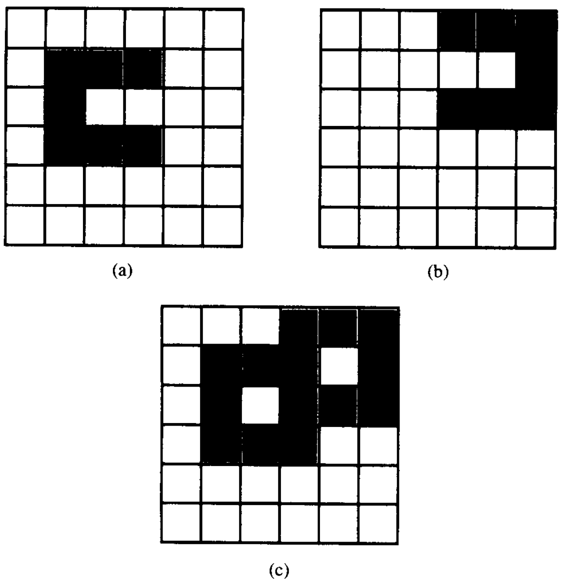

Fig. 4. (a), (b) Two incompatible fixed points satisfying (7) for $r = 1$, and (c) the corresponding difference pattern.

Remark: Note that item 1) of the Proposition is general while item 2) applies only if (6) holds. As an example, item 1) applies to networks with nonlinearities of the general type (3). However, item 2) applies if the sum on n is extended only to odd integers and $I_{ij} = 0$ for every i and j. In particular, for CNN's, item 2) applies if there are neither external inputs nor thresholds.

IV. Comments and Conclusions

Some relations of incompatibility between fixed points have been proved for a broad class of recurrent nonlinear neural networks. The property shown herein sheds some light on the role of self-interactions in recurrent neural networks. Without self-interactions, the incompatibility cannot be removed, even if *arbitrary* nonlinearities are introduced in the model.

The results presented in this paper have simple and important implications in the case of associative memory and pattern recognition, where the fixed points correspond to the stored prototypes or learned categories. In this kind of applications, the zero self-feedback assumption is usual, since it corresponds to a reduced number of extraneous (undesired) fixed-points [5], [9]. However, as shown above, the prototypes must satisfy pairwise constraints in order that all of them could be stored as fixed points. In particular, assuming $r = 1$, two of them cannot both be stored if their difference pattern presents one of the configurations shown in Figs. 1 and 2. This holds true irregarding of the learning strategy used. So, before applying any learning rule, we can preliminarily check, by inspection, if the prototypes are compatible.

This incompatibility condition has an effect on the capacity of locally-interconnected neural networks. An estimation of this effect is beyond the scopes of the present paper. However, some comments are necessary. The probability $\mathcal{P}(m, r)$ increases as r decreases. This fact can justify to some extent the experimental results showing that the capacity is a decreasing function of the neighborhood size [5], [10]. Note that the incompatibility is a consequence of locality. In the case of partially interconnected networks with sparse interactions, the result does not apply. So, using the same overall number of interactions, the capacity should be larger when these links are randomly distributed among the units. This is reasonable taking into account that locality is a further constraint imposed on the network architecture. However, from the practical realization viewpoint, local links are preferable.

Finally, it is worth noting that even if the prototypes are compatible, they cannot necessarily be stored. Compatibility is only a prerequisite. Thus, the probability of storing a set of m random prototypes is less than $1-\mathcal{P}(m, r)$.

