

# Models of Musical Instruments from Chua's Circuit with Time Delay

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**Abstract**—We study a new version of Chua's circuit where the linear elements are replaced by a delay line. We show that this circuit is a model of an interesting class of musical instruments, namely, those, like the clarinet, consisting of a massless reed coupled to a passive linear system. The properties and behaviors of this circuit are studied with or without a filter in the feedback loop. In particular, the oscillation conditions are determined, and the respective role of the nonlinearity and of the linear elements are explained. Real-time simulations of the time-delayed Chua's circuit on a digital workstation makes possible easy experimentation with signals and sounds. A surprisingly rich and novel family of periodic and chaotic musical sounds has been obtained. The audification of the local properties of the parameter space permits easy determination of very complex structures that could not be computed analytically and would be difficult to determine by other methods.

## I. INTRODUCTION

CONTEMPORARY music creation relies increasingly on electronic circuits for synthesis of sounds. For an electronic instrument to be useful in music creation, it should be possible to easily modify the sounds for such effects as expressivity or different playing techniques. Having an instrument is not enough to make good music; one has to be able to play it to produce extremely precise results. The *synthesis model* chosen determines not only the possible sound effects and their specificity but also the characteristics of the *controls* offered to the musician. The *physical model* approach to sound synthesis consists of an explicit simulation of a physical system that produces sound [1], [2]. However, the final aim goes beyond the strict imitation of a specific instrument. Rather, it aims at providing new simulated *instruments* with extended properties, such as a broader range of sounds, improved playability, or other properties sought by musicians.

In Section II, we derive a simple model of an interesting class of musical instruments, namely those, like the clarinet, consisting of a massless reed coupled with a passive linear system, where the delay line plays an important role. We then show in Section III that our model is identical to a new version of Chua's circuit where the linear part is replaced by a delay line. The properties and behaviors of this circuit without a filter in the feedback loop are studied in Section IV. In particular, a stability condition is determined, and some relations between the nonlinearity and the periodic and chaotic solutions are

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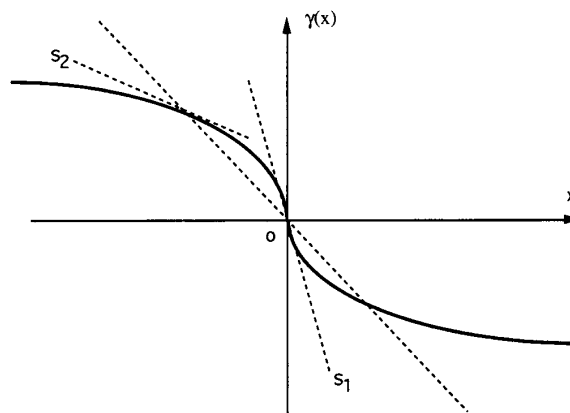


Fig. 1. Cubic smooth map with slopes  $s_1$  and  $s_2$ .

established. The next section looks at the case where a filter is introduced in the feedback loop. It also proposes a simple interpretation of the role of the delay-plus-filter element. In Section VI, we show that for a very large class of models, the Graphical Hopf Theorem gives the conditions for a unique oscillatory solution. Finally, Section VIII explains our real-time digital simulation and the surprisingly large variety of sounds obtained with this circuit.

## II. A SIMPLE PHYSICAL MODEL

Many *physical models* of sustained musical instruments (strings, bass, reeds, flutes, and voice) can be described by autonomous retarded functional difference and differential equations. One of the simplest is written for  $x \in R$  with a memoryless nonlinearity  $\gamma$ :

$$\alpha x'(t) = x(t) + \gamma(x(t - \tau)) \quad (1)$$

or

$$x(t) = h * \gamma(x(t - \tau)) \quad (2)$$

where  $\alpha \in R$ ,  $\gamma: R \rightarrow R$  (e.g., Fig. 1),  $\tau \in R$  is some time delay,  $h: R \rightarrow R$  is the impulse response, and  $*$  is the convolution operator.

