Generation of n-Double Scrolls ($n = 1, 2, 3, 4, \ldots$)

Johan A. K. Suykens and Joos Vandewalle

***Abstract*— Chua's double scroll is probably the best known and most extensively studied example of chaotic behaviour generated by electrical circuits. It is illustrated in this paper that by modifying the characteristic of the nonlinear resistor with additional break points even more "complicated" attractors can be obtained, called n-double scrolls (n=1,2,3,4,...). The new circuit can be seen as a generalization of Chua's circuit such that the 1-double scroll corresponds to the classical double scroll. The construction of the attractors was partially based on a combination of linearization around equilibrium points and an alternative method for studying nonlinear systems that we called a quasilinear approach. This method is heuristic and qualitative but may give additional global insight into the state space behaviour and may open new views towards the construction of attractors.**

***Index Terms*—Chua's circuit, n-double scroll, bifurcations.**

I. Introduction

One of the simplest systems known up to date exhibiting chaotic behaviour with a rich variety of bifurcation phenomena is Chua's circuit [1], [3], [6]. Depending on the parameter values of the system several portraits can be obtained between order and chaos ranging from sinks to a double scroll through period-doubling. In this paper it will be shown how Chua's circuit can be generalized to a new circuit by modifying the characteristic of the nonlinear resistor, leading to more complex attractors. Additional break points are introduced in the nonlinearity and the description is parameterized. The obtained attractors will be called n-double scrolls ($n = 1, 2, 3, 4, \ldots$) according to the specific parameterization. The well known double scroll will correspond then to the 1-double scroll in this framework. It is also illustrated how a transition between multiple sink portraits and the n-double scrolls can be achieved as a function of a bifurcation parameter of the circuit.

The viewpoint that we take here is rather experimental. Instead of a giving a rigorous mathematical analysis of all bifurcation phenomena, which would become a very complicated task for this generalized circuit, a local stability analysis at the equilibrium points and a quasilinear interpretation [9] of the state space behaviour is given. The approximate analysis is confirmed by computer simulations.

This paper is organized as follows. In Section II, the new circuit is described. In Section III, analysis of equilibrium points is done, and finally, in Section IV, a quasilinear analysis is given with respect to a bifurcation parameter of the system.

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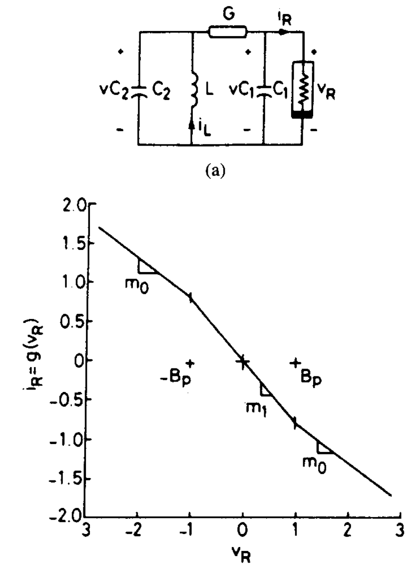

Fig. 1. Chua's circuit for generation of the double scroll attractor.

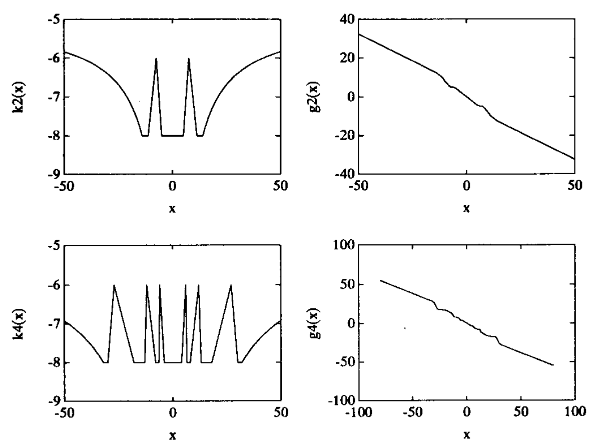

Fig. 2. Nonlinearities $k_2(x), k_4(x)$ and corresponding $g_2(x) = C_1 k_2(x)x$, $g_4(x) = C_1 k_4(x)x$.

II. Generalization of Chua's Circuit

The electrical circuit of Fig. 1 with circuit dynamics described as

$$\begin{cases} C_1 \frac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - g(v_{C_1}) \\ C_2 \frac{dv_{C_2}}{dt} = G(v_{C_1} - v_{C_2}) + i_L \\ L \frac{di_L}{dt} = -v_{C_2} \end{cases} \quad (1)$$