References

[1] L. O. Chua and L. Yang, "Cellular neural networks: Theory," *IEEE Trans. Circuits Syst.*, vol. CAS-35, pp. 1257–1272, 1988.
[2] *IEEE Trans. Circuits Syst.—I: Fundamental Theory and Appl.*, Special Issue on Cellular Neural Networks, vol. 40, Mar. 1993.
[3] *IEEE Trans. Circuits Syst.—II: Analog and Digital Signal Process.*, Special Issue on Cellular Neural Networks, vol. 40, Mar. 1993.
[4] G. Martinelli and R. Perfetti, "Associative memory design using space-varying cellular neural networks," *CNNA-92*, Munich, Germany, IEEE New York, 1992, pp. 117–122.
[5] J. M. Karlholm, "Associative memories with short-range, higher order couplings," *Neural Networks*, vol. 6, pp. 409–421, 1993.
[6] L. Personnaz, I. Guyon, and G. Dreyfus, "High-order neural networks: Information storage without errors," *Europhys. Lett.*, vol. 4, pp. 863–867, 1987.
[7] M. Morita, "Associative memory with nonmonotone dynamics," *Neural Networks*, vol. 6, pp. 115–126, 1993.
[8] H. Harrer and J. A. Nossek, "Discrete-time cellular neural networks," *Int. J. Circuit Theory and Applicat.*, vol. 20, pp. 453–467, 1992.
[9] Y. Kamp and M. Hasler, *Recursive Neural Networks for Associative Memory*. Chichester: Wiley, 1990.
[10] M. Bo, G. Martinelli, and R. Perfetti, "Estimating the storage capacity of space-varying cellular neural networks," *Electronics Lett.*, vol. 30, no. 20, pp. 1691–1693, 1994.

Is the Colpitts Oscillator a Relative of Chua's Circuit?

G. Sarafian and B. Z. Kaplan

***Abstract*—The topological resemblance of the Colpitts oscillator to that of Chua's circuit suggests that the two circuits may also be similar in their dynamic behavior. The similarity in the dynamics is demonstrated in the present communication by resorting to computer simulation. We deal in particular with the unknown chaotic behavior of the Colpitts oscillator. It is shown by referring to recently published articles that the Colpitts oscillator and Chua's circuit are in fact topologically conjugate one to another.**

I. Introduction

A recent analysis of practical RF VCO based on Colpitts oscillator [1] seems to add a new interpretation to the rich and interesting dynamic behavior of the well known Chua's family of circuits [2]–[3]. It is interesting that there exists direct similarity between one of

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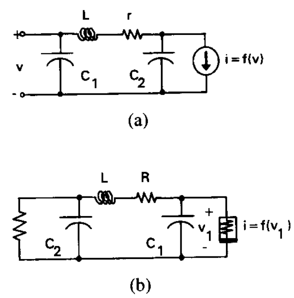

Fig. 1. (a) Model of Colpitts oscillator, see [1]. (b) The canonical realization of Chua's circuit family, see [3]. We have retained here the same symbols that appear in the references.

Chua's circuits [2] and the well-known practical Colpitts oscillator circuit [1]. The small difference between the circuits, see Fig. 1(a) and (b), is due to the slightly different nonlinear device, while the passive network is the same in both cases. The Chua's circuit nonlinear device consists of a two- terminal conductance, whose current is a nonlinear function of its own voltage (v_1). The nonlinear device in the Colpitts oscillator, on the other hand, is a three-terminal nonlinear element, where the corresponding current is determined by the voltage on the second capacitor (v). We show in the present communication that the relationship between the systems is not merely due to the topological aspect, but is due also to their similar dynamic behavior. This issue is partially supported by a recent investigation of VCO's based on practical Colpitts oscillator [1]. It indicates that under certain conditions, the Colpitts oscillator dynamics reveals chaotic behavior [4]. One of the aims of the present work is to deal with the unknown chaotic aspect of the well known Colpitts oscillator.

Besides the small difference in topology between the Colpitts oscillator and the Chua's circuit discussed previously, there exists another difference that is due to the symmetry of the nonlinearity. The nonlinearity of the Chua's circuit possesses an odd symmetry, which is sometimes represented by piecewise-linear characteristics [2], or in other cases, it is modeled by an antisymmetric cubic polynomial [5]. The nonlinearity of the conventional Colpitts oscillator, on the other hand, is also usually modeled by a third order polynomial [1], but this polynomial contains a square term as well. As a result, the latter characteristic is not symmetric. In order to emphasize further the similarity between the Chua's circuit and the Colpitts oscillator, and in order to emphasize the chaotic response, we have now modeled the Colpitts nonlinearity by symmetric third order polynomial where the square term is omitted. The dynamic behavior of the Colpitts oscillator that results from this kind of modeling is reported in this communication and is strongly related to the well known chaotic behavior of Chua's circuit. Modeling Colpitts oscillator with the aid of pure odd nonlinearity is also of some practical value, since the oscillator is sometimes built in push-pull configuration. The latter configuration is associated with a purely odd nonlinearity [6].