We focus on (2), which is preferred, since  $h$  offers much more flexibility than the  $x'(\cdot)$  term. Unfortunately, the solutions to these equations and their stability are known only partially and in restricted cases ([3], see Section 5; [4], see Section 3; [5]). For musical use, we would like to *guarantee which solution is obtained* among possibly several stable

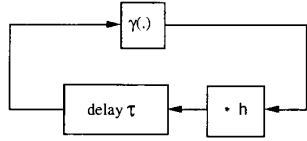


Fig. 2. Simple time-delayed nonlinear system, which is also a basic clarinet model.

solutions. For this purpose, we will accept a broader class of functions  $\gamma$  and of filters  $h$ , provided that they are flexible enough.

For strings, reed-woodwinds, or brass, the delay term plays an essential role [6]. In a simple clarinet model, for instance, the propagation of the sound wave and its reflection at the extremity is represented by a feedback through a delay line and a filter as in (2). The reed is represented by a nonlinear function such as  $\gamma$  above. Similarly, in a simple violin model ([1], [7]), the propagation of the vibration along the string is accounted for by a delay line, and the excitation by the bow is modeled through a possibly discontinuous memoryless nonlinearity. One of the key points for music synthesis is to model the excitation process in such a combination of *nonlinear oscillators that are coupled to passive linear systems*. It is a general model of a large class of musical instruments [8].

Let us examine the reed of a clarinet-like instrument coupled to the bore. Following [8], let us call  $q_o$  and  $q_i$  the outgoing and incoming pressure waves in the bore respectively,  $p$  the pressure in the player's mouth, and  $z$  the characteristic impedance of the bore. The system can be described in a simplified way by the following equations:

$$\begin{aligned} q_o(t) - q_i(t) &= zF(q_i(t) + q_o(t) - p(t)) \\ q_i(t) &= r(t) * q_o(t) = h(t) * q_o(t - T) \end{aligned}$$

where  $h(t - T) = r(t)$  is the *reflection function* of the bore. The most important assumption here is that the reed has no mass, leading to a *memoryless* nonlinearity  $F$ . In the case where this system has a unique solution, then

$$q_o = \gamma(h * q_o(t - T))$$

This is a very simple model to explain the basic oscillatory behavior of the reed in a clarinet-like instrument (Fig. 2). To better understand this behavior, McIntyre [8] and Magenza [9] note that if  $h(t)$  is simplified into a dirac impulse generalized function  $\delta_t$  (the sign inversion is included in  $\gamma$ ), then

$$q_o(t) = \gamma(q_o(t - T)),$$

and similarly for  $q_i$ . The signal value  $q_o(t)$  depends only on the value at  $t - T$ . If  $q_o(t) = Q_o$  is constant on  $[-T, 0]$ , then it is constant on any interval  $[(n - 1)T, nT]$  with a value

$$Q_n = \gamma(Q_{n-1}).$$

We now relate this system, found as a basic model of a clarinet-like instrument, to the time-delayed Chua's circuit.

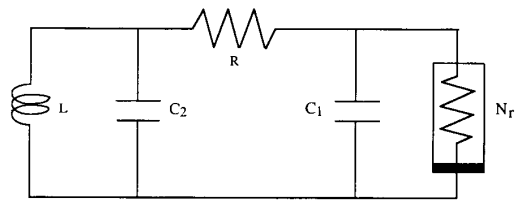


Fig. 3. Chua's circuit.

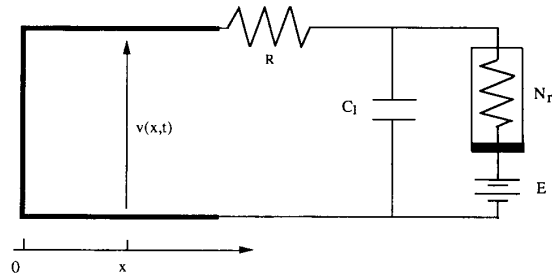


Fig. 4. Time-delayed Chua's circuit.