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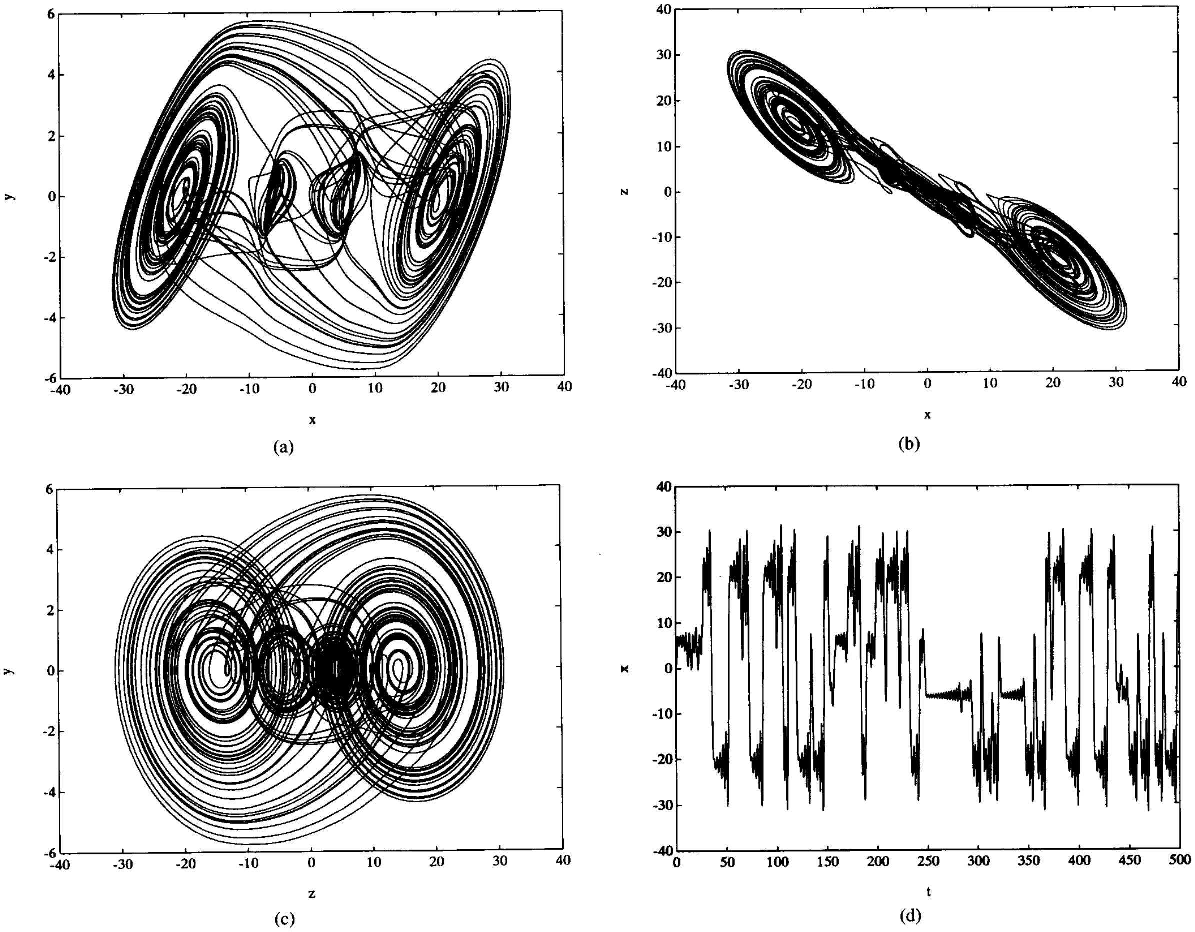

Fig. 3. 2-double scroll attractor: (a) $(x - y)$, (b) $(z - y)$, (c) $(x - z)$, (d) $x(t)$.

is known as Chua's circuit [1], [3], [6] where v_{C_1}, v_{C_2}, i_L denote respectively the voltage across C_1 and C_2 and the current through L and $g(v_{C_1})$ is the piecewise-linear function of Fig. 1 consisting of two breakpoints

$$g(v_{C_1}) = m_0 v_{C_1} + 0.5(m_1 - m_0)|v_{C_1} + B_p| + 0.5(m_0 - m_1)|v_{C_1} - B_p|. \quad (2)$$

By setting the parameters $1/C_1 = 9$, $1/C_2 = 1$, $1/L = 7$, $G = 0.7$, $m_0 = -0.5$, $m_1 = -0.8$, $B_p = 1$ chaotic behaviour is obtained (double scroll attractor). Equation (1) can be written in the form

$$\dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} \quad (3)$$

as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a - k(x) & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (4)$$

or $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ where $\boldsymbol{x}(t) = [x(t)y(t)z(t)]^t$, $\boldsymbol{f}: \mathbb{R}^3 \mapsto \mathbb{R}^3$, $x = v_{C_1}$, $y = v_{C_2}$, $z = i_L$, $a = G/C_1$, $b = G/C_2$, $c = 1/L$, $k(x) = (1/C_1)g(x)/x (x \neq 0)$ and $C_2 = 1$.