II. A Model for the Chaotic Colpitts Oscillator and Its Behavior

The main items of the model are described in [1]. This is also shown in Fig. 1(a). The system equations are

$$\begin{aligned} C_1 \cdot \frac{dv}{dt} - i_L &= 0 \\ r \cdot i_L + L \cdot \frac{di_L}{dt} + v + v_2 &= 0 \\ -i_L + C_2 \cdot \frac{dv_2}{dt} - f(v) &= 0. \end{aligned} \tag{1}$$

The nonlinearity is modeled here by

$$f(v) = a_1 \cdot v + a_3 \cdot v^3. \tag{2}$$

The system equation is therefore [1]

$$\frac{d^3v}{dt^3} + \alpha_1 \cdot \frac{d^2v}{dt^2} + \alpha_2 \cdot \frac{dv}{dt} + \alpha_3 \cdot v + k_3 \cdot v^3 = 0. \tag{3}$$

The coefficients $\alpha_1, \alpha_2, \alpha_3, k_3$ are defined in [1], (11). The system has been investigated by simulating the system on a digital computer. The results are presented in Figs. 2 and 3. The phase plane figure in Fig. 2(a) illustrates the chaotic behavior of the Colpitts oscillator. It seems to indicate a scrolling phenomenon. It may even represent a "scroll tripling" type of behavior that has some structural resemblance to the one that is described in [3, Fig. 13]. Both the time waveform in Fig. 2 and the related Poincare map seem to exhibit strong chaotic features. It is interesting to notice the similarity of the attractor in Fig. 2(a) to those in plate no. 24 and 44 in the gallery of attractors in [7].

The route to chaos of the system dynamics is illustrated in Fig. 3 by relying on the power spectra of the $v(t)$ signal for four values of r. Fig. 3(a) demonstrates an ordinary periodic behavior of Colpitts oscillator. Fig. 3(b) exhibits a power spectrum related to a period doubling phenomenon in the oscillator dynamics. The spectrum in Fig. 3(c) exhibits a period-four phenomenon. The power spectrum in Fig. 3(d) appears to be chaotic related.

III. The Helpful Contribution of the Bias Variable in Explaining the System Dynamics

The original Colpitts oscillator system variables in (1) were replaced in [1] by slow variables that enabled us to employ the Harmonic Balance method. One of the slow variables entitled by us as a bias (the $z(t)$ variable in [1]) is one of the main items of the present interpretation. The bias variable was helpful both for the linear and especially for the nonlinear approximations in [1]. This variable assists in interpreting the system output as consisting of a relatively rapid oscillating term that is superimposed on a slowly alternating bias. Even a simple consideration of (3) already reveals the helpful interpretation gained by emphasizing the bias contribution. One can suggest that

$$v(t) = z(t) + \text{rapidly oscillatory variable}. \tag{4}$$

As a result, (3) may be modified to two equations; one that governs the rapidly oscillatory term and another that governs the slow variable $z(t)$. In addition, we are also interested in identifying the equalibria positions of $z(t)$, which seem to serve as the foci of the rapid oscillations as is demonstrated in Fig. 2(b). Hence, the equation representing the equalibria points of z is (z_0 indicates an equilibrium point of $z(t)$)

$$\alpha_3 \cdot z_0 + k_3 \cdot z_0^3 = 0. \tag{5}$$

As a result, three solutions can be obtained

$$z_{01} = 0, \quad z_{02} = -\sqrt{-\frac{\alpha_3}{k_3}}, \quad z_{03} = +\sqrt{-\frac{\alpha_3}{k_3}}. \tag{6}$$

This existence of three solutions is helpful in explaining the tendency of the system to reveal a quasi triple scrolling phenomenon. The three equilibria points in (6) can be expressed in terms of the original nonlinearity coefficients in (2)

$$z_{01} = 0, \quad z_{02} = -\sqrt{-\frac{a_1}{a_3}}, \quad z_{03} = +\sqrt{-\frac{a_1}{a_3}}. \tag{7}$$