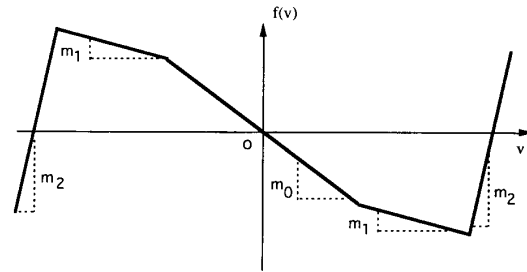


Fig. 5. Chua's diode characteristic.

### III. THE TIME-DELAYED CHUA'S CIRCUIT

The Chua's circuit displayed in Fig. 3 is now well known [10]–[16]. Sharkovsky *et al.* [17] add a dc bias voltage source in series with the Chua's diode and replace the capacitor  $C_2$  and the inductance  $L$  by a lossless transmission line. The resulting time-delayed Chua's circuit is shown in Fig. 4. In a first simplification, the slopes  $m_0$  and  $m_2$  of the characteristic of  $N_r$  (Fig. 5) are set equal. The study of this dynamical system is difficult, but with  $C_1 = 0$ , it reduces to a nonlinear difference equation. The solution consists of the sum of an incident wave  $a(t - x/\nu)$  and a reflected wave  $b(t + x/\nu)$ , such that

$$a(t - x/\nu) = -b(t - x/\nu) = \Phi(t - x/\nu)$$

$$\Phi(t) = \gamma(\Phi(t - 2T))$$

where  $T$  is the time delay in the transmission line and  $\gamma$  is a piecewise linear 1-D function that can be computed from the parameters of the circuit [17]. This is the same delay-equation that we have obtained for the basic clarinet model, except for the filter  $h$ , which we will examine later.

By a proper affine change of variable, the invariant interval of the function can be set to the interval  $[0, 1]$ . For certain

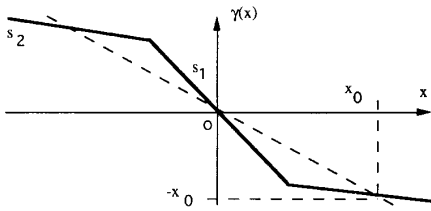


Fig. 6. Piecewise-linear map with slopes  $s_1$  and  $s_2$ .

parameter values, the function  $\gamma$  is composed of two segments only in the invariant interval with slopes  $s_1$  and  $s_2$ . In this particular case, Sharkovsky *et al.* have shown analytically that the time-delayed Chua's circuit exhibits a remarkable period-adding and chaos phenomenon (see Fig. 11 of [17]). In the  $(s_1, s_2)$  space, regions are found where the system has stable limit cycles with periods equal, respectively, to 2, 3, 4, etc. In between every two consecutive stable regions, the system exhibits a chaotic behavior. From the sound synthesis point of view, this is very interesting. Period adding corresponds to successively lowering pitch. In the case of chaos, the signal sounds like noise added to a periodic tone of the instrument, but with some relationship between partials and noise.

#### IV. STABILITY AND PROPERTIES WITHOUT FILTER $h$

We now examine this system as the basic model of a clarinet-like instrument without the filter  $h$  and assume, for simplicity, that  $\gamma' < 0$ ,  $x\gamma''(x) \geq 0$  and that there are only three solutions to the equation  $\gamma(x) = -x$  as in Fig. 1. If the blowing pressure  $p = 0$ , then the pressure in the instrument  $q_0$  should remain zero. Thus, the origin  $O$  is a reasonable fixed point of the system. In order for the system to oscillate around  $O$ , as we expect a musical instrument to do, the slope  $s_1$  of a smooth function about  $O$  has to be less than  $-1$  (Fig. 1). In order for the signal not to grow to infinity, the slope of  $\gamma$  has to become greater than  $-1$  at some distance from  $O$ . Hence, for our purpose, the simplest odd function  $\gamma$  has one segment with slope  $s_1$  around the origin and two segments with slopes  $s_2 > -1$  further from the origin (Fig. 6). Remarkably, this is the same function as in the time-delayed Chua's circuit.