A new circuit will be defined now in terms of the state space description (4) where the nonlinear function $k(x)$ will be *parameterized* as $k_q(x) (q \in \mathbb{N}, q \geq 1)$. The nonlinear function $g(x)$ is adapted accordingly into $g_q(x) = C_1 k_q(x) x$. The circuit is described by

$$\begin{aligned} \dot{x} &= (-a - k_q(x))x + ay \\ \dot{y} &= bx - by + z \\ \dot{z} &= -cy \end{aligned} \quad (5)$$

with the following algorithmic description for $k_q(x)$

$$\begin{aligned}
&\text{if } 0 < |x| \leq \delta_1\text{: } k_q(x) = \alpha_1 \\
&\text{for } i = 2 \text{ to } q \\
&\quad j = 2 + 3(i - 2) \\
&\quad \text{if } \delta_{j-1} < |x| \leq \delta_j\text{: } k_q(x) = \alpha_2 \frac{|x| - \delta_{j-1}}{\delta_j - \delta_{j-1}} + \alpha_1 \\
&\quad \text{if } \delta_j < |x| \leq \delta_{j+1}\text{: } k_q(x) \\
&\qquad = -\alpha_2 \frac{|x| - \delta_j}{\delta_{j+1} - \delta_j} + \alpha_1 + \alpha_2 \\
&\quad \text{if } \delta_{j+1} < |x| \leq \delta_{j+2}\text{: } k_q(x) = \alpha_1 \\
&\text{end}
\end{aligned} \quad (6)$$

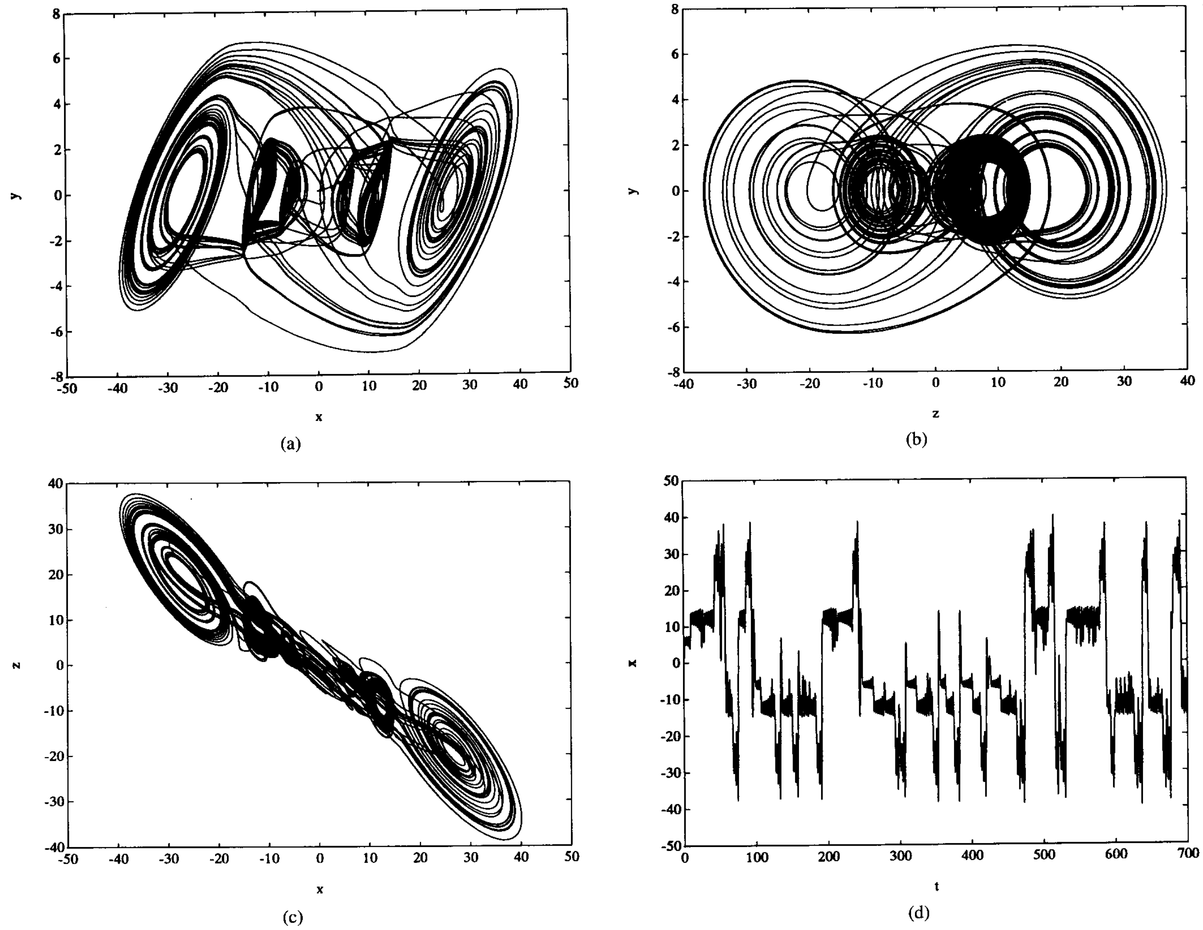

Fig. 4. 3-double scroll attractor: (a) $(x - y)$, (b) $(z - y)$ (c) $(x - z)$, (d) $x(t)$.

if $\delta_{4+3(q-2)} < |x|$:
$k_q(x) = (\alpha_3/|x|)(\beta_1|x| + \beta_2||x| + \delta_{4+3(q-2)}|$
$+ \cdots \beta_3||x| - \delta_{4+3(q-2)}|)$

and by definition

$$\Delta_q = [\delta_1 \delta_2 \cdots \delta_{4+3(q-2)}]^t. \quad (7)$$

The graphical description of some $k_q(x)$ and $g_q(x)$ is given in Fig. 2. It is easily verified that for $q = 1$ the circuit (5) and (6) corresponds to Chua's circuit (4) and $k_1(x) = k(x)$. Throughout the text we will only consider bifurcations with respect to the parameter a and take $b = 0.7$, $c = 7$, $\alpha_1 = -8a/7$, $\alpha_2 = 2a/7$, $\alpha_3 = a/0.7$, $\beta_1 = -0.5$, $\beta_2 = -0.15$, $\beta_3 = 0.15$.

