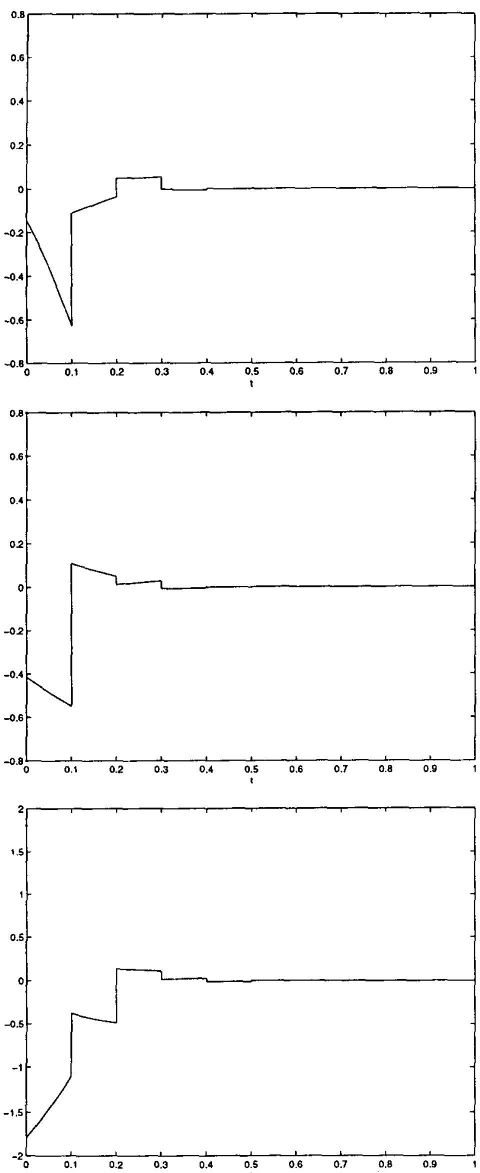

Figure 2: *Error signals $e = x - z$ between the master and the slave double scroll attractors. (Top) $e_1(t)$; (Middle) $e_2(t)$; (Bottom) $e_3(t)$.*

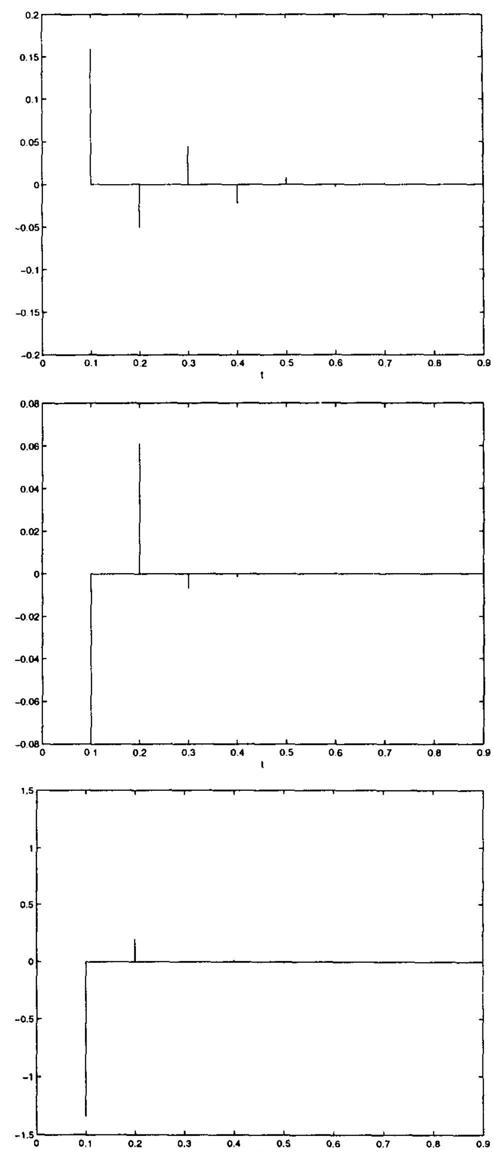

Figure 3: *Control signals $u = K(x-z)$ applied to the slave system. (Top) $u_1(t)$; (Middle) $u_2(t)$; (Bottom) $u_3(t)$.*