Spiral Waves on a 2-D Array of Nonlinear Circuits

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***Abstract*—Spatio-temporal patterns formed in a 2-D array of Chua's circuits have been studied numerically. It has been found that spiral wave solutions can appear over a large range of parameters and some of their properties have been measured. This demonstrates that spiral wave dynamics can be studied in arrays of discrete electronic circuits, such as a 2-D array of Chua's circuits, where real-time results can be obtained. We also study the influence of small differences in the parameters of the circuits, as is the case in real electronic components, where a 5% device tolerance is typical.**

I. Introduction

The tendency of excitable media to organize themselves into highly structured periodic waves or spirals has been the object of intense research interest. Rotating spiral waves (or vortex) have been observed in various excitable media, including cardiac muscle [1] (their formation is one of the fundamental mechanism of dangerous arrhythmias which often leads to sudden death), retinae [2] (their appearance is a manifestation of some pathology such as Leao's spreading depression), cultures of the slime mould *Dyctiostelium discoideum* [3], [4] (here, spirals play a constructive role in a morphogenetic process), chemical waves on the surface of platinum catalyzers [5], and chemical oscillators such as the Belousov–Zhabotinsky (BZ) reaction [6], [7].

Most of these systems have been successfully modeled (e.g., Oregonator's model for the BZ chemical reaction [8], [9]) and this spiral dynamics can be studied by tedious time-consuming computer simulations. Nevertheless, all these simulations share the fact that they are represented by a continuum model via partial differential equations, even though many real systems are more realistically modeled as a set of discrete coupled cells (cardiac muscle or the *Dyctiostelium discoideum* bacterias, are some of the most well-known examples).

Recently, it has been shown that a 1-D array of Chua's circuit can support traveling waves propagating through the resistively coupled cells [10]–[12]. Properties such as the dispersion relation [13], [14] or the propagation failure phenomena [15] have been measured in arrays of active nonlinear circuits and compared with their equivalents in continuous dynamical systems.

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In many cases, the global system can be viewed as an assembly of a large number of identical local subsystems which are coupled to each other by diffusion. Here, the local subsystems are defined as those obeying the Chua's circuit equation [16]. From this point of view, spatio-temporal structures, such as those mentioned above, can be understood on the basis of a synchronization of the local periodic processes occurring in the global system [17].

In this paper, we will show how spiral waves can be reproduced numerically in a 2-D CNN array of resistively coupled globally unfolded Chua's circuits [18] henceforth referred to as Chua's circuit for simplicity. Descriptions of the tip of the spiral as well as some properties (period and wavelength) of the spiral wave have been measured.

These results show that it is possible to build an electronic circuit that enables us to study the spiral wave dynamics in real time and where all the parameters of the medium can be easily controlled.

II. Model of the Two-Dimensional Array of Chua's Circuit

The basic unit (cell) of our 2-D CNN array is a Chua's circuit, a simple active nonlinear circuit which exhibits a variety of bifurcation and chaotic phenomena. The circuit contains three linear energy-storage elements (an inductor and two capacitors), a linear conductance, and a locally active nonlinear resistor. Every cell is coupled to its four nearest adjacent neighbors through linear resistors, thereby simulating a diffusion process.

The equations modeling the 2-D array of resistively coupled Chua's circuit were presented elsewhere [11] and here we rewrite for each circuit cell at the position (i, j) of the array as

$$\begin{aligned} \dot{x}_{i,j} &= \alpha(y_{i,j} - x_{i,j} - f(x_{i,j})) \\ &\quad + D(x_{i+1,j} + x_{i-1,j} + x_{i,j+1} + x_{i,j-1} - 4x_{i,j}) \\ \dot{y}_{i,j} &= x_{i,j} - y_{i,j} + z_{i,j} \\ \dot{z}_{i,j} &= -\beta y_{i,j} - \gamma z_{i,j} \end{aligned} \tag{1}$$

where $1 \leq \{i, j\} \leq n$, n is the size of the array, and $f(x_{i,j})$ is a three-segment piecewise-linear voltage–current characteristic of the nonlinear resistor described by

$$f(x) = \begin{cases} m_1 x + m_0 - m_1, & x \geq x_2 \\ m_0 x, & x_1 \leq x \leq x_2 \\ m_2 x - m_0 + m_1, & x \leq x_1. \end{cases} \tag{2}$$