III. Local Stability Analysis

3.1. Equilibrium Points and Jacobians

The equilibrium points of (5) and (6) are identified by setting $\dot{x} = \dot{y} = \dot{z} = 0$ which yields the conditions

$$\begin{cases} y = 0 \\ bx + z = 0 \\ x = 0 \text{ or } k_q(x) = -a \end{cases} \quad (8)$$

This results into $4q - 1$ equilibrium points: $\mathbf{eq}_0 = 0$ and $\mathbf{eq}^{\pm}_{j,j-1}$, $\mathbf{eq}^{\pm}_{j+1,j}$, $\mathbf{eq}^{\pm}_{4+3(q-2)}$ with $j = 2 + 3(i-2)$ and $i = 2, \cdots, q$ with $q \geq 1$. The latter appear in pairs ($\pm$ refers to the sign of the x component) with x components satisfying

$$\begin{aligned} &\text{case } \delta_{j-1} < |x| \leq \delta_j: \\ &\quad x_{\mathbf{eq}^{\pm}_{j,j-1}} = \pm 0.5(\delta_j + \delta_{j-1}) \\ &\text{case } \delta_j < |x| \leq \delta_{j+1}: \\ &\quad x_{\mathbf{eq}^{\pm}_{j+1,j}} = \pm 0.5(\delta_{j+1} + \delta_j) \\ &\text{case } \delta_{4+3(q-2)} < |x|: \\ &\quad \alpha_3(\beta_1|x_{\mathbf{eq}^{\pm}_{4+3(q-2)}}| + \beta_2||x_{\mathbf{eq}^{\pm}_{4+3(q-2)}}| + \delta_{4+3(q-2)}| \\ &\quad + \cdots \beta_3||x_{\mathbf{eq}^{\pm}_{4+3(q-2)}}| - \delta_{4+3(q-2)}|) \\ &\quad = -a|x_{\mathbf{eq}^{\pm}_{4+3(q-2)}}| \end{aligned} \quad (9)$$