We show now that this  $(s_1, s_2)$  structure for  $\gamma$  is even more justified in terms of the independent controls of sound quality. It can easily be seen that  $|s_1|$  controls the *transient onset velocity*: The greater  $|s_1|$ , the faster the onset. We have here a clear control parameter for the onset behavior of our instrument. If  $h(t) = \delta$ , the signal is a square wave. If  $h(t)$  is a low-pass impulse response, then the signal is *rounded*. This smoothing can be controlled by  $|s_1|$  and  $|s_2|$ : The closer they are to unity, the less high frequencies are in  $q_0$ . This can also be viewed as follows: In the square wave case, the system uses only two points of the graph of  $\gamma$ , whereas in the rounded case, it uses more points spread more regularly on the graph of  $\gamma$ . We have found that two important characteristics of the sound, transient onset velocity and spectral richness, are controlled by the slopes  $s_1$  and  $s_2$ .

However, such a piecewise-linear function has a drawback. Consider the onset of the signal, i.e., the transient from zero.

Observe that before a certain amplitude is reached, only the linear part of  $\gamma$  is used. The system behaves therefore like a linear system; that is, there is no change in the short time spectrum of the signal, other than an amplitude growth (this can be observed very easily in the short time spectrum display of our real-time implementation, detailed below). On the contrary, the nonlinearity of the reed of a real instrument can be more realistically approximated by a quadratic function [8]. Therefore, during the transient, there is a constant transfer of energy between frequency components. As a consequence, we favor a quadratic or cubic nonlinearity of the form  $\gamma(x) = ax^3 + s_1x$  (Fig. 1), where  $a$  is determined according to the slopes  $s_1$  at  $O$  and  $s_2$  at the point  $(x_0, y_0)$ , such that  $y_0 = -x_0 = ax_0^3 + s_1x_0$ . Note that as we vary  $s_1$ , we determine the amplitude and spectral richness of the sound simultaneously. Then a greater amplitude leads to a richer sound (i.e., more high-frequency components and with larger amplitude) as generally happens with natural instruments. However, by varying  $a$  and  $s_1$ , we can still provide independent control of the two first sound qualities mentioned above.

It should be noted that such a polynomial function may introduce other fixed points than the origin, thereby complicating the dynamics of the circuit.<sup>1</sup> An ideal function for our purpose should have the origin as the only fixed point. To guarantee this, the function should not cross the line  $y = x/H(0)$ , where  $H(0)$  is the transfer function of  $h$  at dc.

#### V. CIRCUIT WITH FILTER $h$

For musical purposes, we expect to have control of the period duration, since its inverse is the *pitch*, and control of the waveform itself, since it determines the timbre of the sound. In the continuous case, Chow *et al.* [3] have studied similar equations of the form

$$x(t) = h * \gamma(x(t - \tau))$$

where  $*$  denotes the convolution operation. It is shown that under some fairly general conditions on the function  $\gamma$  and the impulse response  $h$ , the period  $2$  (corresponding to the first *mode* in a clarinet, i.e., to a period of  $2\tau$ ) is asymptotically stable. This means that we can expect to play and keep some steady tone from the instrument. It is also shown that if  $\gamma$  is odd, the signal  $x(t)$  has the symmetry  $x(t + \tau) = -x(t)$ . Then, the signal is composed of odd harmonics only. This is an essential characteristic of the clarinet sound. Under some conditions, periods having durations that are integer fractions of  $2\tau$  are also possible.