The quasi-triple scrolling behavior is interpreted as related to essentially three types of oscillations burst. Some of the oscillations

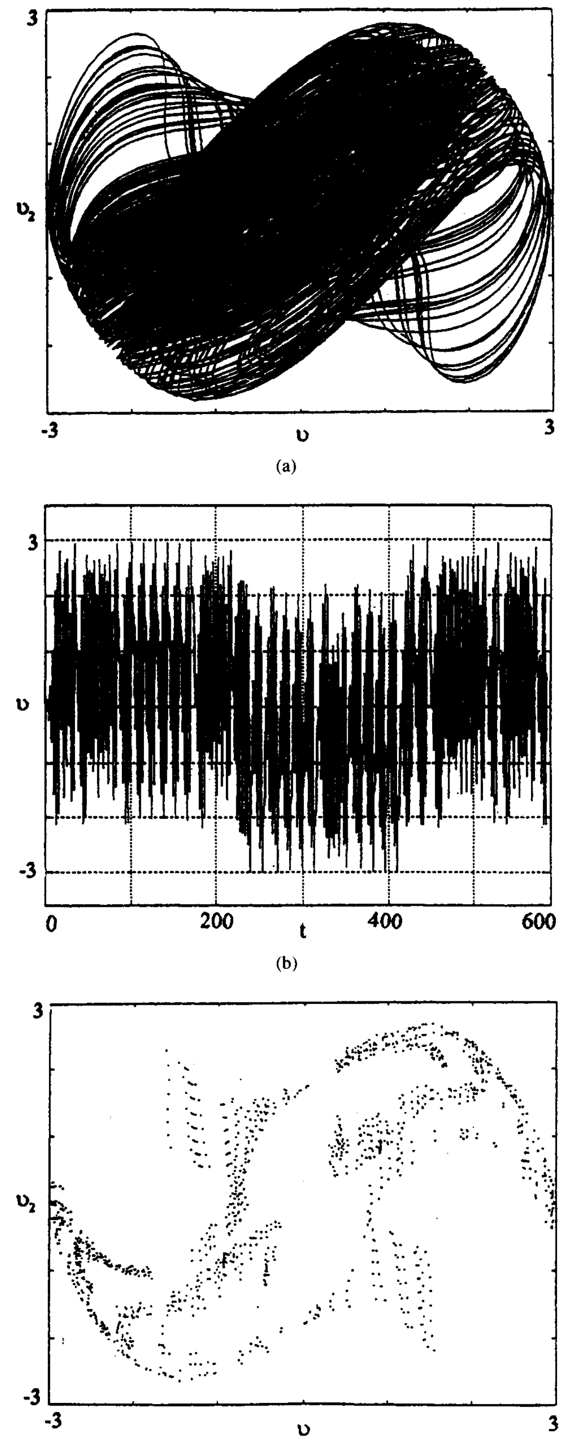

Fig. 2. Numerical evaluations related to the dynamics of the circuit in Fig. 1 for the following normalized circuit elements: $1/C_2 = 2.2, 1/C_1 = 2.4, L = 1, r = 0.249, a_1 = 1, a_3 = -0.2$. The initial values for the numerical evaluation are $v(0) = 0.05, v_2(0) = 0.05, i_L(0) = 0.01$. (a) Chaotic attractor in $(v - v_2)$ plane. (b) The related waveform versus time $v(t)$. (c) The related Poincare map in $(v - v_2)$ plane when $i_L = 0$.