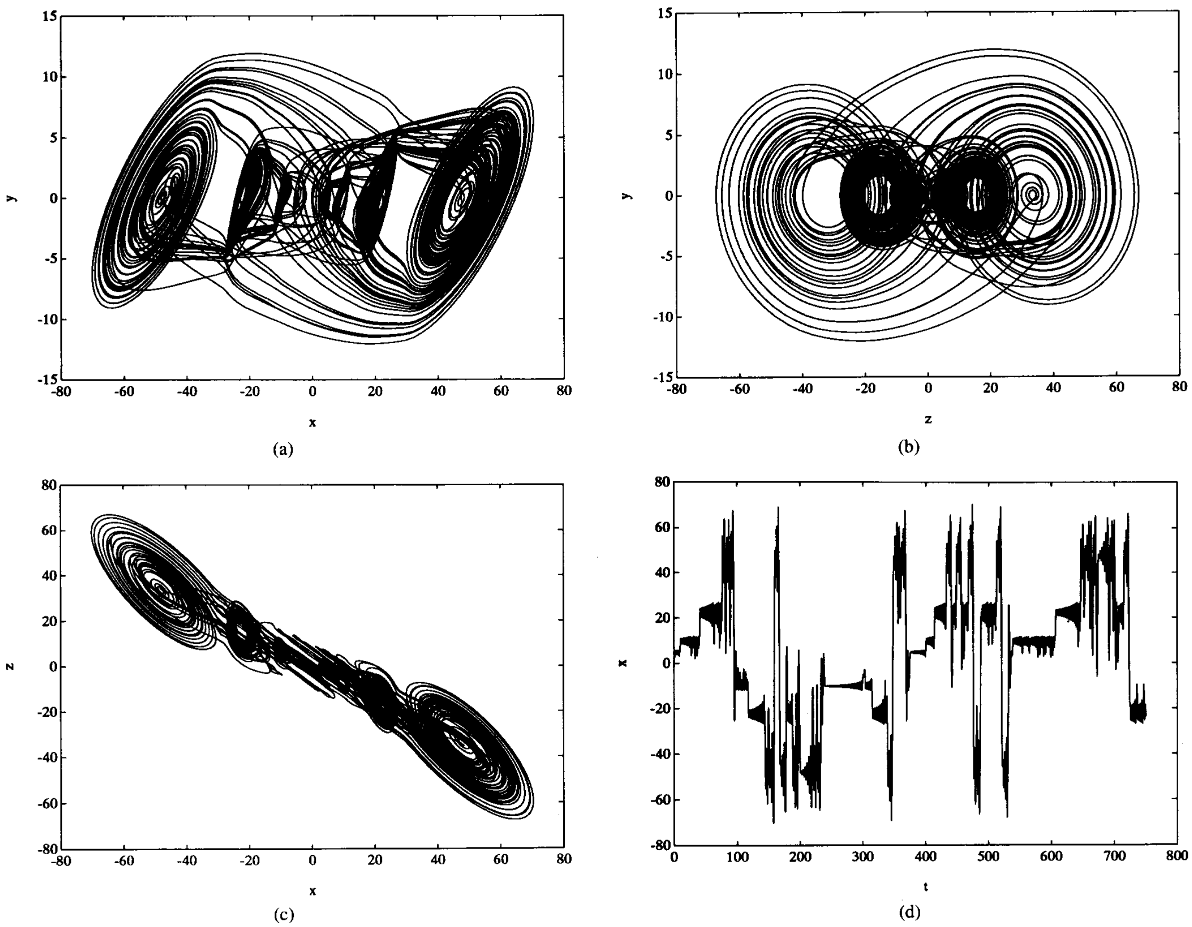

Fig. 5. 4-double scroll attractor: (a) $(x - y)$, (b) $(z - y)$, (c) $(x - z)$, (d) $x(t)$.

The Jacobian matrix for (5) and (6) is equal to

$$J(\boldsymbol{x}) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} -a - \frac{\partial(k_q(x)x)}{\partial x} & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix}. \quad (10)$$

Evaluated at the equilibrium points this gives

$$J(_{\text{eq}0}) = \begin{bmatrix} -a - \alpha_1 & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix},$$

$$J(_{\text{eq}j,j-1}^{\pm}) = \begin{bmatrix} -(a/7)\gamma_{j,j-1} & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix}$$

$$J(_{\text{eq}j+1,j}^{\pm}) = \begin{bmatrix} (a/7)\phi_{j+1,j} & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix},$$

$$J(_{\text{eq}4+3(q-2)}^{\pm}) = \begin{bmatrix} -a - \alpha_3(\beta_1 + \beta_2 + \beta_3) & a & 0 \\ b & -b & 1 \\ 0 & -c & 0 \end{bmatrix}$$

with $\gamma_{j,j-1} = (\delta_j + \delta_{j-1})/(\delta_j - \delta_{j-1})$ and $\phi_{j+1,j} = (\delta_{j+1} + \delta_j)/(\delta_{j+1} - \delta_j)$ for $j = 2 + 3(i-2)$ and $i = 2, \cdots, q$.

3.2. Generation of n-Double Scrolls

In order to generate the n-double scroll attractors ($n = 1, 2, 3, 4, \ldots$) the parameter a is fixed at $a = 7$. The attractors are obtained by setting $\gamma_{j,j-1} = 5$ (for $j = 2 + 3(i-2)$, $i = 2, \cdots, q$). Some examples are listed below

$$\begin{aligned} \text{case } q = 2: &\quad \Delta_2 = [5 \;\; 7.5 \;\; 11.25 \;\; 14]^t \\ &\quad \rightarrow 2 - \text{double scroll} \\ \text{case } q = 3: &\quad \Delta_3 = [5 \;\; 7.5 \;\; 8 \;\; 10 \;\; 15 \;\; 16 \;\; 18]^t \\ &\quad \rightarrow 3 - \text{double scroll} \\ \text{case } q = 4: &\quad \Delta_4 = [4 \;\; 6 \;\; 6.5 \;\; 8 \;\; 12 \;\; 13 \;\; 18 \;\; 27 \;\; 30 \;\; 32]^t \\ &\quad \rightarrow 4 - \text{double scroll} \end{aligned} \quad (11)$$

The functions $k_q(x)$ and $g_q(x) = C_1 k_q(x)x$ are plotted in Fig. 2 for $q = 2$ and $q = 4$. The associated equilibrium points and eigenvalues of the Jacobians are shown in (12)–(14) at the bottom of the next page.

For simulations a trapexoidal integration rule with constant step length equal to 0.05 was used and initial state $x(0) = y(0) = z(0) = 0.1$. The simulation results are shown in Figs.

3–5 for the 2-, 3-, and 4-double scroll, respectively. Finally, in Fig. 6, a simulation result is presented of a 6-double scroll with

$$\Delta_6 = [4\ 6\ 6.4\ 8\ 12\ 12.8\ 16\ 24\ 25.6\ 32\ 48\ 51.3\ 62\ 93\ 99.4\ 110]^t. \quad (15)$$

3.3. Bifurcations with Respect to a

Now, the eigenvalues of the Jacobians (10) at $\mathbf{eq}_0$, $\mathbf{eq}^{\pm}_{j,j-1}$, $\mathbf{eq}^{\pm}_{j+1,j}$, $\mathbf{eq}^{\pm}_{4+3(q-2)}$ are studied as a function of the bifurcation parameter a. In Fig. 7, these results are shown for $\gamma_{j,j-1} = 5$ and $\phi_{j+1,j} > 0$ with $j = 2 + 3(i-2)$ and $i = 2, \cdots, q$. From this diagram the existence of multiple sinks can be derived. In Fig. 8 an example is given for $q = 2$ and $a = 4$ (2-double sink). Hence transitions occur between ordered n-double sink portraits and n-double scrolls for increasing parameter a. The rich variety of phenomena observed in between n-double sinks and n-double scrolls is not discussed in this paper.