The class of filters considered by Chow *et al.* is defined by  $h(t) = 1/2\epsilon$  for  $-\epsilon < t < \epsilon$  and  $h(t) = 0$  elsewhere. For a more general filter  $h$ , the first point we consider is the condition for oscillation around the origin when such a filter  $h$ , with transfer function  $H$ , is introduced in the feedback loop. The open-loop transfer function is now

$$G(j\omega) = e^{-j\omega\tau} H(j\omega).$$

<sup>1</sup> Such other fixed points exist for natural instruments. For a high enough blowing pressure, the reed of a clarinet will keep the mouthpiece closed, but this is usually an unwanted effect.

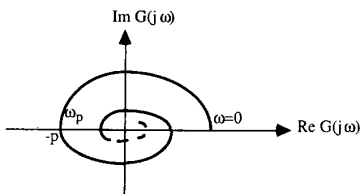


Fig. 7. Nyquist plot of  $G(j\omega)$ .

Since this represents the transfer function of the physical instrument, we naturally suppose that its impulse response belongs to  $L_1$ . We can apply the *graphical stability test* [18] to find the value of the slope  $s_1$  above which the system is stable. The limit value  $1/s_1$  should lie to the left of all intersections of the Nyquist plot of  $G(j\omega)$  with the real axis (Fig. 7). Let  $-p + j0$  denote the intersection point with the smallest value, and let  $\omega_p$  be the value such that  $G(j\omega_p) = -p$ . Then the system becomes unstable when  $s_1 < -1/p$ . Note that this indicates only that the system may eventually oscillate. The proof of this property is more involved and is provided in the next section.

We can extract more information from this diagram. Suppose for the moment that  $H(j\omega)$  is real positive (without loss of generality we can at least choose the delay  $\tau$  such that  $H(j\omega_p) = p$ ). Then, the intersection of  $G(j\omega)$  with the negative real axis occurs for  $\omega_k\tau = \pi + 2k\pi$ , i.e., for frequencies  $f_k = (1 + 2k)/2\tau$ . Observe that  $f_0 = 1/2\tau$  is the frequency corresponding to twice the delay  $\tau$  necessary for a sound wave to propagate from the reed to the end of the bore and back to the reed. The values  $f_k, k = 0, 1, 2, \dots$ , are the frequencies of the modes of the instrument. Therefore,  $G(j\omega_p)$  and  $\omega_p/2\pi$  can be simply interpreted as the amplitude and the frequency of the strongest mode of the instrument.<sup>2</sup> Observe that the frequencies  $f_k$  are the odd harmonic partials of the fundamental  $f_0$ , but that the oscillation frequency may be different from  $f_0$ , since it generally is the frequency of the strongest mode  $\omega_p/2\pi$ . Suppose, for simplicity, that the oscillation frequency is  $\omega_p/2\pi$  and is equal to  $f_0$ . Then,  $G(j2k\omega_p)$  is real positive and the corresponding even partials usually vanish. Assume now that the argument of  $G(j2k\omega_p)$  is different from zero and that the oscillation frequency remains the same. Then, the even partials can appear, and the amplitude of the odd partials can be damped if  $\text{Arg}\{G(j(2k+1)\omega_p)\}$  is not zero. However, in both cases, we have to make sure that the oscillation frequency remains the same. Another way to look at the influence of  $\text{Arg}\{G(jk\omega_p)\}$  is to say that it can move modes away from harmonic positions.

VI. HOPF BIFURCATION AND PERIODIC SOLUTIONS

The graphical stability test given above is valid as long as we can partition our system into a memoryless nonlinearity and a linear feedback loop. This encompasses more models than the simple one we have studied here. But since we are interested in periodic oscillation, we mention here a more

<sup>2</sup>In the case of the trumpet, for instance, the mouthpiece acts as a resonator that boosts some modes with numbers greater than 1, thereby allowing an easy oscillation at the frequency of one of these modes [6].