Notice that $f(x)$ is nonsymmetric in this paper because $m_1 \neq m_2$,

$$x_1 = -1 \quad x_2 = \frac{m_0 - m_1}{m_0 - m_2}.$$

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The relevant nonlinear boundary-value problem we are concerned with is defined by (1) and (2) and by imposing zero-flux (Neumann) boundary conditions. A uniform time step of 5×10^{-4} was used throughout as the differential equations were integrated using the explicit Euler method. The spatial step size is kept at a constant value equal to one as a consequence of our assumption of a discrete array.

The parameters that appear in these equations can be related to the physical parameters of the circuit through the relations $\alpha = C_2/C_1$, $\beta = C_2/(LG^2)$, $\gamma = (C_2 r_0)/(LG)$, $m_0 = G_0/G$, $m_1 = G_1/G$, and $m_2 = G_2/G$, where C_1 and C_2 are the capacitances of the circuit, L is the inductance, r_0 is the resistance in series with the inductor, G_0, G_1, and G are the slopes of the piecewise-linear segments, and G is the conductance of the linear resistor in Chua's circuit. In (1), D represents the diffusion coefficient of the variable x and is given by $D = \alpha/(GR)$ in its dimensionless form, where R is the coupling resistance between two Chua's circuits. However, a diffusion coefficient which is a function of the position, $D = D(i,j)$ is necessary in order to model the influence of small differences in the coupling resistances which cannot be matched exactly in practice.

In case we wish to study the influence of small differences in the nominal values of the internal parameters, (1) and (2) are no longer valid since the scaling process used to derive them fails. In this case, the nonscaled differential equations should be used.

III. Bifurcation Diagram and Limit Cycle

Spiral waves can be described in polar coordinates by the equation $\varphi(r) = \kappa(r) - \omega t$, where ω is the rotation frequency. This simplest description of an archimedian spiral becomes more complicated when we deal with a full reaction-diffusion system. In this case, the waves are not isolines but have some finite thickness and amplitude depending on the properties of the medium. In this case, if we look at any radial direction, it is possible to observe how the property under observation periodically oscillates in space. The distance between two consecutive maxima is called the *wavelength*. The time period between two consecutive waves at any test point of the medium is considered to be the *period* of the spiral wave. For the archimedian spiral described by the equation $\varphi(r)$, the spiral period is exactly equal to $1/\omega$.

Numerically, those points which belong to the wave are considered to be excited and those nonexcited cells are considered to be at the rest state. A wave propagates because an initial excitation spreads out through the whole array by diffusion, exciting its neighborhood (see coupling term in (1)) and, after a given time, decaying to its fundamental state where it reamins until a new wave arrives. An array of oscillators having the above properties is called an *excitable media*. Spiral waves are considered to rotate around a hole consisting of unexcitable cells which is called the *core* of the spiral. The end of the spiral wave, henceforth called the *tip*, follows a trajectory that can be circular or a more complex pattern [19], and in this case the period of the spiral becomes more complicated to define since the distance between waves is not conserved anymore [7].

On the other hand, we can consider each single circuit of the grid to be operating along a limit cycle. This limit cycle should have a fast and a slow regime. In the slow regime, the beginning state of the limit cycle must remain at an almost constant value for some period of time τ_{st}, after which the cycle returns rapidly to the beginning in a significantly shorter period of time, $\tau_{ex} \ll \tau_{st}$. Let us consider a single grid point which has been externally excited from its rest state to the excited state. This means that we move from the slow regime of the limit cycle to the fast one. After a short time period τ_{ex} the state will begin its slow regime (fundamental state) and remain there until a time period equal to τ_{st}, where another revolution of the limit cycle will begin autonomously, or a new perturbation arrives.