IV. A Quasilinear Study

In order to get a global view onto the state space behaviour of (5) and (6) a quasilinear approach is applied, as described in [9]. This method is heuristic but may give additional insight compared to a local stability analysis, provided one keeps in mind certain limitations of the method. Namely with respect to stability analysis of nonlinear systems counterexamples to the method exist like to the Aizerman conjecture [7], [9]. The idea is simply to study the eigenvalues of $A(\boldsymbol{x})$ in $\dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x}$. Suppose

$$\lambda(A(\boldsymbol{x})) = \sigma(\boldsymbol{x}) + j\omega(\boldsymbol{x}). \quad (16)$$

For the system (5) and (6) only the following situation occurs

$$\begin{aligned} \lambda_1 &= \sigma_1 \\ \lambda_{2,3} &= \sigma_{2,3} + j\omega. \end{aligned} \quad (17)$$

Regions S, U_r, $U_i \subset \mathbf{R}^3$ are now defined as

$$\begin{aligned} &\text{if } \sigma_1(x_P) < 0 \text{ and } \sigma_{2,3}(\boldsymbol{x}_P) < 0 \rightarrow x_P \in S \\ &\text{if } \sigma_1(x_P) > 0 \text{ and } \sigma_{2,3}(\boldsymbol{x}_P) < 0 \rightarrow x_P \in U_r \\ &\text{if } \sigma_1(x_P) < 0 \text{ and } \sigma_{2,3}(\boldsymbol{x}_P) > 0 \rightarrow x_P \in U_i. \end{aligned} \quad (18)$$

In Fig. 9 these regions are plotted for the cases $q = 2$ (Fig. 9(a)), $q = 3$ (Fig. 9(b)), $q = 4$ (Fig. 9(c)) as a function of the parameter a between 3 and 8. Only positive x values are shown because of symmetry with respect to the origin. Some intuitive but nonrigorous conclusions can be formulated from these figures

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- 2-double scroll:

$$\begin{array}{lll} \mathbf{eq}_0 = [0\ 0\ 0]^t & J(\mathbf{eq}_0) \rightarrow \lambda_1 = 1.76 & \lambda_{2,3} = -0.73 \pm 1.85j \\ \mathbf{eq}^{\pm}_{2,1} = [\pm 6.25\ 0\ \mp 4.38]^t & J(\mathbf{eq}^{\pm}_{2,1}) \rightarrow \lambda_1 = -5.58 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{3,2} = [\pm 9.38\ 0\ \mp 6.56]^t & J(\mathbf{eq}^{\pm}_{3,2}) \rightarrow \lambda_1 = 5.65 & \lambda_{2,3} = -0.67 \pm 2.40j \\ \mathbf{eq}^{\pm}_{4} = [\pm 21\ 0\ \mp 14.7]^t & J(\mathbf{eq}^{\pm}_{4}) \rightarrow \lambda_1 = -3.05 & \lambda_{2,3} = 0.18 \pm 2.14j \end{array} \quad (12)$$

- 3-double scroll:

$$\begin{array}{lll} \mathbf{eq}_0 = [0\ 0\ 0]^t & J(\mathbf{eq}_0) \rightarrow \lambda_1 = 1.76 & \lambda_{2,3} = -0.73 \pm 1.85j \\ \mathbf{eq}^{\pm}_{2,1} = [\pm 6.25\ 0\ \mp 4.38]^t & J(\mathbf{eq}^{\pm}_{2,1}) \rightarrow \lambda_1 = -5.78 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{3,2} = [\pm 7.75\ 0\ \mp 5.43]^t & J(\mathbf{eq}^{\pm}_{3,2}) \rightarrow \lambda_1 = 31.15 & \lambda_{2,3} = -0.43 \pm 2.6j \\ \mathbf{eq}^{\pm}_{5,4} = [\pm 12.5\ 0\ \mp 8.75]^t & J(\mathbf{eq}^{\pm}_{5,4}) \rightarrow \lambda_1 = -5.78 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{6,5} = [\pm 15.5\ 0\ \mp 10.85]^t & J(\mathbf{eq}^{\pm}_{6,5}) \rightarrow \lambda_1 = 31.15 & \lambda_{2,3} = -0.43 \pm 2.6j \\ \mathbf{eq}^{\pm}_{7} = [\pm 27\ 0\ \mp 18.9]^t & J(\mathbf{eq}^{\pm}_{7}) \rightarrow \lambda_1 = -3.05 & \lambda_{2,3} = 0.18 \pm 2.14j \end{array} \quad (13)$$