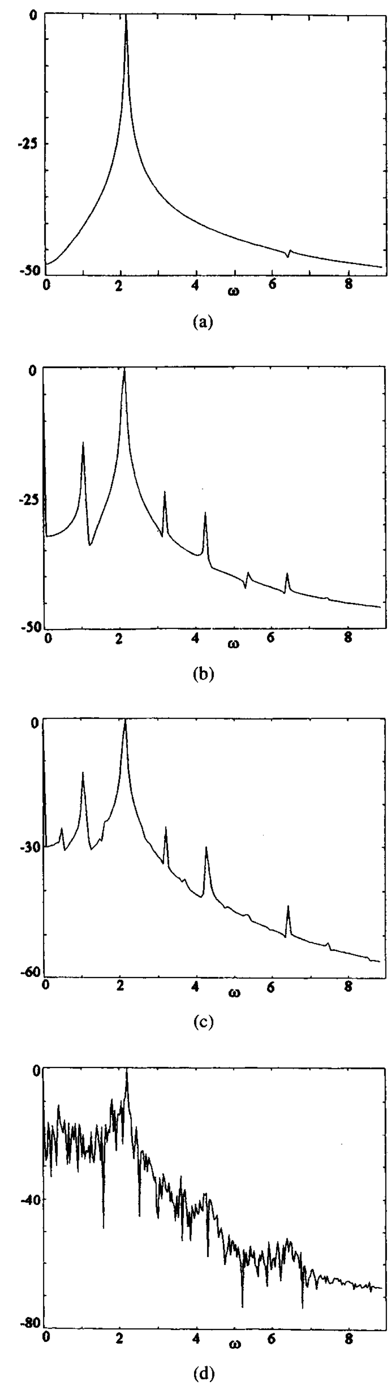

Fig. 3. Power spectra for $v(t)$ of the Colpitts oscillator for various values of r. (The variation of r is related to respective changes in the resonant circuit Q.) The value of r is employed as the parameter for inducing the system's route to chaos. (a) $r = 1.00$, (b) $r = 0.30$, (c) $r = 0.28$, (d) $r = 0.249$. Horizontal axis: frequency (rad./s), vertical axis: power (mean squared amplitude of $v(t)$) (dB).

seem to be centered around zero, while the two other seem to be centered around the positive and negative values z_{02} and z_{03} in (7) (see Fig. 2(a)).

IV. Conclusion

The various models associated with the Chua's circuit have been recently found helpful in demonstrating and explaining the various facets of chaos [3]. Attempts are made in the literature to discover new members of the large family of Chua's circuits and to find their close relatives [8], [9]. An example for such a recently discovered relative is described in [10], where a nonautonomous related circuit is discussed. (The latter circuit can also be regarded as a relative of the RL diode circuit). The present communication deals with the well known Colpitts oscillator, which is shown to be topologically similar to Chua's circuit. It is also shown here that when the nonlinearity of the active device in the Colpitts oscillator is modified to be purely odd, then the circuit exhibits chaotic phenomena closely related to those exhibited by the classical [3] Chua's circuit. Hence, yet another relative of Chua's circuit has been discovered.

A reviewer has pointed out that recent works of Chua *et al.* [8], [9] have established mathematically the exactly detailed relationship between Chua's oscillator and relatively many other 3-D systems. The works prove that such 3-D systems are topologically conjugate to Chua's oscillator (or in circuit terms, they are equivalent to Chua's oscillator [11]). A Chua's oscillator is obtained by adding a resistor in series with the inductor in Chua's circuit [11]. The classical Chua's oscillator [11] is, therefore, conjugate to the circuit in Fig. 1(b). Hence, by demonstrating that there exists a robust relationship between the Colpitts oscillator in Fig. 1(a) and the Chua's oscillator [11], one can show that the two systems of Fig. 1 are not simply loose relatives, but they are even strongly related and can be regarded as being conjugate one to the other. Reference [9], which is strongly related to [8], cites an example due to Arneodo *et al.* [12] of a 3-D system that is strictly proved in [9] as being conjugate to the Chua's oscillator. It is interesting that the latter example ((10) in [9]) is in fact the same equation as the one that represents the presently discussed Colpitts oscillator (3). Hence, we can conclude that the two member systems of Fig. 1 are not merely related, but they are even conjugate one to the other. Therefore, due to the helpful remarks of the reviewer, we are assured that the answer to the question posed in the title of the present communication is strongly yes.