If we consider a grid of such circuits, a spiral wave propagating in this medium is a set of excited circuits, where each one excites its neighbor circuits by diffusion and after a time period τ_{ex} will decay to its fundamental state. This system will be stable only when τ_{st} is much larger than the spiral period because otherwise the grid points can be spontaneously excited before a new wave arrives, thereby destroying the spiral structure. It is also important that the decaying time, which corresponds to the "fast" time period τ_{ex} in the limit cycle, is not too small because the spiral wave's thickness depends on this time.

This is the reason why not all limit cycles available in Chua's circuit can support a spiral wave solution, but only when the condition $\tau_{ex} \ll \tau_{st}$ is satisfied.

In order to obtain spiral wave solutions, it is necessary to analyze the behavior of a single circuit as a function of the model parameters L, G, r_0, C_1, and C_2. After a systematic study of the eigenvalues of a single circuit, we found a region in the parameter space in which the circuit behaves as desired. The values of the parameters used were $L = 0.33$ H, C_1=10 nf, $C_2 = 100$ nF, $r_0 = 0\ \Omega$, $1/G_0 = -1.14 \times 10^{-3}$ kΩ, $1/G_1 = 0.07 \times 10^{-3}$ kΩ, and $1/G_2 = 50.71 \times 10^{-3}$ kΩ.

Notice that we are working with a highly asymmetric characteristic function $f(x)$ for the circuit. This is not an arbitrarily imposed condition, but a necessary condition for obtaining a spiral wave solution. Working with a highly asymmetric characteristic function $f(x)$, it is possible to modify drastically the rate τ_{st}/τ_{ex}, and it is this rate which is mainly responsible for the existence of spiral wave solutions. If $\tau_{st}/\tau_{ex} \simeq 1$ then the spiral wave solution will not be stable because it will be destroyed by the spontaneous oscillations of the media.

Fig. 1 shows the behavior of a single grid point for different values of the resistance $(1/G)$ and the capacitance (C_1). In the vertical axis, the ratio between τ_{st} and τ_{ex} is plotted as a function of the resistance $(1/G)$. Different curves correspond to different values of the capacitance, $C_1 \in [1, 100]$ nF. Three different regions can be observed; regions I and III correspond to limit cycles, whereas region II corresponds to stable equilibrium points and hence is an excitable media. In regions I and III the ratio τ_{st}/τ_{ex} increases as we get closer to region II and, hence, the existence of spiral waves can be expected. When considering a 2-D array of Chua's circuits chosen from the "excitable" region, each having the same

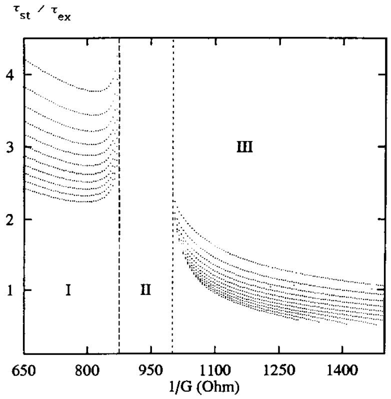

Fig. 1. One-parameter bifurcation diagram in the parametric space for a Chua's circuit. The ratio τ_{st}/τ_{ex} is shown as a function of the linear internal resistance $1/G$. Different curves correspond to different values of the capacitor C_1 (the same behavior was observed when L is changed and C_1 is kept constant). Three different regions are considered: regions I and III correspond to a "high-relaxation" limit cycle, while region II corresponds to an excitable system.

circuit parameters, spiral waves were obtained for all cases as expected, and the results will be discussed later.

A typical limit cycle from the region III is analyzed in Fig. 2. Fig. 2(a) shows the behavior of the y variable versus the x variable, and Fig. 2(b) shows the behavior of the three state variables of our system of differential equations as a function of time. In this last graph it is possible to observe that $\tau_{st} \gg \tau_{ex}$ (in this case, $\tau_{st} \approx \tau_{\mathrm{ex}}$).

We will now study two different cases. One of them will be a 2-D array of Chua's circuits where each circuit is chosen from region III of Fig. 1. In the other case, the circuits are chosen from region II, and therefore, an excitable system will be investigated.