- 4-double scroll:

$$\begin{array}{lll} \mathbf{eq}_0 = [0\ 0\ 0]^t & J(\mathbf{eq}_0) \rightarrow \lambda_1 = 1.76 & \lambda_{2,3} = -0.73 \pm 1.85j \\ \mathbf{eq}^{\pm}_{2,1} = [\pm 5\ 0\ \mp 3.5]^t & J(\mathbf{eq}^{\pm}_{2,1}) \rightarrow \lambda_1 = -5.78 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{3,2} = [\pm 6.25\ 0\ \mp 4.38]^t & J(\mathbf{eq}^{\pm}_{3,2}) \rightarrow \lambda_1 = 23.2 & \lambda_{2,3} = -0.45 \pm 2.6j \\ \mathbf{eq}^{\pm}_{5,4} = [\pm 10\ 0\ \mp 7]^t & J(\mathbf{eq}^{\pm}_{5,4}) \rightarrow \lambda_1 = -5.78 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{6,5} = [\pm 12.5\ 0\ \mp 8.75]^t & J(\mathbf{eq}^{\pm}_{6,5}) \rightarrow \lambda_1 = 25.19 & \lambda_{2,3} = -0.44 \pm 2.6j \\ \mathbf{eq}^{\pm}_{8,7} = [\pm 22.5\ 0\ \mp 15.75]^t & J(\mathbf{eq}^{\pm}_{8,7}) \rightarrow \lambda_1 = -5.78 & \lambda_{2,3} = 0.04 \pm 2.46j \\ \mathbf{eq}^{\pm}_{9,8} = [\pm 28.5\ 0\ \mp 19.95]^t & J(\mathbf{eq}^{\pm}_{9,8}) \rightarrow \lambda_1 = 19.24 & \lambda_{2,3} = -0.47 \pm 2.59j \\ \mathbf{eq}^{\mp}_{10} = [\pm 48\ 0\ \mp 33.6]^t & J(\mathbf{eq}^{\mp}_{10}) \rightarrow \lambda_1 = -3.05 & \lambda_{2,3} = 0.18 \pm 2.14j \end{array} \quad (14)$$

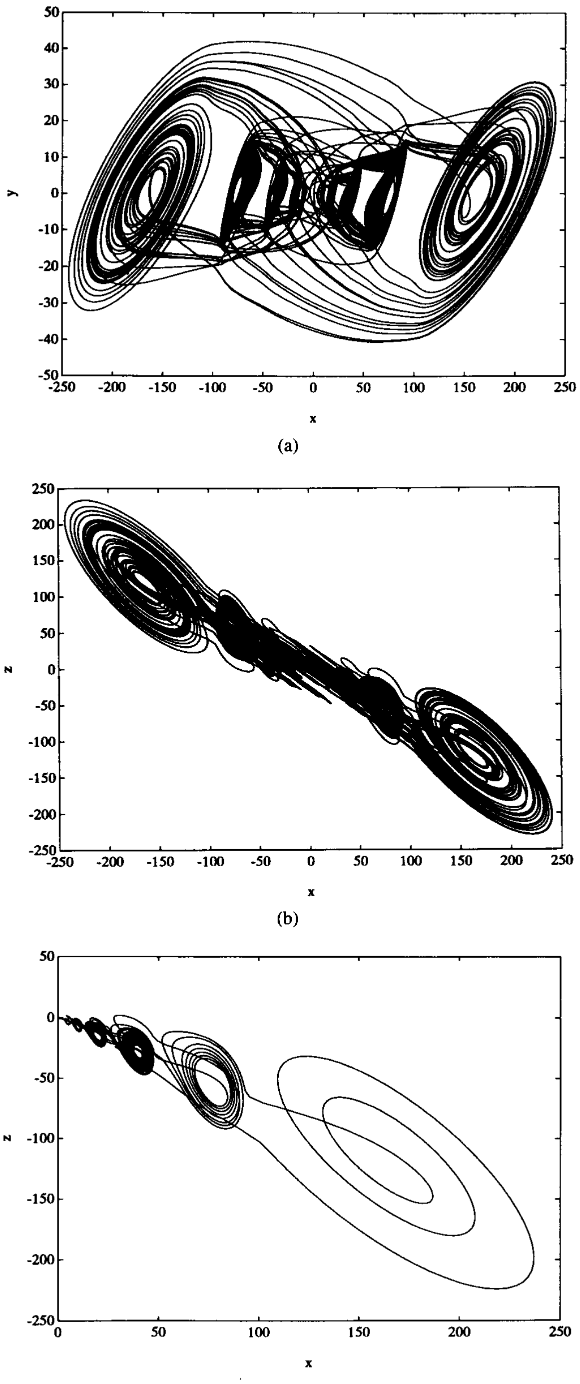

Fig. 6. 6-double scroll attractor: (a) $(x-y)$, (b) $(x-z)$, (c) enlarged part of (b).