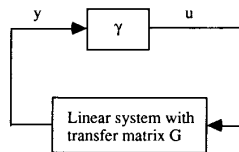


Fig. 8. Nonlinear multiple feedback loop system  $y \in R^M, u \in R^l$ :

general method that allows us to prove the existence of a periodic solution when it occurs, and provides estimates for the frequency and amplitude of the oscillation. It also applies to an even more general class of systems encountered with the most sophisticated physical models of the instruments, such as those in [2] and [19]. The graphical Hopf theorem and its algebraic version [20] apply to a nonlinear multiple feedback loop system, as shown in Fig. 8, where  $\gamma$  is  $C^4$ . Note in particular that  $G$  may include delays. Then, under certain conditions on  $\gamma$  and  $G$ , the system has a unique stable periodic solution. Even though it is straightforward, we will not state this theorem in detail, since it is rather lengthy. We merely emphasize that it provides the existence, uniqueness, and stability test of the solution required for our application. Furthermore, the graphical interpretation is analogous to the graphical test applied in the previous section. However, the periodic solution is guaranteed only in a limited neighborhood of the bifurcation value. Therefore, other stable solutions may appear under more general playing conditions. This occurs in natural instruments [21] but can be a serious inconvenience for an electronic instrument.

VII. DIGITAL SIMULATION

For more flexibility, we have simulated the time-delay Chua's circuit on a digital computer. We have implemented our simulation on a Silicon Graphics Indigo workstation, which is very well adapted for this purpose. It has good quality 16-b audio ports and good graphics capabilities for the user interface (Xwindow and Motif). Furthermore, it is fast enough for real-time simulation of the various circuits that we have studied.

Real-time simulations were implemented by using HTM, which is a tool for rapid prototyping of musical sound synthesis algorithms and control strategies [22]. We have written a Motif-C++ graphical-user interface that allows for easy experimentation with the parameter values [23]. Faders control the two slopes of the piecewise linear map  $\gamma$ , the output amplitude, the fundamental frequency, and the amount of filtering  $h$ . The extreme values of the faders can be adjusted by editing the corresponding number fields. Various graphs are displayed in real-time: the output signal, its short time Fourier transform (STFT) or the function  $\gamma$ . In particular, the possibility of looking at the STFT in real-time is very useful for better understanding of the circuit and of the role of the various parameters [24].

The structure of the periodic and chaotic regions in the  $(s_1, s_2)$  space as displayed in Fig. 9 of [17] is interesting from a sonic point of view. The analytical computation is possible because the characteristic of the nonlinear element is

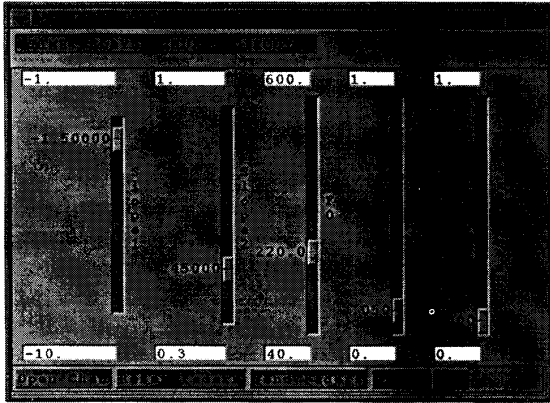


Fig. 9. User interface for the time-delayed Chua's circuit.

piecewise linear. The computation would not be possible for more complex characteristics. But by listening to the sound of the circuit, one can easily determine these regions and their frontiers. Let us take as an example the values for which histograms have been represented in [17], i.e.,  $s_1 = -18$  and  $s_2$  varies from 0.04 to 0.49 or more. One can listen to the sound while changing parameter  $s_2$ . In  $\pi_n$  regions, the periodic signal is clearly heard as a harmonic sound, and the changes in periodicity are easily found by ear. In the intermediate chaotic regions, the sound is unstable or even noisy, and it is not difficult to find approximate values for the frontiers between these regions. It is remarkable that this audification of the local properties of the space allows an easy determination of very complex structures, which in some cases cannot be computed analytically and are not simple to determine by other ways.