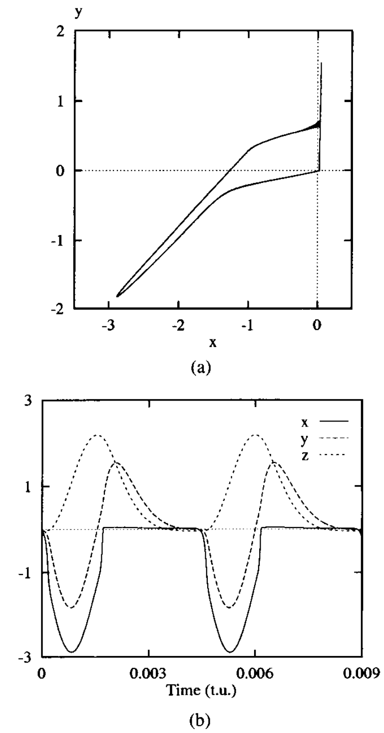

Fig. 2. (a) "High-relaxation" limit cycle (x versus y) for $1/G = 1050\ \Omega$. Notice the strongly asymmetric shape of the limit cycle; (b) behavior of the x, y, and z variables in (1) as a function of time recorded over nearly two periods of the limit cycle corresponding to the same limit cycle as in Fig. 2(a).

IV. Numerical Results

A grid of 300×300 oscillators was used in our numerical calculations. The model parameters C_1, C_2, r_0, L, $1/G_1$, $1/G_2$, and $1/G_0$ were assigned the values 10 nF, 100 nF, 0 Ω, 0.33 H, 0.07, 50.71, and -1.14 kΩ, respectively, for both the excitable and the high relaxation limit cycle cases. The only difference between the two cases is the value of the linear resistance, $1/G$, which was set to 950 and 1050 Ω, respectively.

In order to obtain spiral waves, special care must be given to the choice of the initial conditions. Generalizing the set of initial conditions described by Jahnke and Winfree for the Oregonator model [19] to our system, all variables were reset to their fundamental (slower part of the limit cycle) values in all circuits except for a front of excited circuits in the x and y variables, followed by a smoothly declining circular gradient for the z variable. This set of initial conditions is shown in Fig. 3.

Generalizing the method described in [18] to three dimensions, we determine the path of the spiral tip by identifying the points having a large cross product in the x and y gradients. In order to avoid boundary problems, we restrict our calculations of the tip to a small region around the last tip position.

Fig. 4 shows several snap shots depicting the dynamic process from the initial pattern to a fully developed spiral wave (only the x variable is shown for the high relaxation limit cycle case). Fig. 4(a) shows the initial conditions where all points inside the "wedge-like" region correspond to the excited value in x (in this case, x=-2.9, coresponding to the fast part in the limit cycle), and all other points correspond to the equilibrium value (the slow part of the limit cycle). Observe that the tip of the wave front in Fig. 4(b) begins to increase its curvature. This process is seen to continue in Fig. 4(c) and (d) where the number of turns of the spiral increases until it reaches the equilibrium. The configuration shown in Fig. 4(e) has reached the stationary state but continued to rotate steadily another 150 time units (t.u.) as shown in Fig. 4(f). Fig. 5 shows a fully developed spiral wave (the same snapshot as in Fig. 4(f)) in a 3-D view where the vertical axis corresponds to the x variable (all points on top of the spiral wave have the same x value—highly excited region).

Fig. 6 shows different spiral periods obtained when different diffusion coefficients were used. Observe that increasing the diffusion coefficient results in decreasing the rotation period and its wavelength, as expected. This result reproduces well-known results obtained in continuous media. The behavior is qualitatively the same for arrays of Chua's circuits whose parameters are in regions II or III.