- In the 2-double scroll case (e.g., $a = 7$) one has the configuration

$$U_r - S - U_i - S - U_r - S - U_i$$

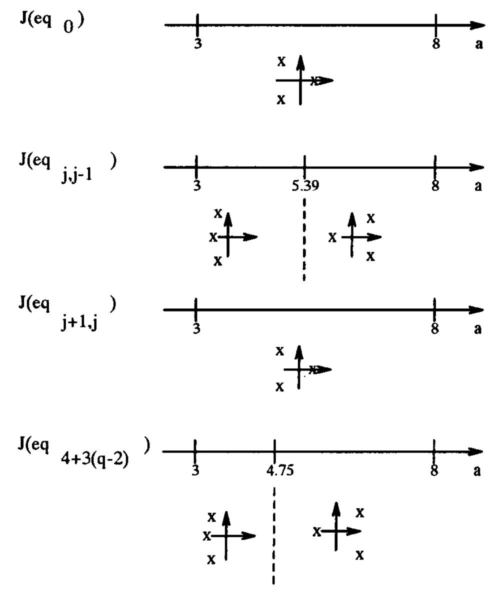

Fig. 7. Eigenvalue configurations of the Jacobians at $\mathbf{eq}_0$, $\mathbf{eq}^{\pm}_{j,j-1}$, $\mathbf{eq}^{\pm}_{j+1,j}$, $\mathbf{eq}^{\pm}_{4+3(q-2)}$, as a function of the parameter a with $\gamma_{j,j-1} = 5$ and $\phi_{j+1,j} > 0$ (for $j = 2 + 3(i-2)$, $i = 2, \cdots, q$). The figure should be read rowwise as follows: each row gives the location of the eigenvalues of a Jacobian for the region of interest for $a (a \in [3, 8])$.

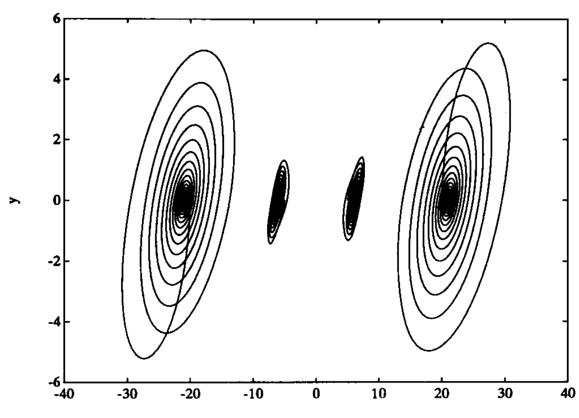

Fig. 8. Multiple sink portrait (2-double sink) for $a = 4$.

The configuration $S - U_i - S$ is responsible for the two scrolls closest to the origin.

- The configuration U_r causes the transition between the scrolls.

V. Conclusions

In this paper Chua's circuit was generalized by introducing additional breakpoints in the characteristic of the nonlinear resistor, leading to n-double scroll attractors ($n = 1, 2, 3, 4, \ldots$). These attractors serve as an example to illustrate that a combination of a local stability analysis with a

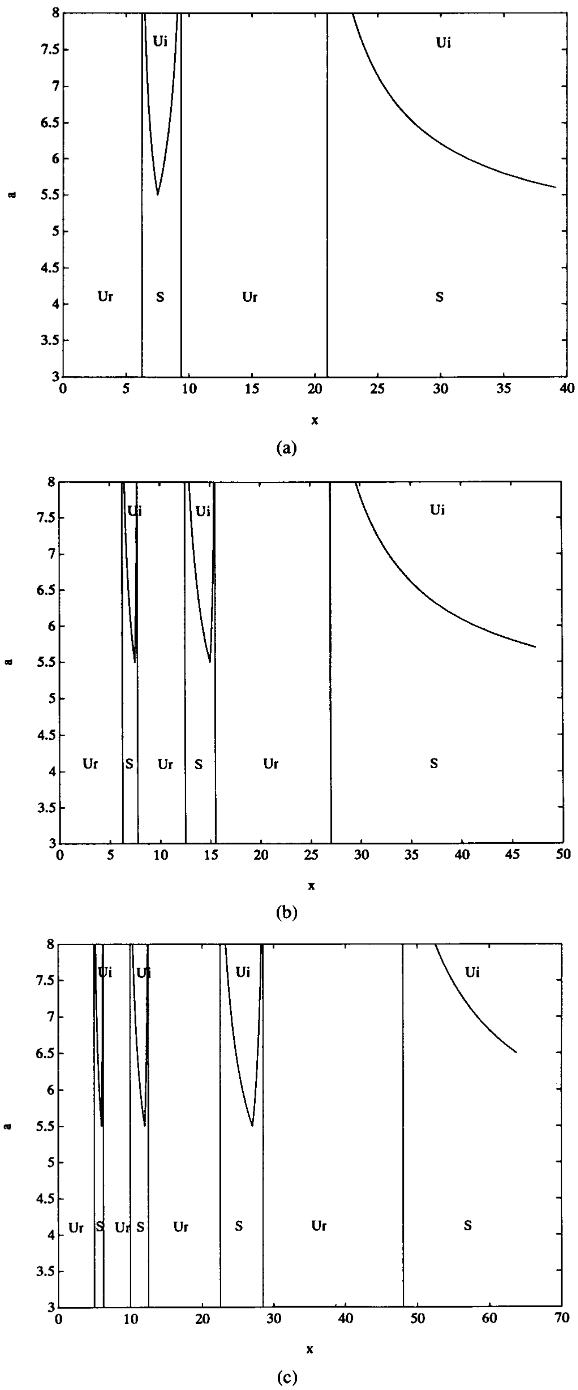

Fig. 9. Quasilinear study of the nonlinear circuit for generation of the 2-double scroll, 3-double scroll and 4-double scroll (slice at $a = 7$ in (a), (b), (c) respectively).

quasilinear approach may be an easy way to get a quick and rough insight into complex behaviour revealed by nonlinear systems. Hence the method may contribute as a tool towards the construction of dynamics.

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