A very large variety of sounds can be produced by the system because of the combination of the rich dynamics of the nonlinear function, together with the number of states represented by the delay line  $\tau$ . As an example, one can hear remarkable sounds by use of  $s_2 = 0.99$  or 0.6 and  $s_1$  between  $-1$  and  $-10\,000$ . Fig. 10 shows the short-time spectrum of signals from the digital system for some of these values.

### VIII. CONCLUSION

We have studied here some problems stemming from physical models of musical instruments for the purpose of sound synthesis. In particular, we have shown that the time-delayed Chua's circuit is a model of the basic behavior of an interesting class of musical instruments, namely those, like the clarinet, consisting of a massless reed coupled with a passive linear system. Simple as it is, this circuit exhibits a surprisingly large variety of bifurcations and chaos. In the different regions of the parameter space, periodic and chaotic signals provide novel musical sounds. We have found conditions for periodic oscillations and relationships between parameter values and important properties of the produced signal such as onset time and spectral balance. We have also proposed an analysis of the role of the linear part of the circuit in terms of the amplitude of the harmonic partials of periodic solutions. In the case of circuits that cannot be reduced to the simple

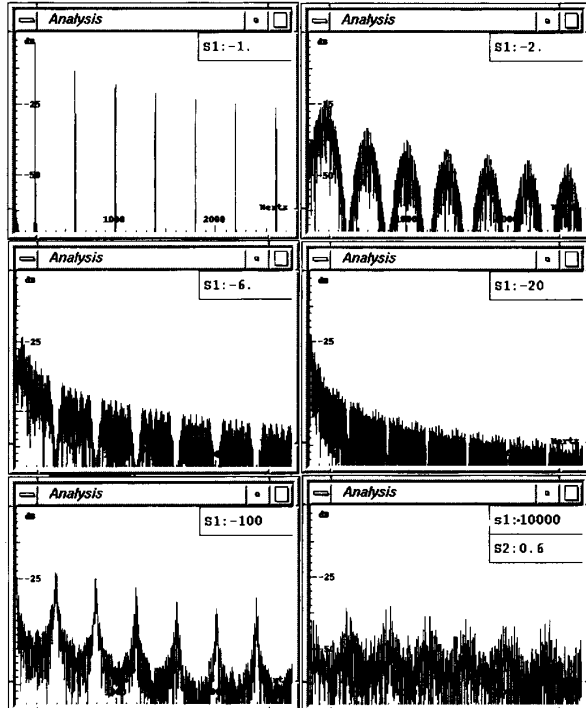


Fig. 10. Short-time spectrum of signals from the digital simulation of the time-delayed Chua's circuit for  $l_0 = 0.99$  and  $l_1$  between  $-1$  and  $-10\,000$ .

form cited above, the Graphical Hopf Theorem provides a test for existence, stability, and uniqueness of periodic solutions.

A real-time implementation of the circuit on an affordable digital workstation has allowed one to make interactive parameter changes while listening to the corresponding sounds, and has allowed easy experimentation with the properties and behaviors of the circuit and sounds. It has revealed a rich and interesting family of sounds for musical applications. The real-time interaction also provides unusual insights on properties of the circuit that would not be as easily discovered by using other means.

We expect to extend Chua's circuit to other instruments such as brass, voice, flute, and strings. It appears that such models are essential for the development and musical use of physical models of classical or new instruments. For instance, we have noted that several stable solutions may appear in general playing conditions. This occurs in natural instruments, but can be a serious inconvenience for an electronic instrument. It would be an interesting achievement to design a system that would model the usual playing behavior of an instrument, but could avoid the other behaviors if requested.

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