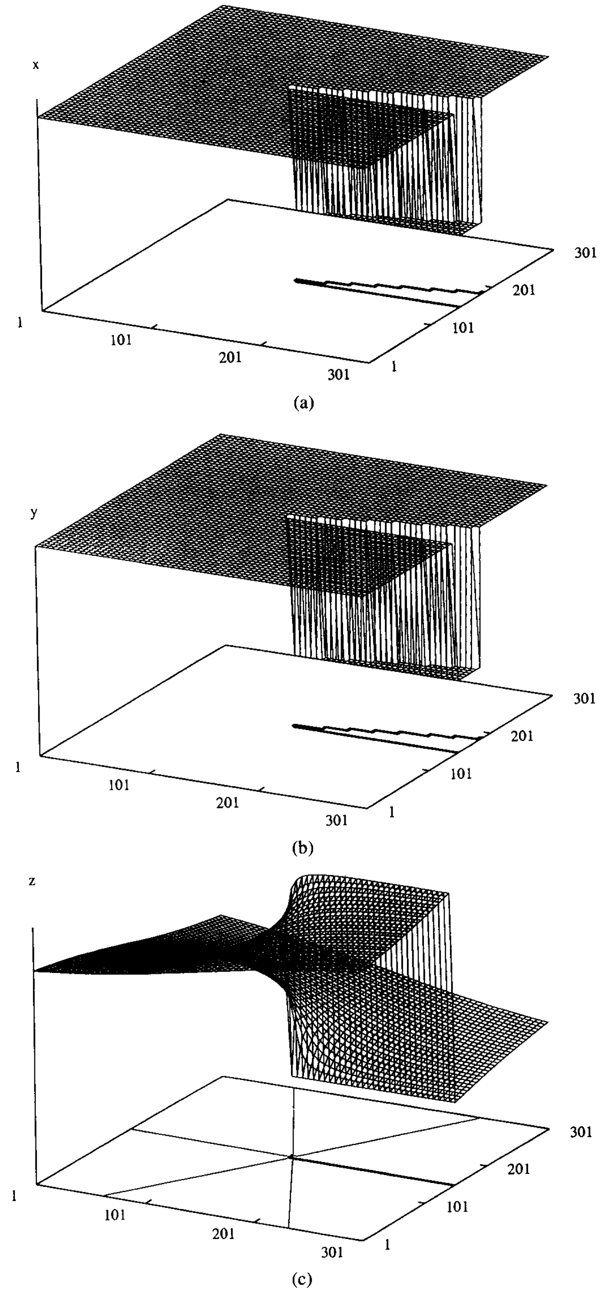

Fig. 3. Set of initial conditions: (a)–(c) correspond to the initial values for the x, y, and z variable, respectively, of each circuit in the grid. (a) and (b) show a "wedge-like" region of excited cells (-2.9 and -1.9 for x and y, respectively), whereas all other circuits are reset to their fundamental value (0.02 and 1.52); (c) initial conditions for the z variable, which is a smoothly declining circular gradient whose maximum value is 2.1 and minimum is 0.

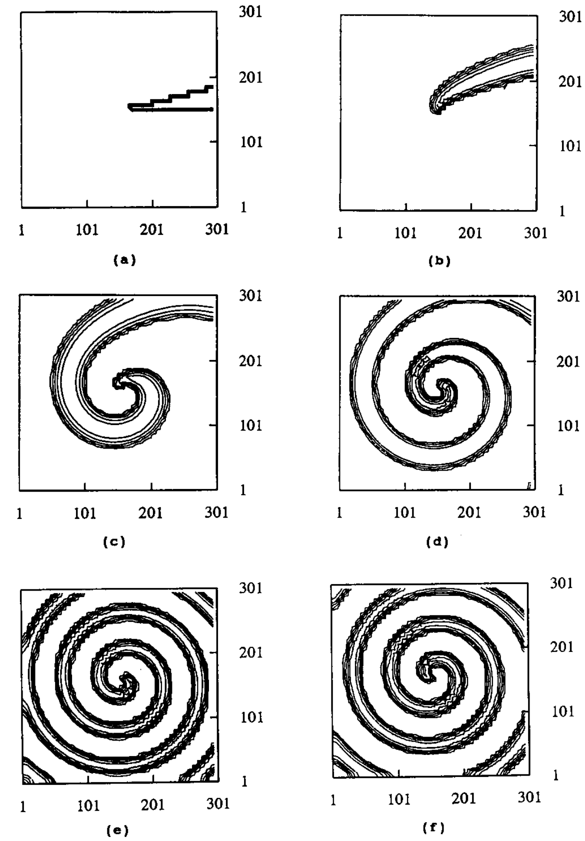

Fig. 4. Process followed to obtain a fully developed spiral wave. All figures show the contour lines connecting those points of the same value of x. (a) to (f) correspond to times 0, 10, 40, 100, 250, and 400 (time units) (each time unit (t.u.) is equal to 0.105 ms in real physical time). For $t = 0$ the initial condition is shown. After $t = 100$, the spiral structure remains while steadily rotating around the center. In this case, the tip of the spiral follows a circular pattern (core) with a typical diameter of 15 circuits ($1/G = 920\ \Omega$) and $D = 1$.