Acknowledgment

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References

[1] G. Sarafian and B. Z. Kaplan, "A new approach to the modeling of the dynamics of RF VCO's and some of its practical implication," *IEEE Trans. Circuits Syst.*, vol. 40, no. 12, pp. 895–901, Dec. 1993.
[2] L. O. Chua and G. Lin, "Intermittency in a piecewise-linear circuit," *IEEE Trans. Circuits Syst.*, vol. 38, no. 5, pp. 510–520, May 1991.
[3] ——, "Canonical realization of Chua's circuit family," *IEEE Trans. Circuit Syst.*, vol. 37, no. 7, pp. 885–902, July 1990.
[4] G. Sarafian, "Practical problems in the dynamics of voltage controlled oscillators," Thesis submitted in partial fulfillment of the requirements for M.Sc. Degree, Ben-Gurion University of the Negev, Beer-Sheva Israel, Faculty of Engineering Sciences, Department of Electrical and Computer Engineering (in Hebrew), Nov. 1992.
[5] E. J. Altman, "Normal form analysis of the Chua's circuit with application for trajectory recognition," *IEEE Trans. Circuits Syst.*, vol. 40, no. 10, pp. 675–682, Oct. 1993.
[6] X. Zhou and A. S. Daryoush, "An injection locked push-pull oscillator at Ku-band," *IEEE Microwave and Guided Wave Lett.*, vol. 3, no. 8, pp. 244–246, Aug. 1993.
[7] R. N. Madan, Ed., *Chua's Circuit: A Paradigm for Chaos.* Singapore: World Scientific, 1993.
[8] L. O. Chua, "Global unfolding of Chua's circuit," *IEICE Trans. Fundamentals*, vol. E76-A, pp. 704–734, May 1993.
[9] L. O. Chua, C. W. Wu, A. Huang, and G. Q. Zhong, "A universal circuit for studying and generating chaos—Part II: Strange attractors," *IEEE Trans. Circuits Syst.*, vol. 40, no. 10, pp. 745–761, Oct. 1993.
[10] K. Murali, M. Lakshmanan, and L. O. Chua, "The simplest dissipative nonautonomous chaotic circuit," *IEEE Trans. Circuits Syst.*, vol. 41, no. 6, pp. 462–463, June 1994.
[11] L. O. Chua, C. W. Wu, A. Huang, and G. Q. Zhong, "A universal circuit for studying and generating chaos-Part I: Routes to chaos," *IEEE Trans. Circuits Syst.*, vol. 40, no. 10, pp. 732–744, Oct. 1993.
[12] A. Arneodo, P.Coullet, and E. A. Spiegel, "Chaos in finite macroscopic systems," *Phys. Lett.*, vol. 92A, no. 8, pp. 369–373, 1982.

On the Relationship Between the Chaotic Colpitts Oscillator and Chua's Oscillator

Michael Peter Kennedy

***Abstract*— In this letter, we show that the two-region third-order piecewise-linear dynamics of the chaotic Colpitts oscillator may be mapped to a Chua's oscillator with an asymmetric nonlinearity.**

I. Introduction

It has recently been shown that the dynamics of a chaotic Colpitts oscillator (shown in Fig. 1(a)) can be captured by a third-order autonomous circuit model containing just one nonlinear element—a two-segment piecewise-linear resistor (Fig. 1(b)), [1].

The circuit is described by a system of three autonomous state equations

$$\begin{aligned} C_1 \frac{V_{CE}}{dt} &= I_L - I_C \\ C_2 \frac{V_{BE}}{dt} &= -\frac{V_{EE} + V_{BE}}{R_{EE}} - I_L - I_B \\ L \frac{I_L}{dt} &= V_{CC} - V_{CE} + V_{BE} - I_L R_L. \end{aligned}$$

We model the transistor as a two-segment piecewise-linear voltage-controlled resistor N_R and a linear current-controlled current source. Thus

$$\begin{aligned} I_B &= \begin{cases} 0 & \text{if} \quad V_{BE} \le V_{TH} \\ \frac{V_{BE} - V_{TH}}{R_{ON}} & \text{if} \quad V_{BE} > V_{TH} \end{cases} \\ I_C &= \beta_F I_B \end{aligned}$$

where V_{TH} is the threshold voltage ($\approx$ 0.75 V), R_{ON} is the small-signal on-resistance of the base-emitter junction, and β_F is the forward current gain of the device.

Fig. 2 shows a chaotic attractor in this two-region piecewise-linear oscillator.

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