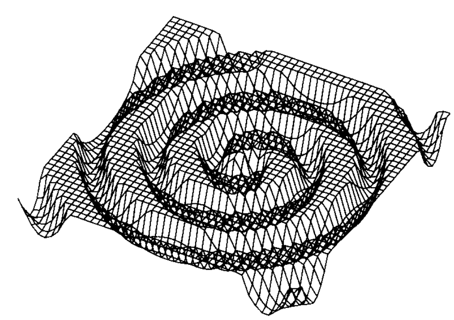

Fig. 5. 3-D view of a fully developed spiral wave. The vertical axis shows the variable x from (1). The parameters correspond to Fig. 4(f).

The tip of the spiral wave in such a medium rotates around a circular region which is called the core of this wave. Inside the core, the medium remains quiescent, i.e., the cells are not excited, despite the fact that the properties of the cells in this region do not differ from the other parts of the same medium. The core looks like an effective hole. The size of the core has been found to depend on the excitability of the cells (in our case the slopes of the nonlinear characteristic of the Chua's circuit in the array).

All results shown above correspond to ideal circuits, i.e., all parameters are equal in every Chua's circuit in the array. This is not the case in real circuits due to uncertainties in the

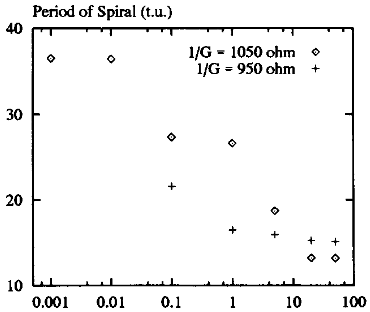

Fig. 6. Spiral period as a function of the diffusion coefficient D. The x axis is plotted in a logarithmic scale. Two different cases were considered: First, all internal resistors were set to $1/G = 1050\ \Omega$, thus describing an oscillatory system (diamonds in the figure), and second $1/G = 950\ \Omega$, thus describing an excitable medium (crosses in the figure).

nominal values of the electronic components. Our numerical simulations have shown that there is no significant influence in the spiral parameters due to small tolerances in the coupling resistor.

We have also checked numerically the influence in the spiral wave's qualitative behaviors due to tolerances in the internal parameters of each cell. No discernible effects have been observed, and for small values of the uncertainty, not even *quantitative* effects were found. When the uncertainty is large enough (above 25%), the velocity of the wave fronts increases as expected from [12].

V. Concluding Remarks

It has been shown that, when a 2-D array of resistively coupled Chua's circuit is considered, it is possible to obtain steadily rotating spiral waves if appropriate initial conditions are used.

A wide range of circuits parameters was studied and it was found that if an array is excitable for a range of continuous excitations, it can support spirals. For oscillatory systems, spiral waves can occur only if each Chua's circuit is sufficiently asymmetric so that the circuit spends most of the time τ_{st} in a resting state, followed by a short time period τ_{ex} of rapid motion per revolution of the limit cycle.

Different values were studied for both cases (excitable and oscillatory systems) and classical results for "continuous" media were reproduced in a "discrete" array of Chua's circuits.

In order to take into account real systems, small differences in the nominal values of the coupling resistors between circuits (diffusion coefficients) and the internal resistances were investigated numerically. No qualitative differences were found for component tolerances below 5%.

Some of these nuemrical results have been confirmed by actual experiments in arrays of Chua's circuits. The complete details will be published elsewhere.

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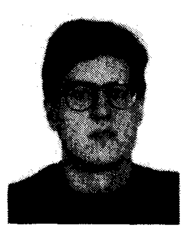

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Leon O. Chua (S'60–M'62–SM'70–F'74), for photograph and biography please see page 791 of this issue of this TRANSACTIONS.